MECHANICS AND MATERIALS I

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4

MECHANICS AND MATERIALS I

Introduction to Statics iv

Distributed Forces: Centroids and Centers of Gravity

Chap. 5

[Beer Johnston et al. 9th edition]

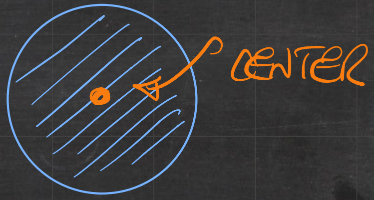
Ali Javili

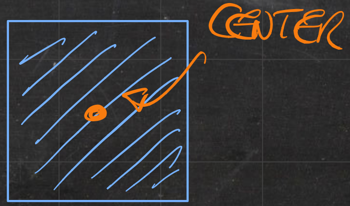
Bilkent University

DISTRIBUTED LOADS, CENTER OF GRAVITY, CENTROID

GRAVITY ⟶ WEIGHT

So far, we assumed the weight of an object as a single force exerted at the center of the object.





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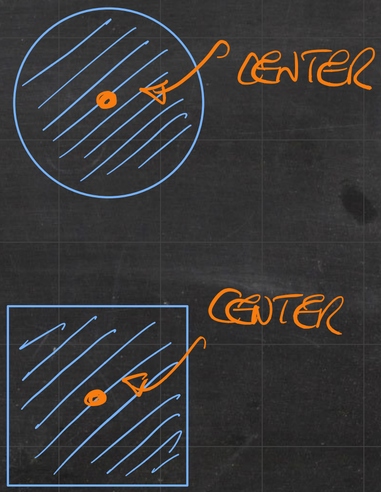
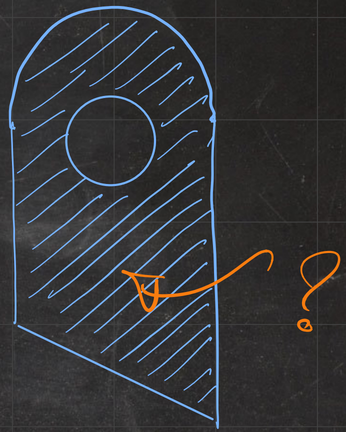
So far, we assumed the weight of an object as a single force exerted at the center of the object.





LOCATION OF CENTER OF GRAVITY

COG

WEIGHT W

IS THE

SUM OF

INCREMENTAL

WEIGHTS ΔW

$$W = \sum_{i=1}^{n} \Delta W_i$$

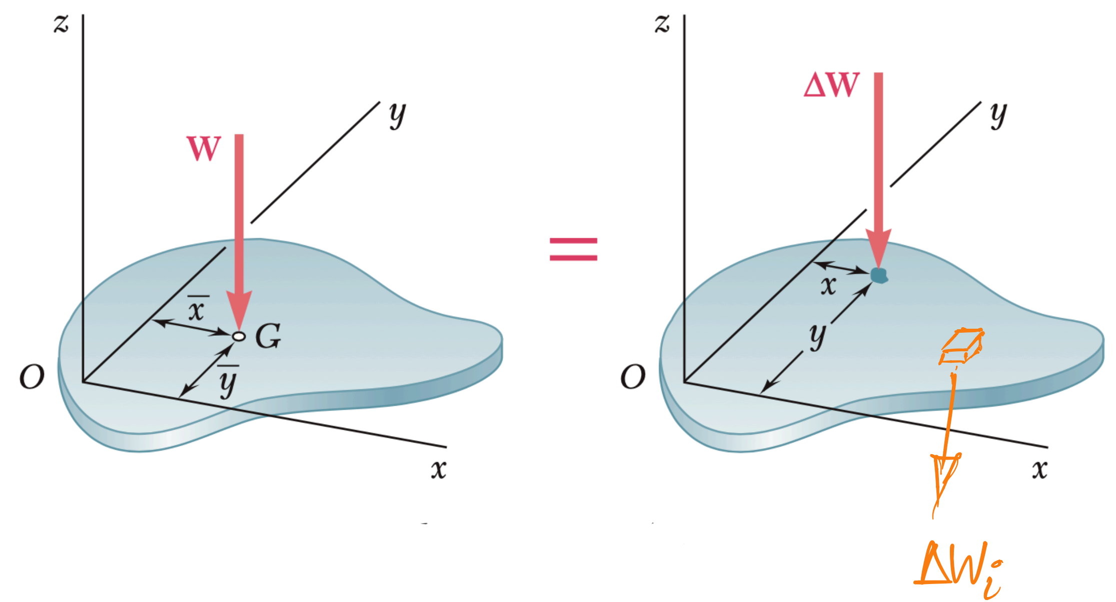


Fig. 5.1 Center of gravity of a plate.

WEIGHT W IS THE SUM OF INCREMENTAL WEIGHTS ΔW

$$W = \sum_{i=1}^{n} \Delta W_i$$

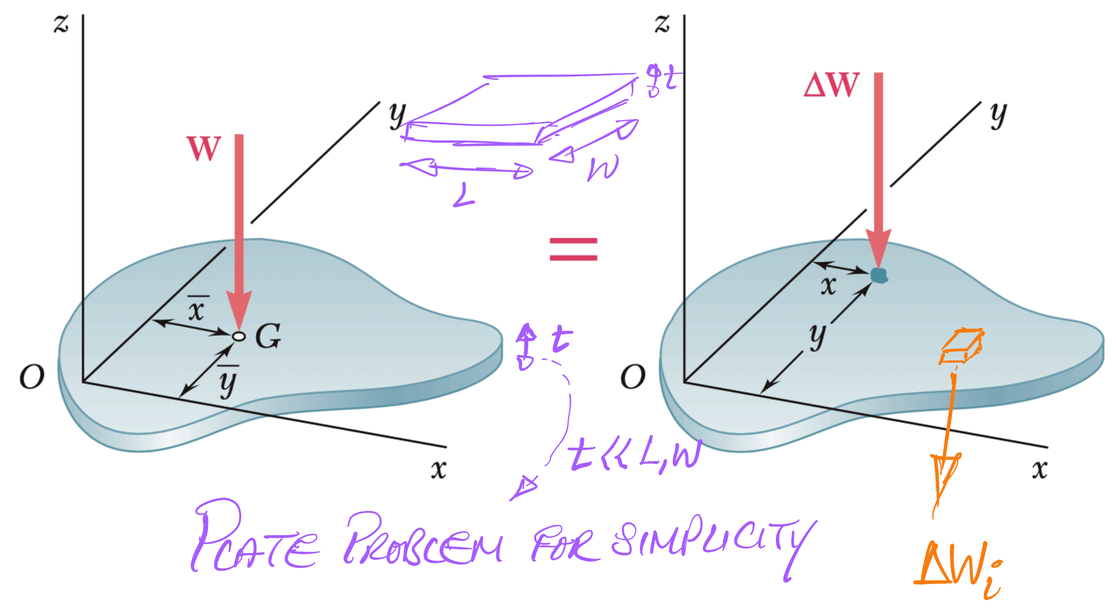


Fig. 5.1 Center of gravity of a plate.

$$\mathbb{F}_R = \sum \mathbb{F}$$

$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

2D

$$\begin{cases} \sum F_z \\ \sum M_{oy} \\ \sum M_{ox} \end{cases}$$

3 EQNS.

$$\mathbb{F}_R = \sum \mathbb{F}$$

$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

2D

$$\begin{cases} \sum F_z \\ \sum M_{oy} \\ \sum M_{ox} \end{cases}$$

3 EQNS.

$$\sum F_z \longrightarrow W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n = \sum_{i=1}^{n} \Delta W_i$$

$$\mathbb{F}_R = \sum \mathbb{F}$$

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2D

$$\begin{cases} \sum F_z \\ \sum M_{oy} \\ \sum M_{ox} \end{cases}$$

3 EQNS.

$$\sum F_z \to W = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n = \sum_{i=1}^{n} \Delta W_i$$

x-coordinate of center of gravity

$$\sum M_{oy} \to \bar{x}\, W = x_1 \Delta W_1 + x_2 \Delta W_2 + \dots + x_n \Delta W_n = \sum_{i=1}^{n} x_i \Delta W_i$$

$$\mathbb{F}_R = \sum \mathbb{F}$$

$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

2D

$$\begin{cases} \sum F_z \\ \sum M_{Oy} \\ \sum M_{Ox} \end{cases}$$

3 EQNs.

$$\sum F_z \rightarrow W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_n = \sum_{i=1}^{n} \Delta W_i$$

x-coordinate of center of gravity

$$\sum M_{Oy} \rightarrow \bar{x}\, W = x_1 \Delta W_1 + x_2 \Delta W_2 + \cdots + x_n \Delta W_n = \sum_{i=1}^{n} x_i \Delta W_i$$

y-coordinate of center of gravity

$$\sum M_{Ox} \rightarrow \bar{y}\, W = y_1 \Delta W_1 + y_2 \Delta W_2 + \cdots + y_n \Delta W_n = \sum_{i=1}^{n} y_i \Delta W_i$$

$$\mathbb{F}_R = \sum \mathbb{F}$$
$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

2D

$$\begin{cases} \sum F_z \\ \sum M_{oy} \\ \sum M_{ox} \end{cases}$$

3 EQNs.

$$\sum F_z \rightarrow W = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n = \sum_{i=1}^{n} \Delta W_i$$

x-coordinate of center of gravity

$$\sum M_{oy} \rightarrow \bar{x} W = x_1 \Delta W_1 + x_2 \Delta W_2 + \dots + x_n \Delta W_n = \sum_{i=1}^{n} x_i \Delta W_i$$

y-coordinate of center of gravity

$$\sum M_{ox} \rightarrow \bar{y} W = y_1 \Delta W_1 + y_2 \Delta W_2 + \dots + y_n \Delta W_n = \sum_{i=1}^{n} y_i \Delta W_i$$

$$W = \sum \Delta W_i \; , \; \bar{x} W = \sum x_i \Delta W_i \; , \; \bar{y} W = \sum y_i \Delta W_i$$

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

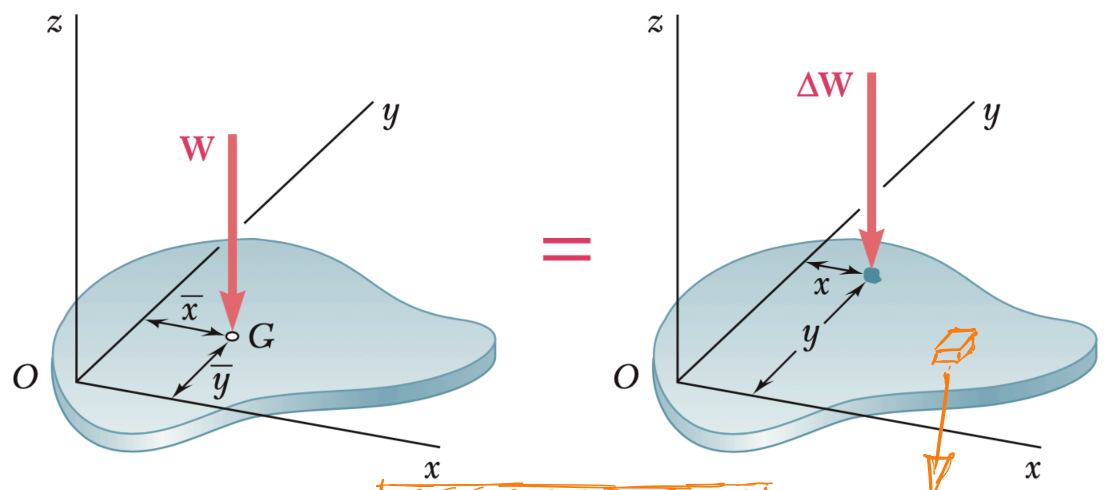
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_O$$

2D

$$\begin{cases} \Sigma F_z \\ \Sigma M_{Oy} \\ \Sigma M_{Ox} \end{cases}$$

$(\bar{x}, \bar{y})$

↳ Coordinates of Center of gravity



ΔW_i

$$\boxed{\begin{aligned} \Sigma M_y: &\quad \bar{x}\,W = \Sigma x\,\Delta W \\ \Sigma M_x: &\quad \bar{y}\,W = \Sigma y\,\Delta W \end{aligned}}$$

Fig. 5.1 Center of gravity of a plate.

$n \to \infty$

$\Sigma \to \int$

$$\Sigma F_z \longrightarrow W = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n = \sum_{i=1}^{n} \Delta W_i$$

x-coordinate of center of gravity

$$\Sigma M_{oy} \longrightarrow \bar{x} W = x_1 \Delta W_1 + x_2 \Delta W_2 + \dots + x_n \Delta W_n = \sum_{i=1}^{n} x_i \Delta W_i$$

y-coordinate of center of gravity

$$\Sigma M_{ox} \longrightarrow \bar{y} W = y_1 \Delta W_1 + y_2 \Delta W_2 + \dots + y_n \Delta W_n = \sum_{i=1}^{n} y_i \Delta W_i$$

$$W = \Sigma \Delta W_i \quad , \quad \bar{x} W = \Sigma x_i \Delta W_i \quad , \quad \bar{y} W = \Sigma y_i \Delta W_i$$

$n \to \infty$

$\rightarrow \sum \to \int$

$$W = \int dW$$

$$\bar{x} = \frac{\int x \, dW}{W}$$

$$\bar{y} = \frac{\int y \, dW}{W}$$

$$\sum F_z \longrightarrow W = \Delta W_1 + \Delta W_2 + \dots + \Delta W_n = \sum_{i=1}^{n} \Delta W_i$$

x-coordinate of center of gravity

$$\sum M_{oy} \longrightarrow \bar{x} W = x_1 \Delta W_1 + x_2 \Delta W_2 + \dots + x_n \Delta W_n = \sum_{i=1}^{n} x_i \Delta W_i$$

y-coordinate of center of gravity

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$$W = \sum \Delta W_i \, , \quad \bar{x} W = \sum x_i \Delta W_i \, , \quad \bar{y} W = \sum y_i \Delta W_i$$

$n \to \infty$

$\to \Sigma \to \int$

$$W = \int dW$$

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OF PARTICULAR INTEREST $\Rightarrow$ UNIFORM PLATE

$n \to \infty$

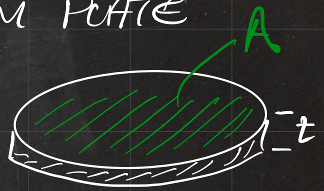
$\to \Sigma \to \int$

$$W = \int dW$$

$$\bar{x} = \frac{\int x \, dW}{W}$$

$$\bar{y} = \frac{\int y \, dW}{W}$$

OF PARTICULAR INTEREST $\Rightarrow$ UNIFORM PLATE



density

$$W = \int \rho \, t \, dA \Rightarrow W = \rho \, t \, A$$

$n \to \infty$

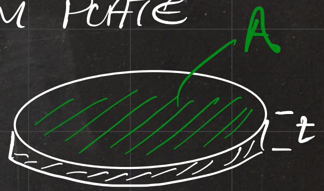
$\Sigma \to \int$

$$W = \int dW$$

$$\bar{x} = \frac{\int x \, dW}{W}$$

$$\bar{y} = \frac{\int y \, dW}{W}$$

OF PARTICULAR INTEREST $\Rightarrow$ UNIFORM PLATE



density

$$W = \int \rho t \, dA \Rightarrow W = \rho t A$$

$$\bar{x} = \frac{\int x \rho t \, dA}{\rho t A} \Rightarrow \bar{x} = \frac{\int x \, dA}{A}$$

$n \to \infty$

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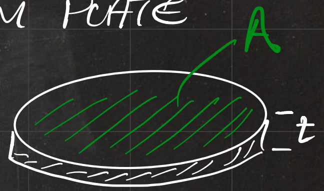
OF PARTICULAR INTEREST ⇒ UNIFORM PLATE



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$$\bar{y} = \frac{\int y \rho t \, dA}{\rho t A} \Rightarrow \bar{y} = \frac{\int y \, dA}{A}$$

$n \to \infty$

$\to \Sigma \to \int$

$$W = \int dW$$

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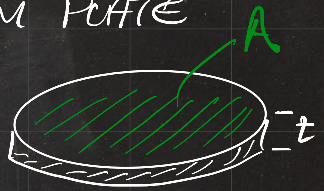
OF PARTICULAR INTEREST ⇒ UNIFORM PLATE



density

$$W = \int \rho\, t\, dA \Rightarrow W = \rho\, t\, A$$

$$\bar{x} = \frac{\int x \rho\, t\, dA}{\rho\, t\, A} \Rightarrow \bar{x} = \frac{\underbrace{\int x\, dA}_{Q_y}}{A}$$

FIRST MOMENT OF AREA W.R.T. Y-AXIS

$$\bar{y} = \frac{\int y \rho\, t\, dA}{\rho\, t\, A} \Rightarrow \bar{y} = \frac{\int y\, dA}{A}$$

$n \to \infty$

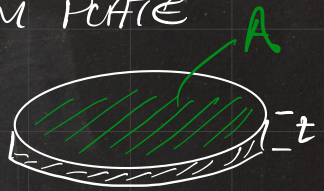
$\Sigma \to \int$

$$W = \int dW$$

$$\bar{x} = \frac{\int x \, dW}{W}$$

$$\bar{y} = \frac{\int y \, dW}{W}$$

OF PARTICULAR INTEREST ⇒ UNIFORM PLATE



density

$$W = \int \rho \, t \, dA \Rightarrow W = \rho \, t \, A$$

$$\bar{x} = \frac{\int x \rho t \, dA}{\rho t A} \Rightarrow \bar{x} = \frac{\overbrace{\int x \, dA}^{Q_y}}{A}$$

FIRST MOMENT OF AREA W.R.T. Y-AXIS

Q_y

$$\bar{y} = \frac{\int y \rho t \, dA}{\rho t A} \Rightarrow \bar{y} = \frac{\overbrace{\int y \, dA}^{Q_x}}{A}$$

FIRST MOMENT OF AREA W.R.T. X-AXIS

Q_x

$n \to \infty$

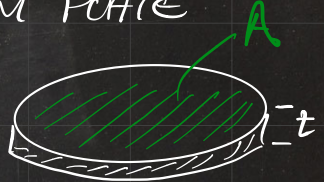
$\Sigma \to \int$

$$W = \int dW$$

$$\bar{x} = \frac{\int x \, dW}{W}$$

$$\bar{y} = \frac{\int y \, dW}{W}$$

OF PARTICULAR INTEREST ⇒ UNIFORM PLATE



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FIRST MOMENT OF AREA W.R.T. Y-AXIS Q_y

GEOMETRICAL QUANTITIES

$$\bar{y} = \frac{\int y \rho t \, dA}{\rho t A} \Rightarrow \bar{y} = \frac{\int y \, dA}{A}$$

FIRST MOMENT OF AREA W.R.T. X-AXIS Q_x

$n \to \infty$

$\Sigma \to \int$

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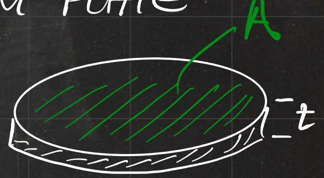
OF PARTICULAR INTEREST $\Rightarrow$ UNIFORM PLATE



density

$$W = \int \rho t \, dA \Rightarrow W = \rho t A$$

$$\bar{x} = \frac{\int x \rho t \, dA}{\rho t A} \Rightarrow \bar{x} = \frac{\int x \, dA}{A}$$

FIRST MOMENT OF AREA W.R.T. Y-AXIS Q_y

GEOMETRICAL QUANTITIES

$$\bar{y} = \frac{\int y \rho t \, dA}{\rho t A} \Rightarrow \bar{y} = \frac{\int y \, dA}{A}$$

FIRST MOMENT OF AREA W.R.T. X-AXIS Q_x

$\Rightarrow (\bar{x}, \bar{y})$: COORDINATES OF CENTROID

IN SUMMARY, FOR UNIFORM DENSITY...

$$A = \int dA$$

$$\bar{x} = \frac{\int x \, dA}{A}$$

$$\bar{y} = \frac{\int y \, dA}{A}$$

IN SUMMARY, FOR UNIFORM DENSITY...

2D 3D

$$A = \int dA$$

$$V = \int dV$$

$$\bar{x} = \frac{\int x \, dA}{A}$$

$$\bar{y} = \frac{\int y \, dA}{A}$$

$$\bar{x} = \int x \, dV / V$$

$$\bar{y} = \int y \, dV / V$$

$$\bar{z} = \int z \, dV / V$$

IN SUMMARY, FOR UNIFORM DENSITY ...

2D 3D

$$A = \int dA \qquad V = \int dV$$

$$\bar{x} = \frac{\int x \, dA}{A} \qquad \bar{x} = \int x \, dV / V$$

$$\bar{y} = \frac{\int y \, dA}{A} \qquad \bar{y} = \int y \, dV / V$$

$$\bar{z} = \int z \, dV / V$$

FOR STANDARD GEOMETRIES

$\Downarrow$

TABLES (HANDBOOKS)

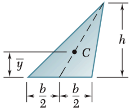
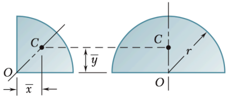
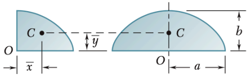
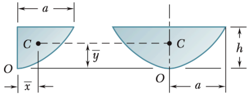
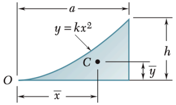
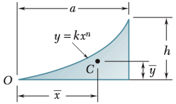
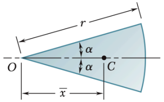
Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3\alpha}$	0	αr^2

Fig. 5.8A Centroids of common shapes of areas.

Fig. 5.8A
P. 225

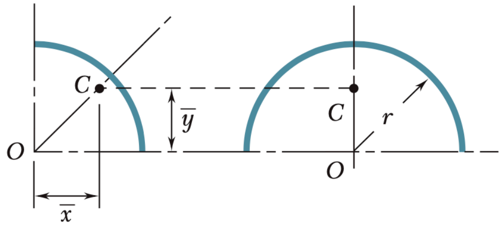
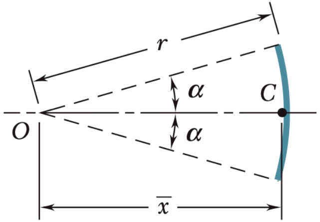
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

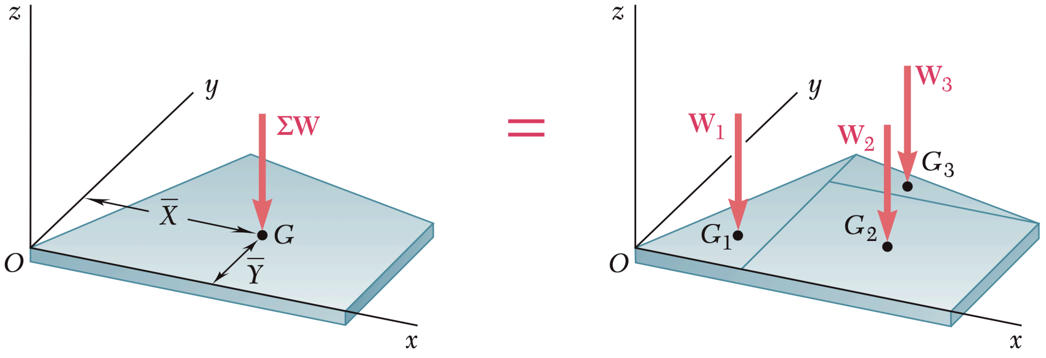
Fig. 5.8B Centroids of common shapes of lines.

Fig. 5.8B P.226

WHAT TO DO WITH COMPLEX SHAPES?

↳ SPLIT THEM INTO MULTIPLE STANDARD SHAPES

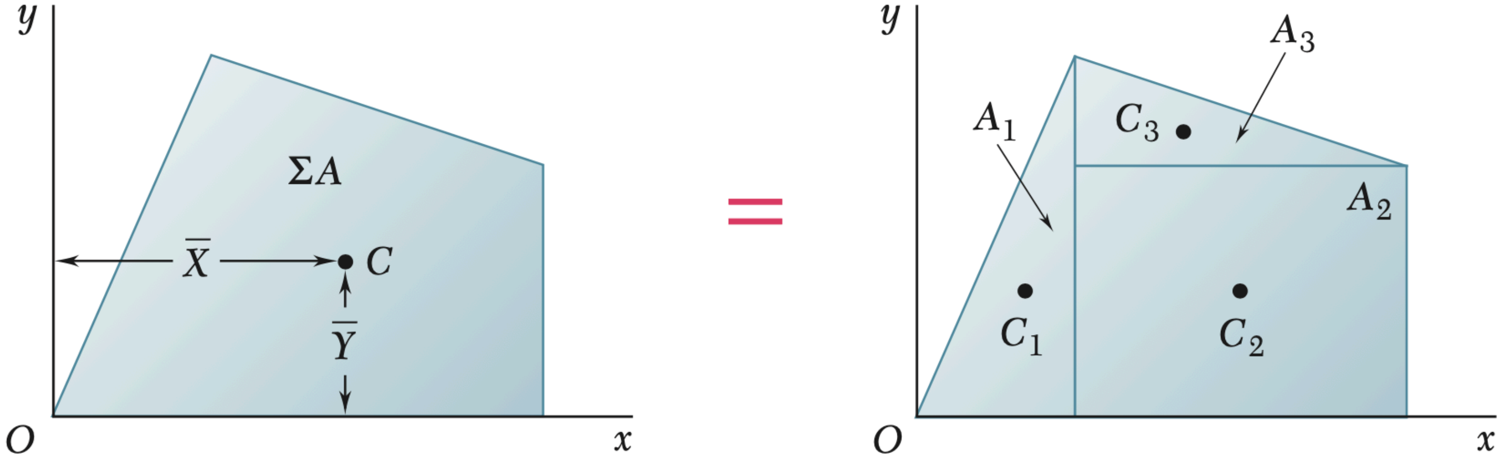
↳ USE TABLES



$$\Sigma M_y: \quad \overline{X}\,\Sigma W = \Sigma \overline{x}\,W$$

$$\Sigma M_x: \quad \overline{Y}\,\Sigma W = \Sigma \overline{y}\,W$$

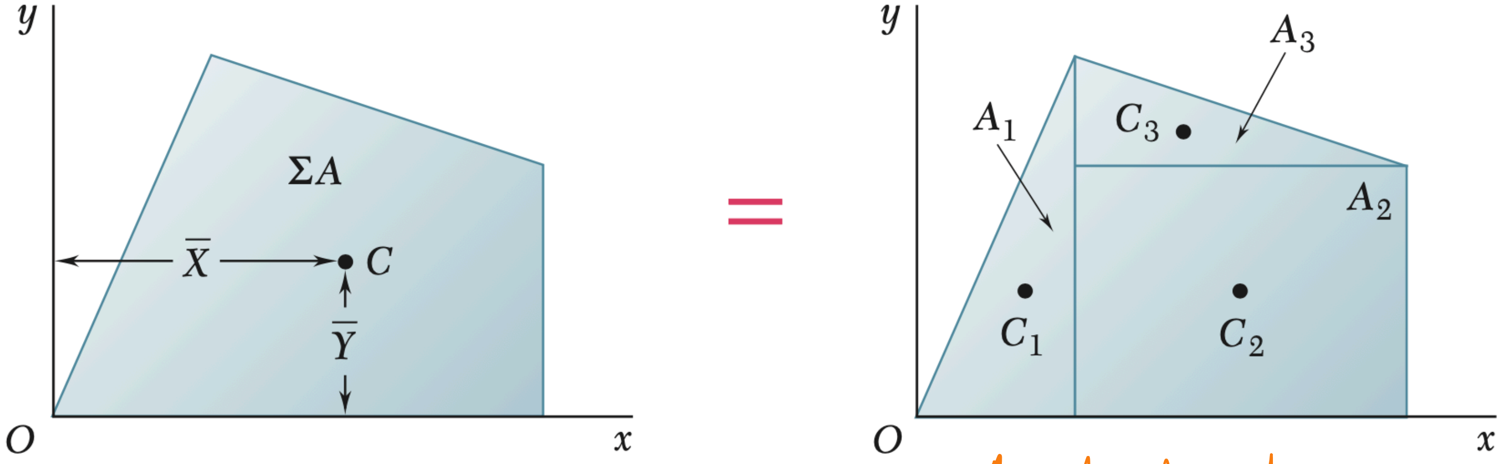
Fig. 5.9 Center of gravity of a composite plate.



$$Q_y = \bar{X}\,\Sigma A = \Sigma\,\bar{x}\,A$$

$$Q_x = \bar{Y}\,\Sigma A = \Sigma\,\bar{y}\,A$$

Fig. 5.10 Centroid of a composite area.



$$A = A_1 + A_2 + A_3$$

$$Q_y = \overline{X}\,\Sigma A = \Sigma\,\overline{x}\,A \qquad \bar{x} = [x_1 A_1 + x_2 A_2 + x_3 A_3]/A$$

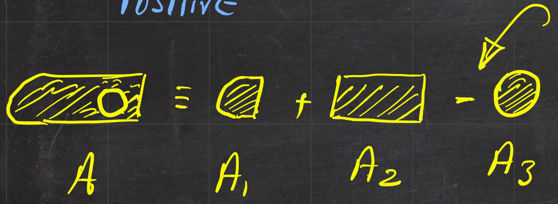
$$Q_x = \overline{Y}\,\Sigma A = \Sigma\,\overline{y}\,A \qquad \bar{y} = [y_1 A_1 + y_2 A_2 + y_3 A_3]/A$$

Fig. 5.10 Centroid of a composite area.

CAREFUL WITH SIGNS !

$\bar{x}_1, \bar{x}_2, \bar{x}_3$ assumed to be distances!

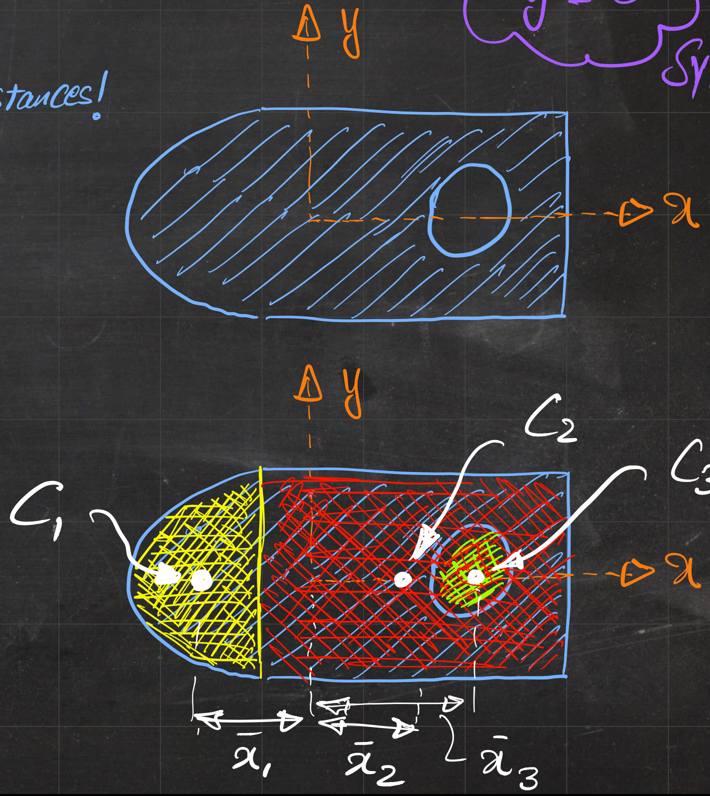
$\underbrace{\bar{x}_1, \bar{x}_2, \bar{x}_3}_{\text{POSITIVE}}$



$$A = A_1 + A_2 - A_3$$

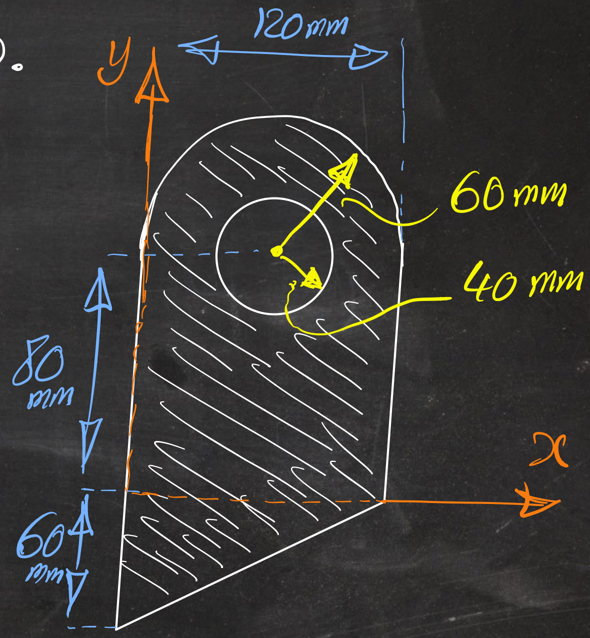
$$\bar{x} = \frac{-\bar{x}_1 A_1 + \bar{x}_2 A_2 - \bar{x}_3 A_3}{A}$$

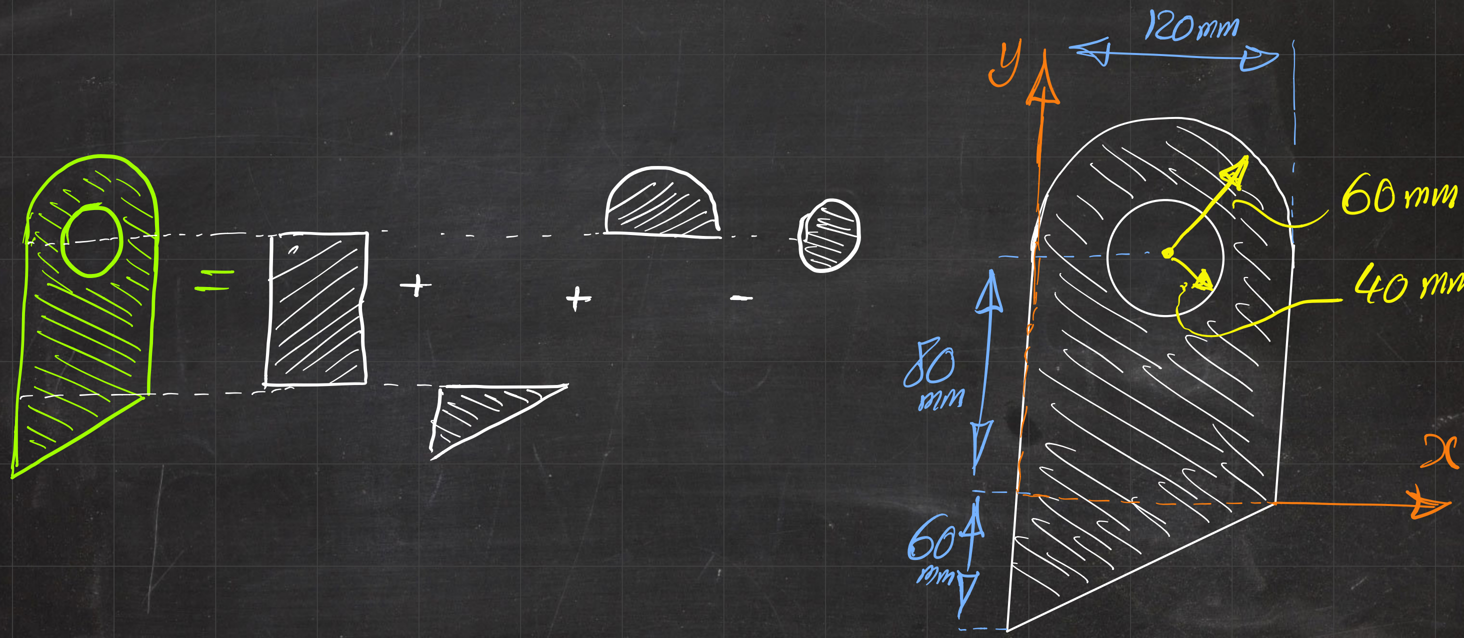
$\bar{y} = 0$ DUE TO SYMMETRY



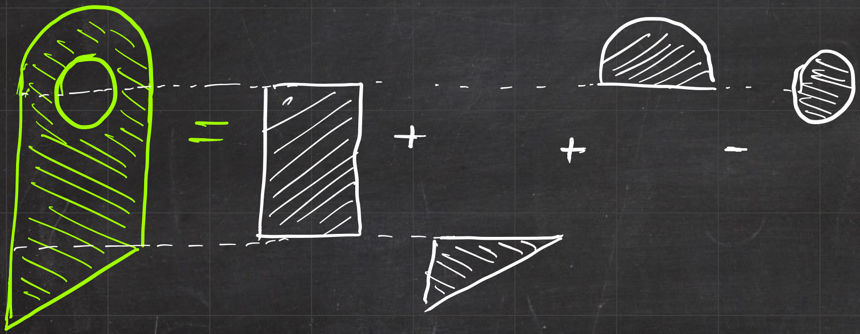
Exercise 1 . [similar to … P. 228 … 5.1]

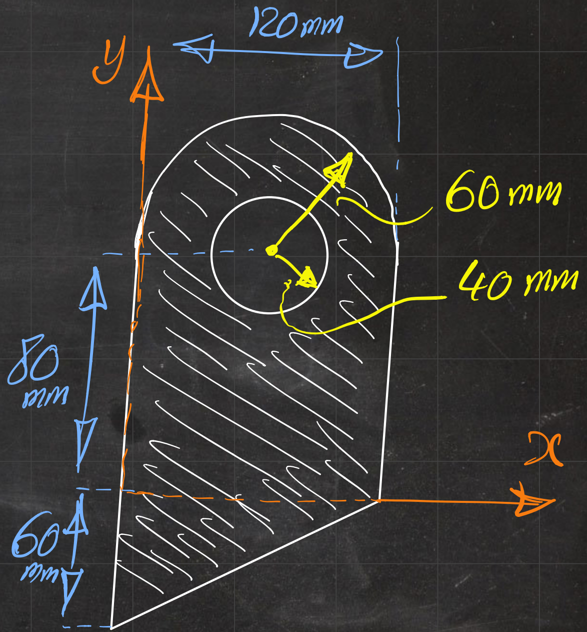
DETERMINE THE LOCATION OF CENTROID.





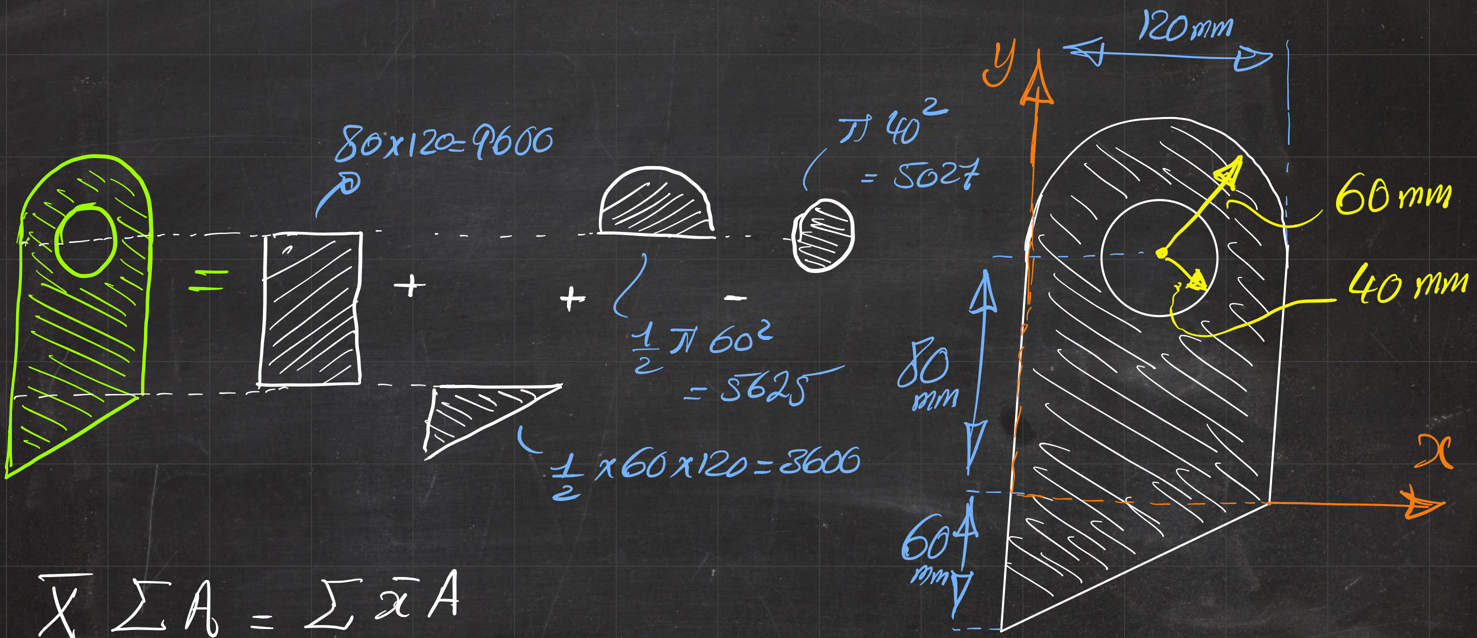
=
+
+
-
y
120 mm
60 mm
40 mm
80 mm
60 mm
x





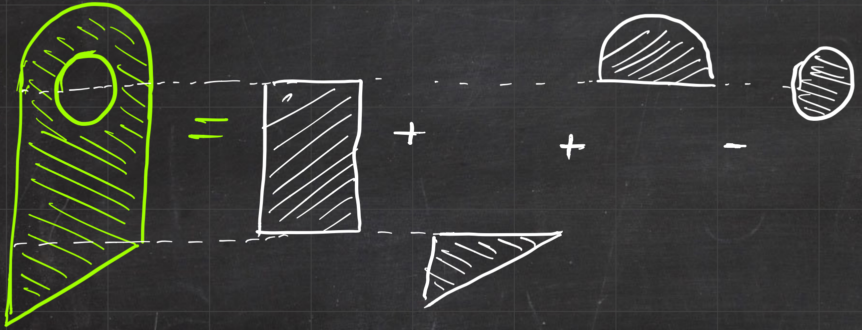
$$\bar{X}\sum A = \sum \bar{x} A$$

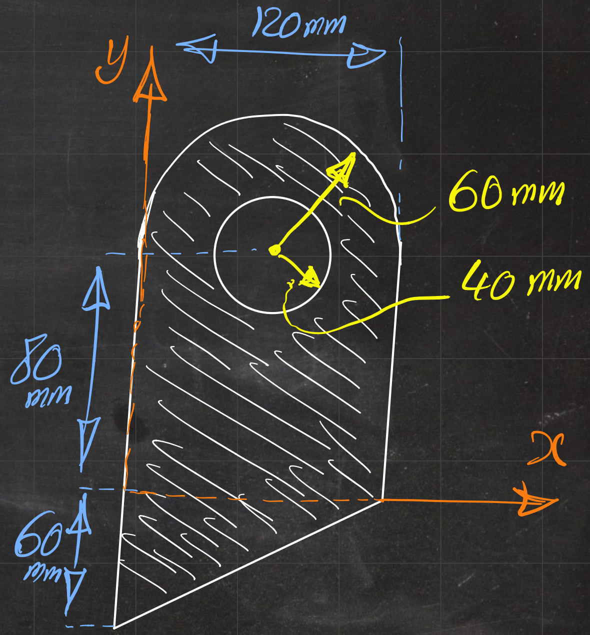
$$\bar{Y}\sum A = \sum \bar{y} A$$



$$\bar{X}\sum A = \sum \bar{x} A$$

$$\bar{Y}\sum A = \sum \bar{y} A$$



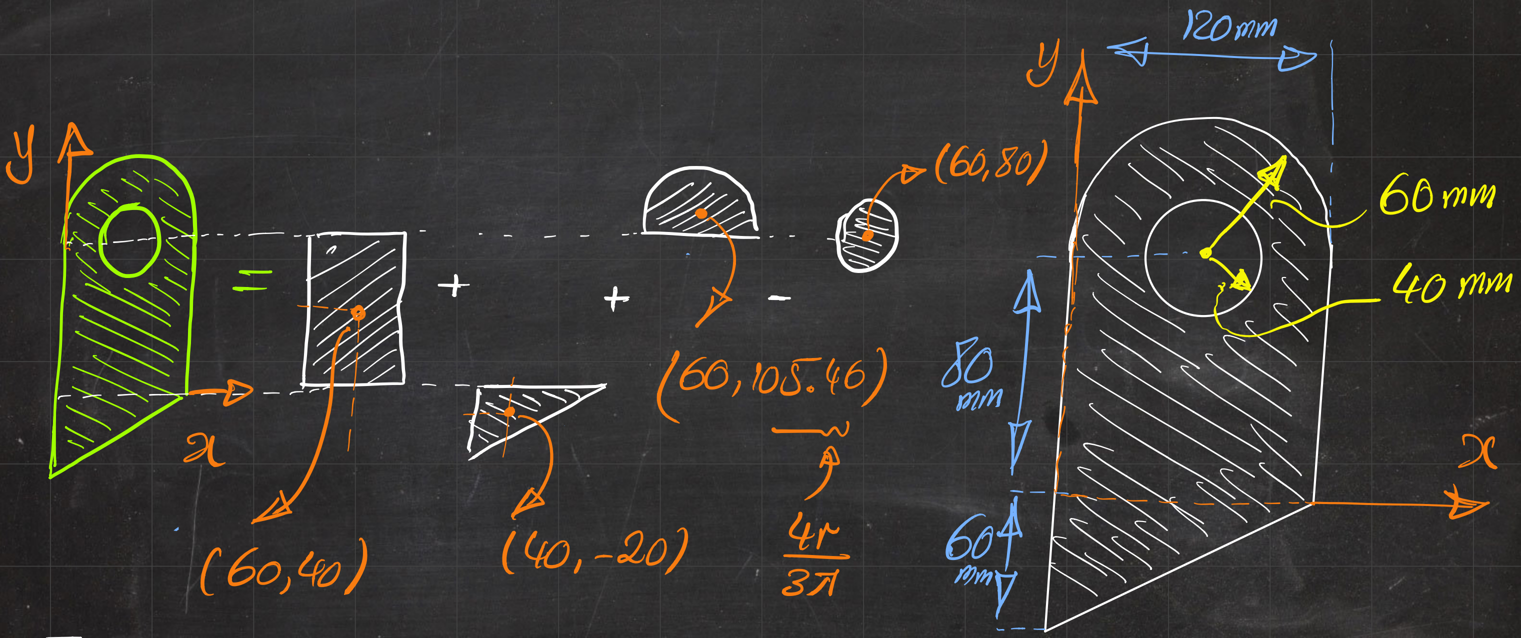


$$\bar{X}\sum A = \sum \bar{x} A$$

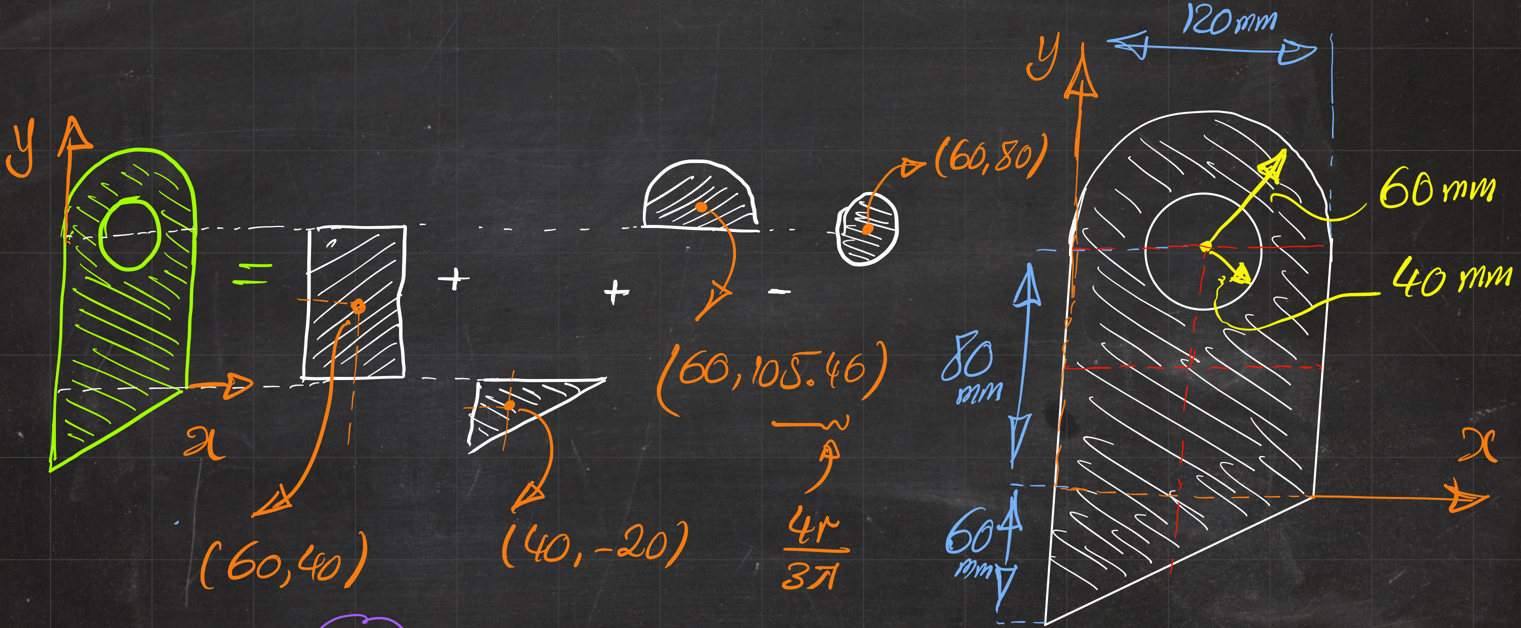
$$\bar{Y}\sum A = \sum \bar{y} A$$

$$\sum A = 9600 + 3600 + 5655 - 5027$$

$$\Rightarrow \sum A = 13828 \ mm^2$$



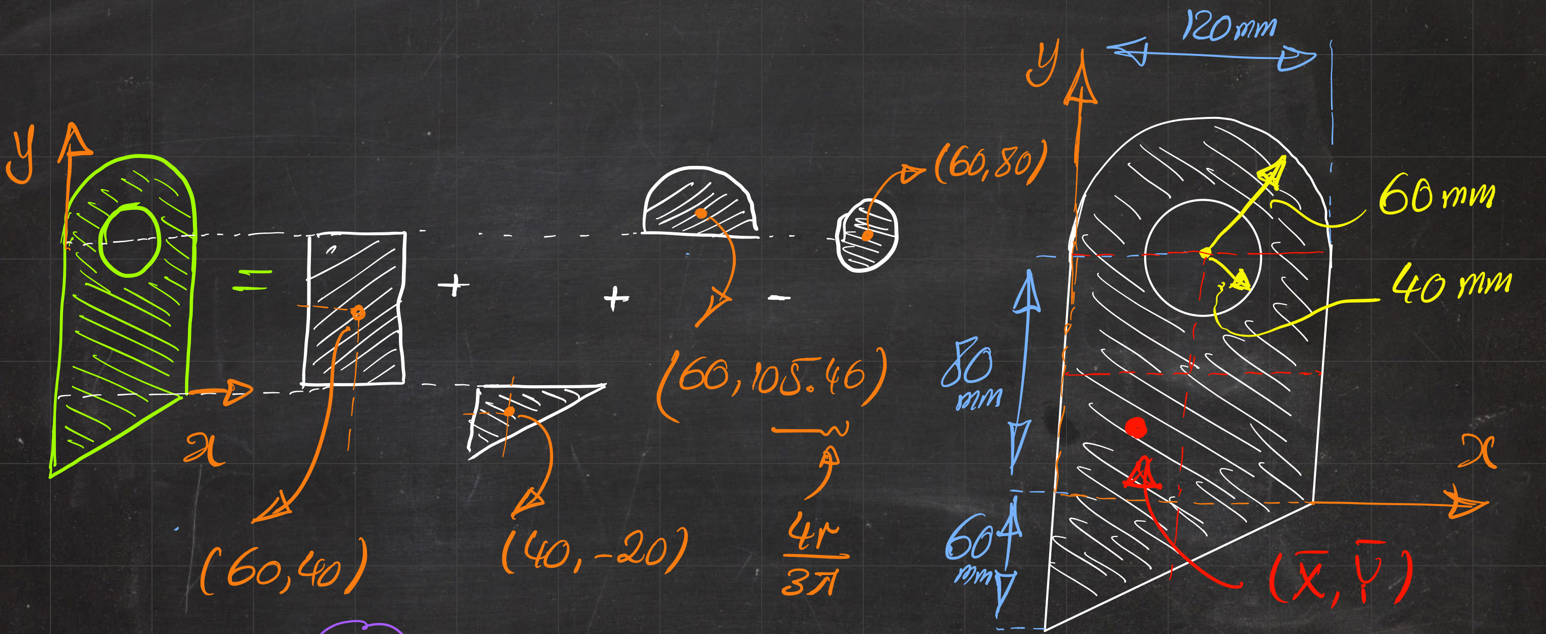
$$\bar{X} = \sum \bar{x} A / \sum A$$
$$\bar{Y} = \sum \bar{y} A / \sum A$$



$$\bar{X} = \sum \bar{x}A / \sum A$$

$$\bar{Y} = \sum \bar{y}A / \sum A$$

$$\sum A = 13828$$



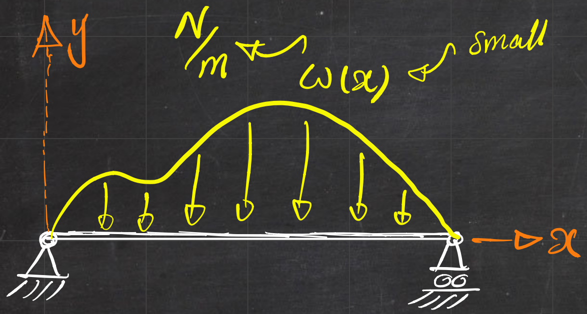
$$\bar{X} = \sum \bar{x}A / \sum A$$

$$\bar{Y} = \sum \bar{y}A / \sum A$$

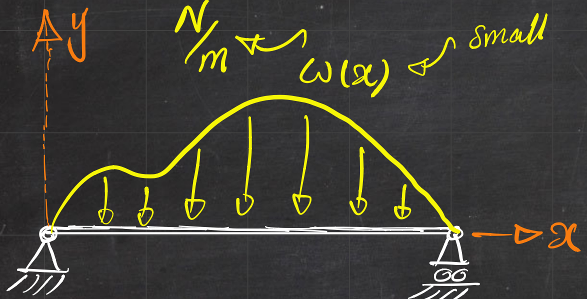
$\sum A = 13828$

$\Rightarrow \bar{X} = 54.8\ mm,\ \bar{Y} = 36.6\ mm$

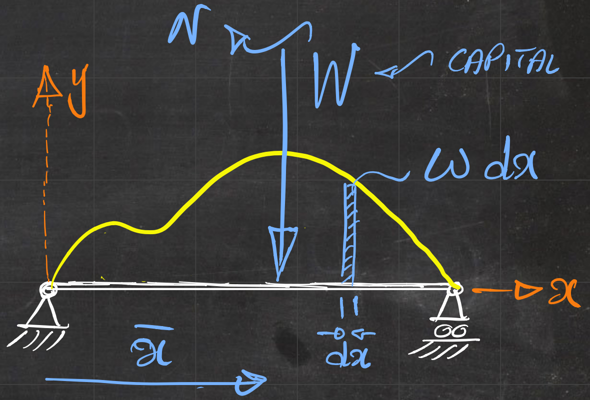
DISTRIBUTED LOADS ON BEAMS $\rightsquigarrow$ e.g. think of snow load!



DISTRIBUTED LOADS ON BEAMS → e.g. think of snow load!

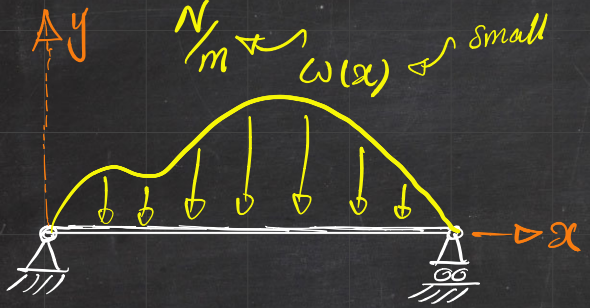


$\equiv$

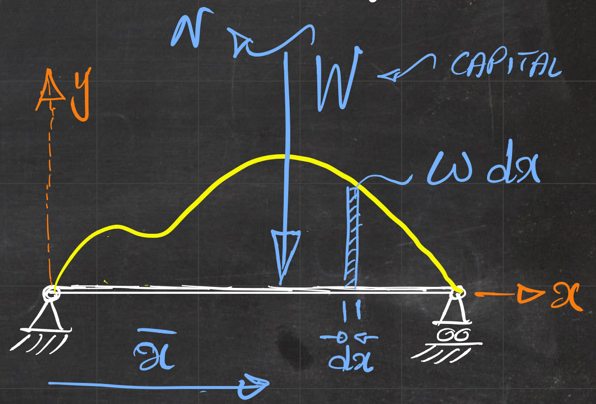


$$W = \int_0^L w \, dx \qquad \bar{x} = \frac{\int_0^L x w \, dx}{W}$$

DISTRIBUTED LOADS ON BEAMS ⟶ e.g. think of snow load!

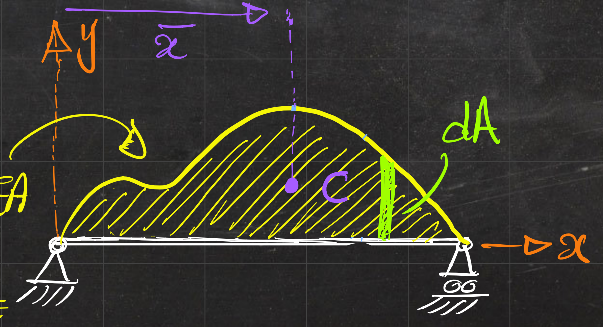


≡



$$W = \int_0^L \omega \, dx \qquad \bar{x} = \frac{\int_0^L \overbrace{x \omega dx}^{dA}}{\underbrace{W}_{A}}$$

↳ AREA UNDER THE CURVE

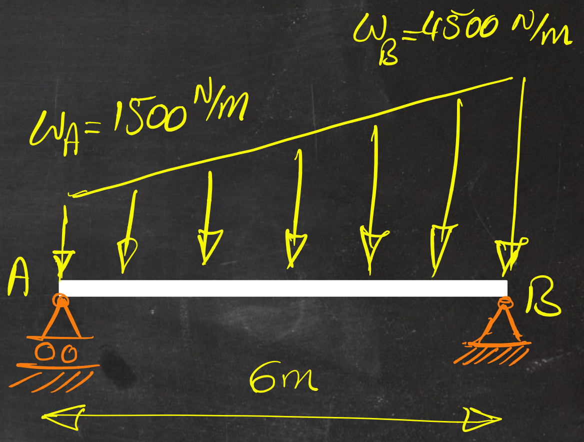


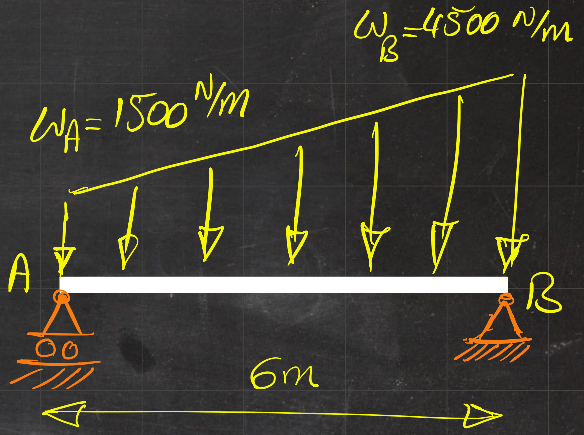
$$\bar{x} = \frac{\int_0^L x \, dA}{A}$$

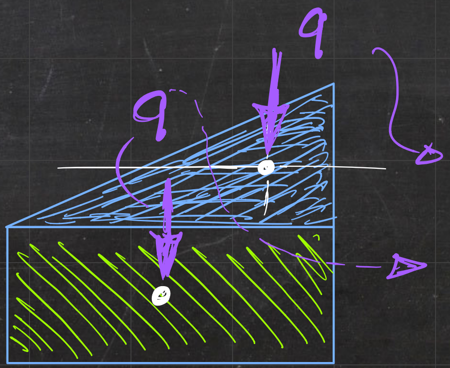
Exercise 2 . [similar to … P. 250 … 5.9]

DETERMINE THE EQUIVALENT CONCENTRATED LOAD.

DETERMINE THE REACTIONS AT THE SUPPORTS.

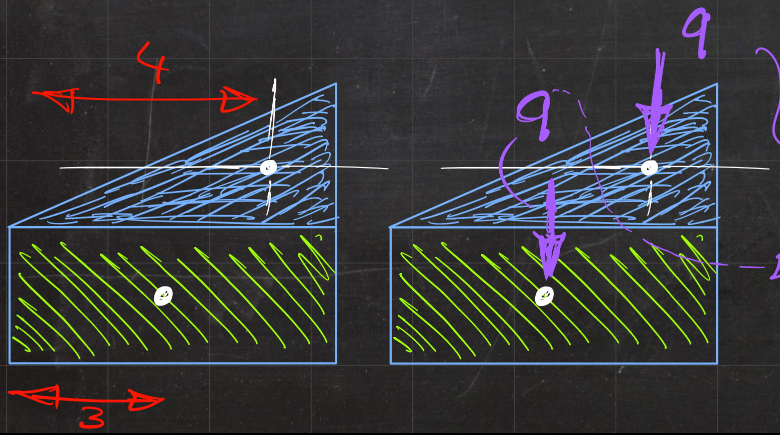






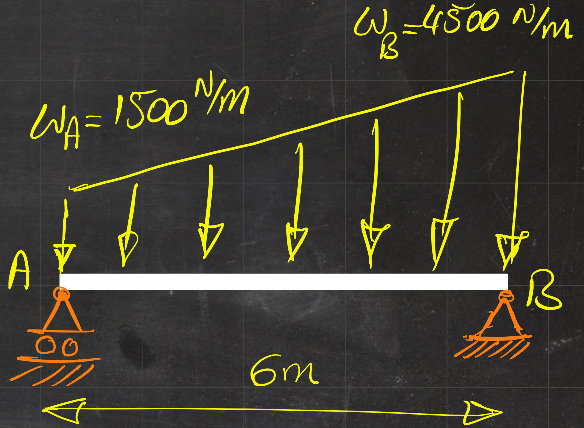
$$3000 \times 6 \times \frac{1}{2} = 9$$

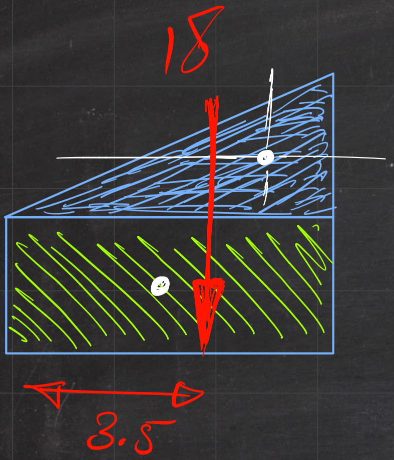
$$1500 \times 6 = 9$$

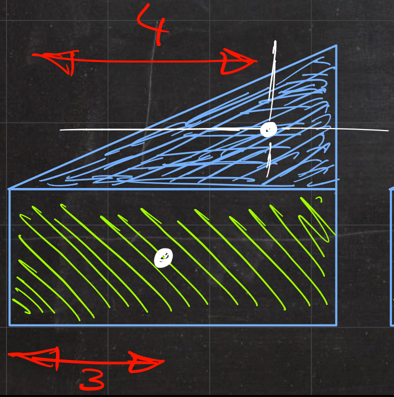


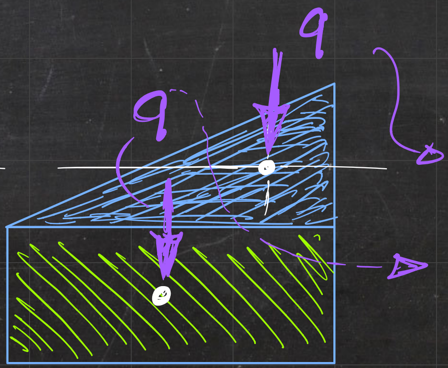
$3000 \times 6 \times \frac{1}{2} = 9$

$1500 \times 6 = 9$



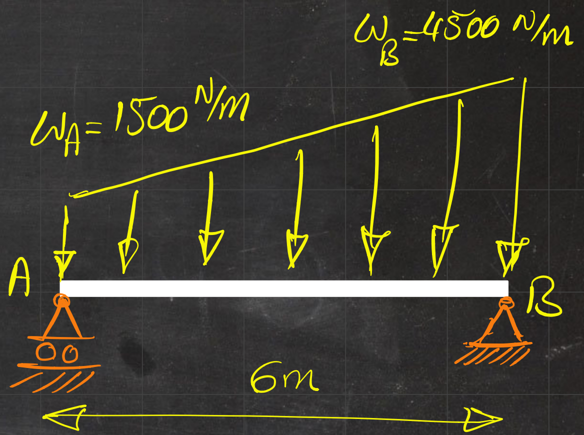


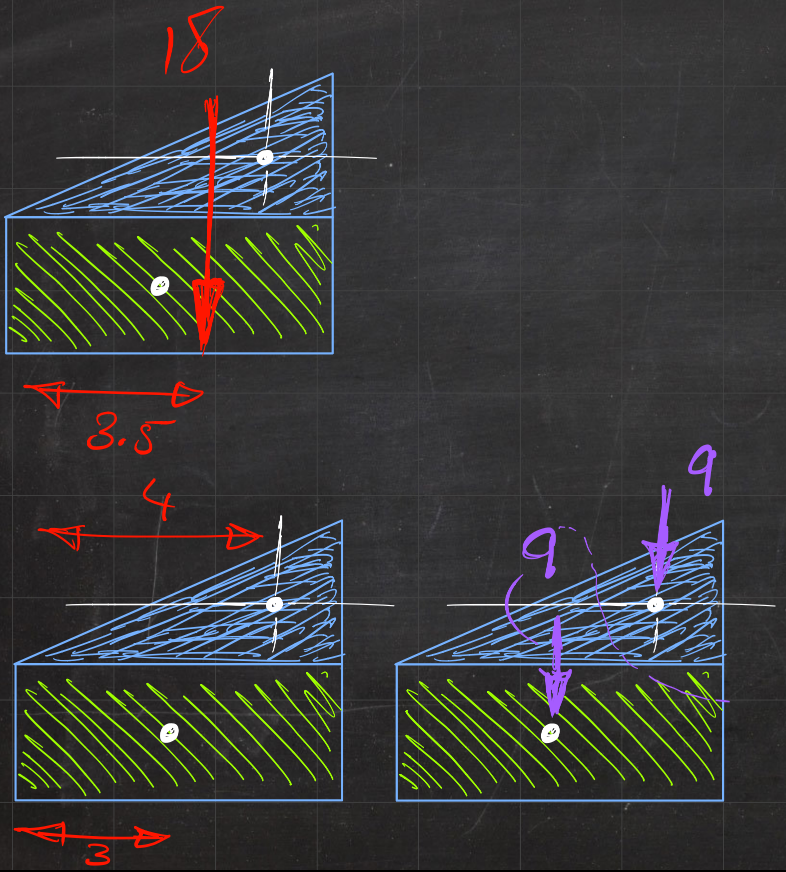


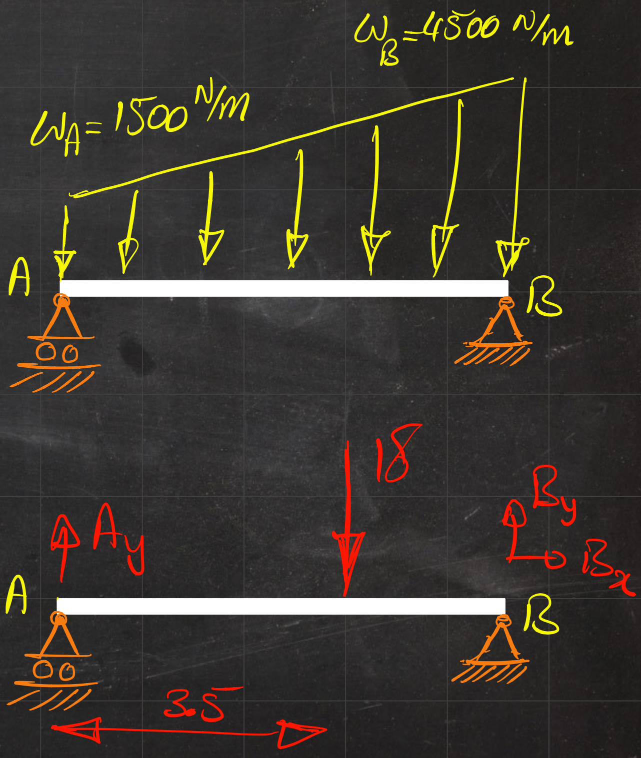


$3000 \times 6 \times \frac{1}{2} = 9$

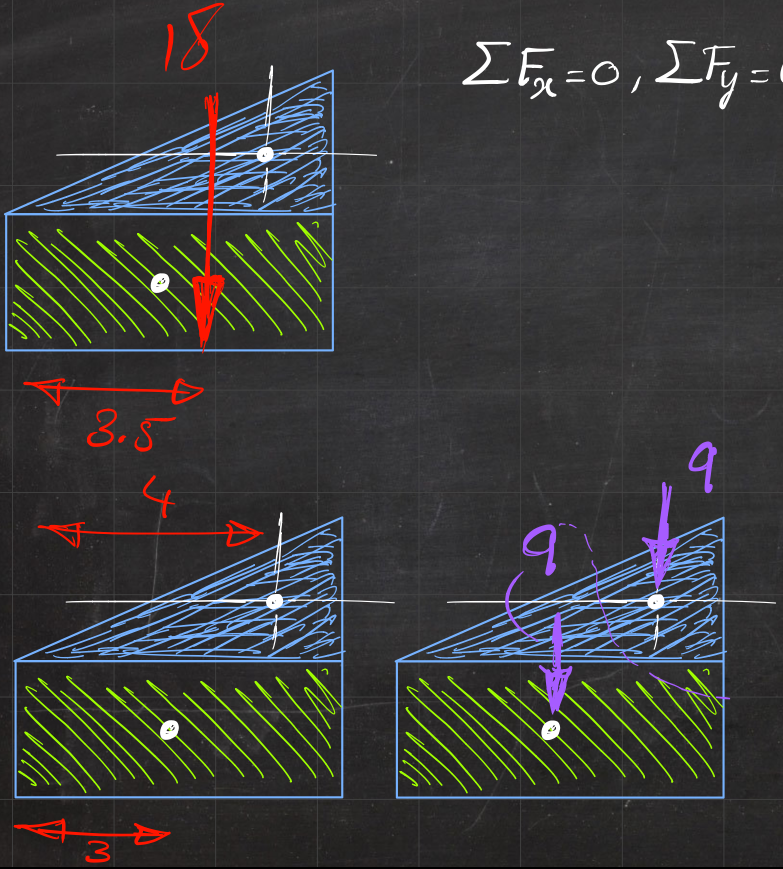
$1500 \times 6 = 9$

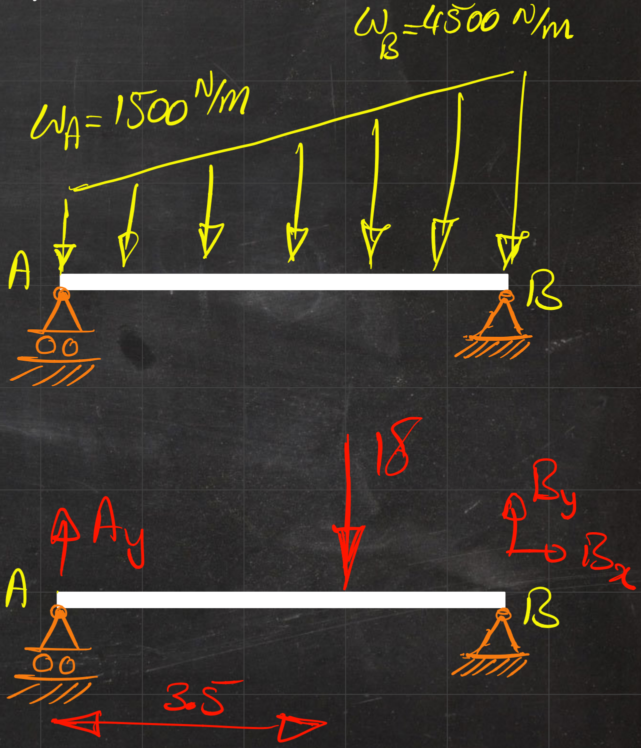



18
3.5
4
3
9
9


$\omega_B = 4500$ N/m
$\omega_A = 1500$ N/m
A
B
18
A_y
B_y
B_x
A
B
3.5

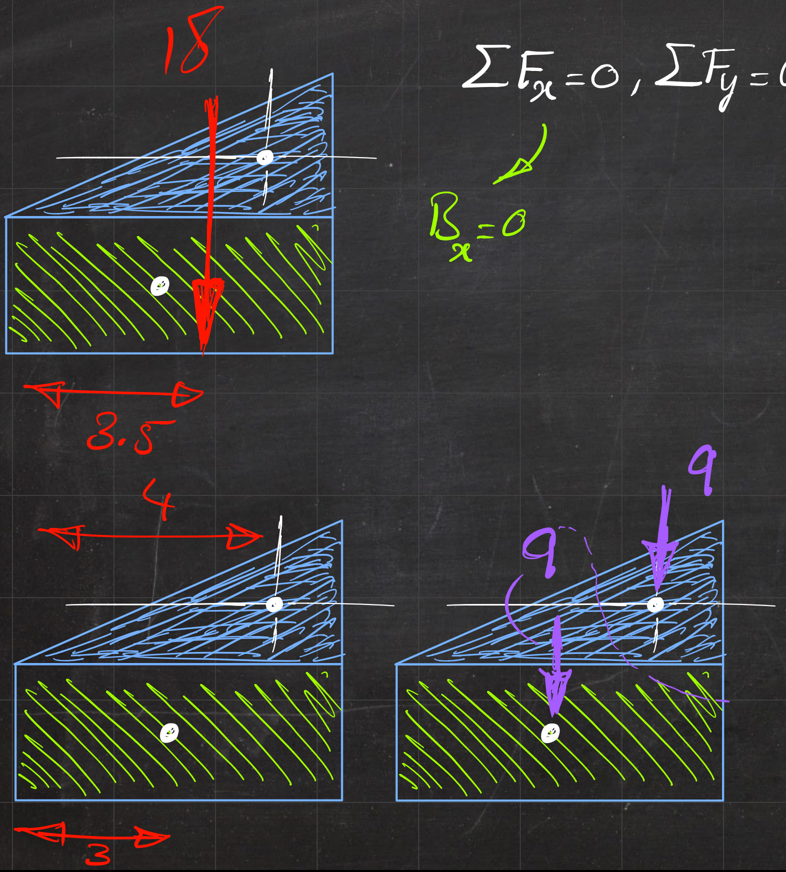
$$\sum F_x = 0, \sum F_y = 0, \sum M_A = 0$$

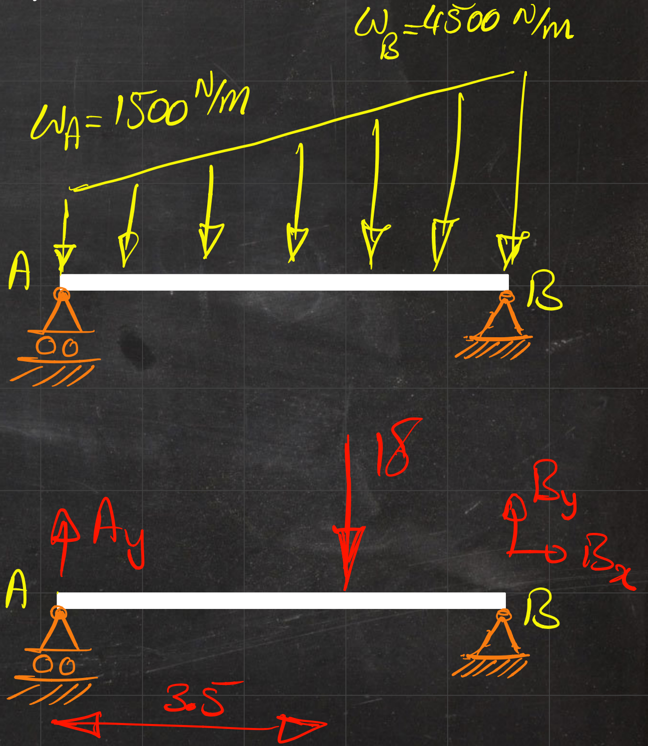


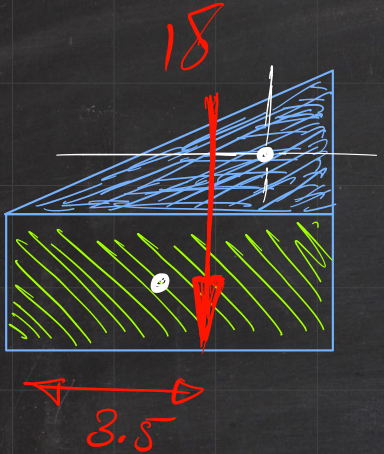


$$\sum F_x = 0, \sum F_y = 0, \sum M_A = 0$$

$$B_x = 0$$



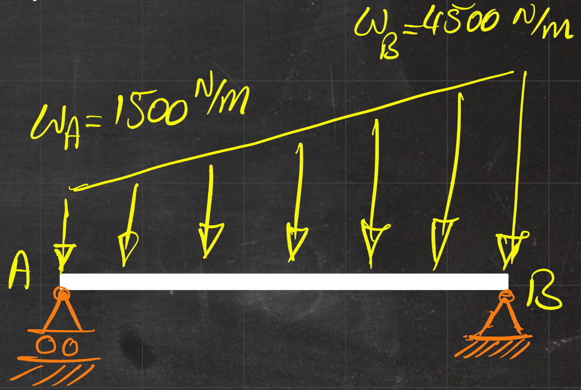


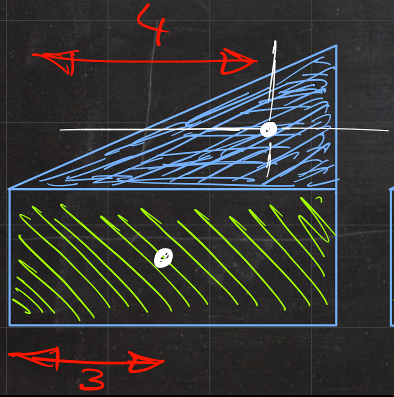


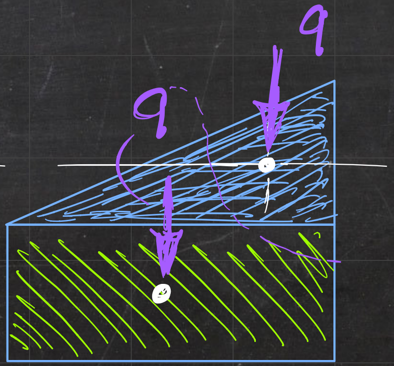
$$\sum F_x = 0, \sum F_y = 0, \sum M_A = 0$$

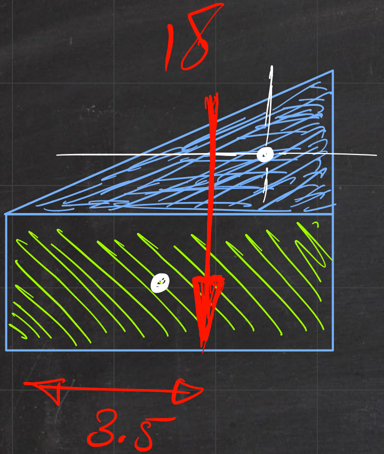
$$B_x = 0$$

$$A_y + B_y = 18$$







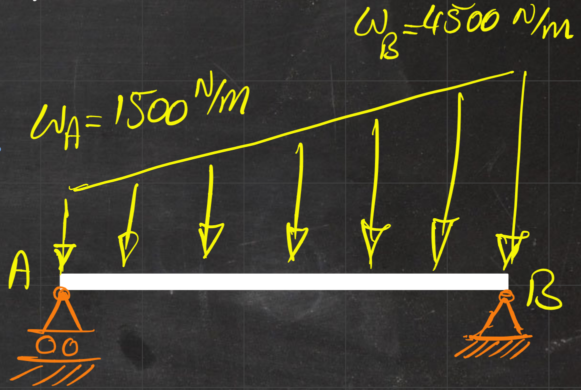


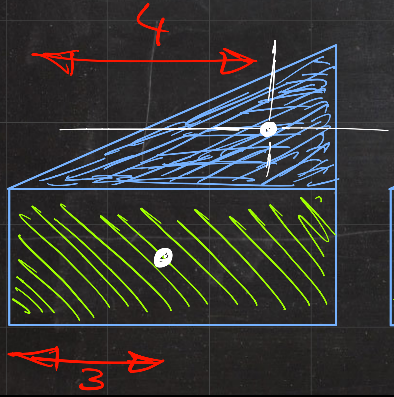
$$\sum F_x = 0, \sum F_y = 0, \sum M_A = 0$$

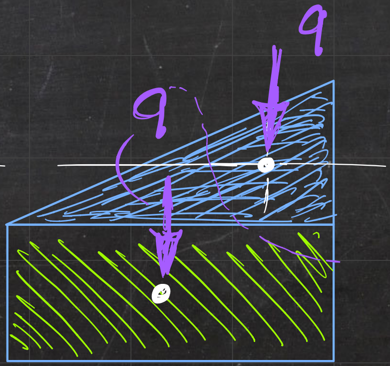
$$B_x = 0$$

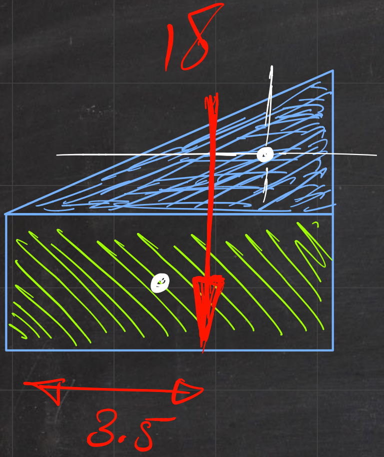
$$A_y + B_y = 18$$

$$6 B_y = 3.5 \times 18$$







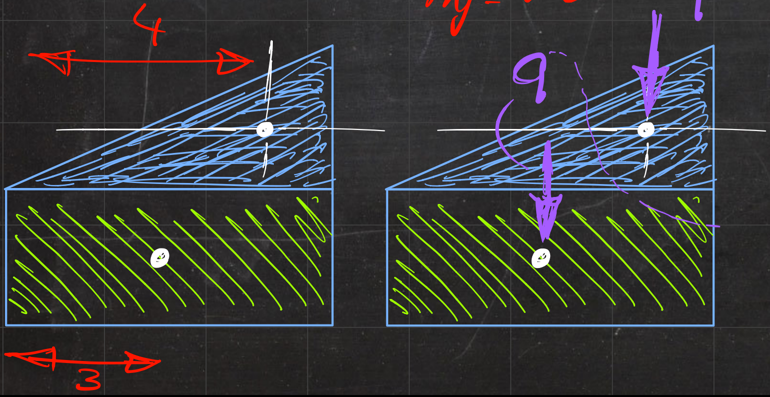


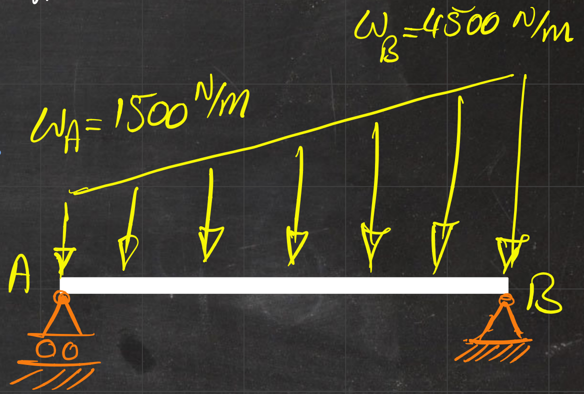
$$\sum F_x=0,\ \sum F_y=0,\ \sum M_A=0$$

$$B_x=0$$

$$A_y+B_y=18$$
$$\Downarrow$$
$$A_y=7.5\ \text{kN}$$

$$6B_y=3.5\times 18$$
$$\Downarrow$$
$$B_y=10.5\ \text{kN}$$





MECHANICS AND MATERIALS I

Introduction to Statics iv

Distributed Forces: Centroids and Centers of Gravity

Chap. 5

[Beer Johnston et al. 9th edition]

MECHANICS AND MATERIALS I

4