MECHANICS AND MATERIALS I

3

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MECHANICS AND MATERIALS I

Introduction to Statics iii

Equilibrium of Rigid Bodies

Chap. 4

[Beer Johnston et al. 9th edition]

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BY NOW ...

Chap. 2

- EQUILIBRIUM OF PARTICLES / POINTS $\rightarrow \sum \mathbf{F} = \mathbf{0}$, $\sum \mathbf{M}_O = \mathbf{0}$

  - EQUILIBRIUM OF A POINT

BY NOW ...

Chap. 2

- EQUILIBRIUM OF PARTICLES / POINTS $\rightarrow \sum \mathbf{F} = \mathbf{0}$, $\sum \mathbf{M}_O = \mathbf{0}$
- EQUIVALENT SYSTEM OF FORCES IN RIGID BODIES
  - TRANSFORM BODY TO POINT

Chap. 3

  - EQUILIBRIUM OF A POINT

BY NOW ...

- EQUILIBRIUM OF PARTICLES / POINTS → $\sum \mathbf{F} = \mathbf{0}$, $\sum \mathbf{M}_O = \mathbf{0}$ ← Chap. 2
- EQUIVALENT SYSTEM OF FORCES IN RIGID BODIES
  - TRANSFORM BODY TO POINT ← Chap. 3
  - EQUILIBRIUM OF A POINT

combine these

NEXT ..

- EQUILIBRIUM OF RIGID BODIES ← Chap. 4

EQUILIBRIUM OF RIGID BODIES

- FBD → DRAW ALL EXTERNAL FORCES & REACTIONS (INCL. WEIGHT; SUPPORTS)
- DRAW AN EQUIVALENT SYSTEM OF FORCES
- EQUILIBRIUM → $\Sigma F = 0$, $\Sigma M_O = 0$

REMARK: BEARINGS USUALLY DO NOT SUPPORT MOMENTS !

Support or Connection	Reaction	Number of Unknowns
Rollers; Rocker; Frictionless surface	Force with known line of action	1
Short cable; Short link	Force with known line of action	1
Collar on frictionless rod; Frictionless pin in slot	90°; Force with known line of action	1
Frictionless pin or hinge; Rough surface	or; α; Force of unknown direction	2
Fixed support	or; α; Force and couple	3

Fig. 4.1 Reactions at supports and connections.

2D

ROLLER (SUPPORT)

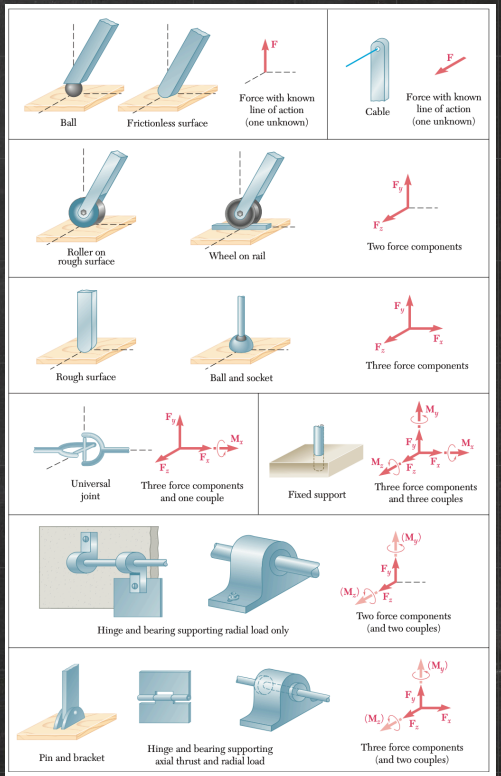

Fig. 4.10 Reactions at supports and connections.

3D

Some remarks on equilibrium (3 remarks)

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1) For 2D problems $\Rightarrow$ Equil.: 3 Eqns. / 3 Unknowns

But not every set of three equations work!

SOME REMARKS ON EQUILIBRIUM (3 REMARKS)

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e.g. $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_A = 0$ — WORKS WELL ✓

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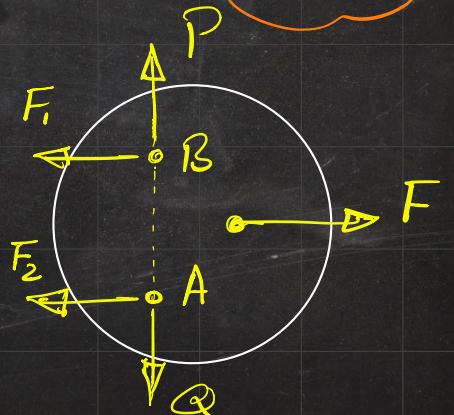

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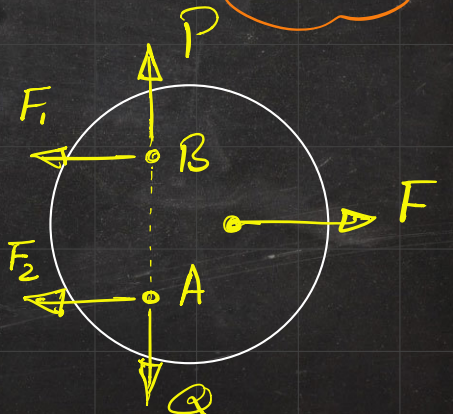

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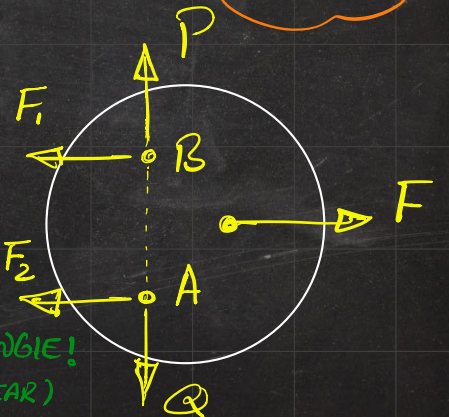
e.g. $\Sigma M_A = 0$, $\Sigma M_B = 0$, $\Sigma M_C = 0$ $\rightarrow$ WORKS ONLY IF ABC FORM A TRIANGLE! (A, B, C NOT COLLINEAR)



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b) # UNKNOWNS < # EQUILIBRIUM EQNS.

c) # UNKNOWNS = # EQUILIBRIUM EQNS.

SOME REMARKS ON EQUILIBRIUM (3 REMARKS)

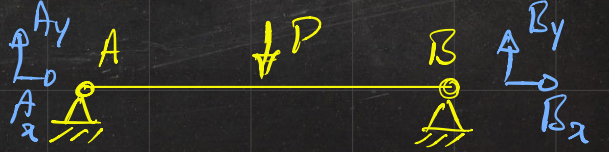
2) THE SYSTEM MUST BE PROPERLY CONSTRAINED

STATICALLY INDETERMINATE

a) # UNKNOWNS > # EQUILIBRIUM EQNS. ⇒ OVERCONSTRAINED

b) # UNKNOWNS < # EQUILIBRIUM EQNS. ⇒ IN EQUILIBRIUM BUT CANNOT BE SOLVED

c) # UNKNOWNS = # EQUILIBRIUM EQNS.



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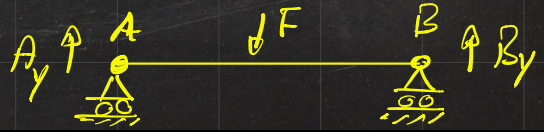
Partially Constrained



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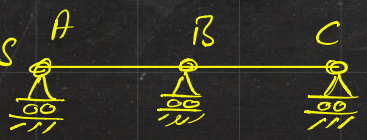
PARALLEL REACTIONS



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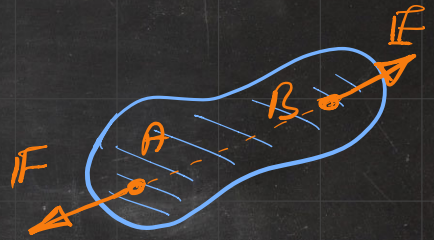
Some remarks on equilibrium (3 remarks)

3) Equilibrium of Multi-Force Bodies

SOME REMARKS ON EQUILIBRIUM (3 REMARKS)

3) EQUILIBRIUM OF MULTI-FORCE BODIES



a) TWO-FORCE BODY → if only two forces applied on a rigid body,

they must be of equal magnitude,

opposite direction,

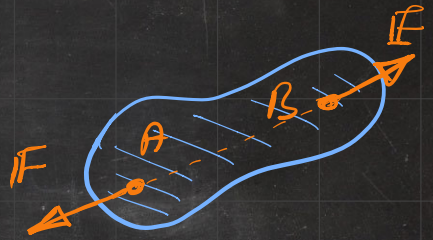
on the same line of action!

↓

TO FULFILL EQUILIBRIUM

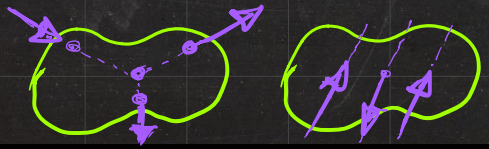
SOME REMARKS ON EQUILIBRIUM (3 REMARKS)

3) EQUILIBRIUM OF MULTI-FORCE BODIES



a) TWO-FORCE BODY

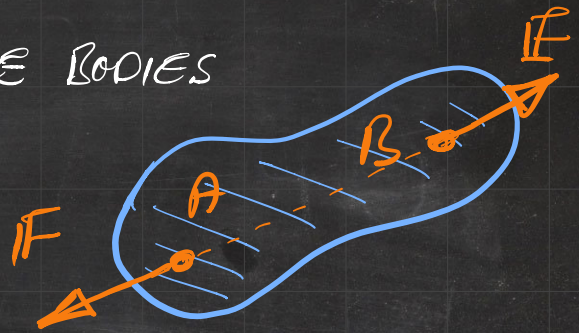
b) THREE-FORCE BODY



⇒ if only three forces are applied on a rigid body, they must be concurrent or parallel ⇒ TO FULFILL EQUILIBRIUM

SOME REMARKS ON EQUILIBRIUM (3 REMARKS)

3) EQUILIBRIUM OF MULTI-FORCE BODIES



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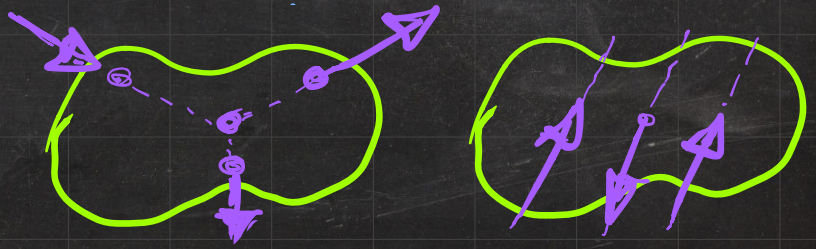
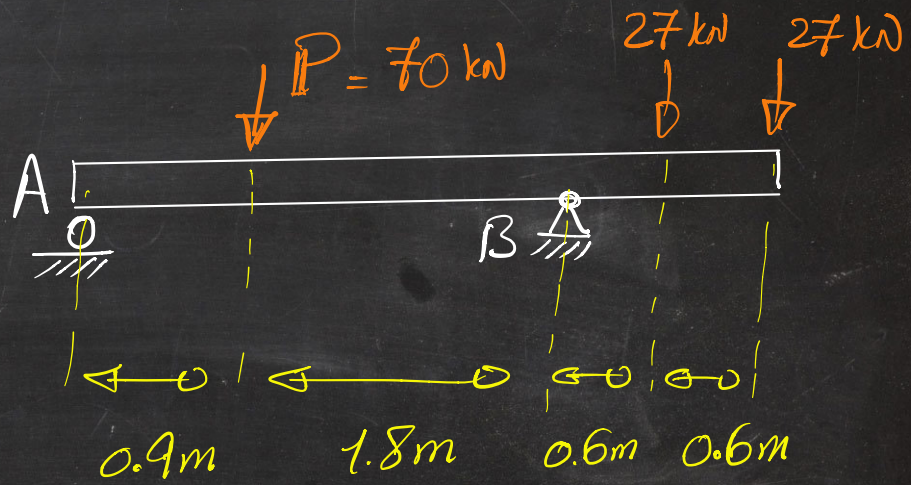


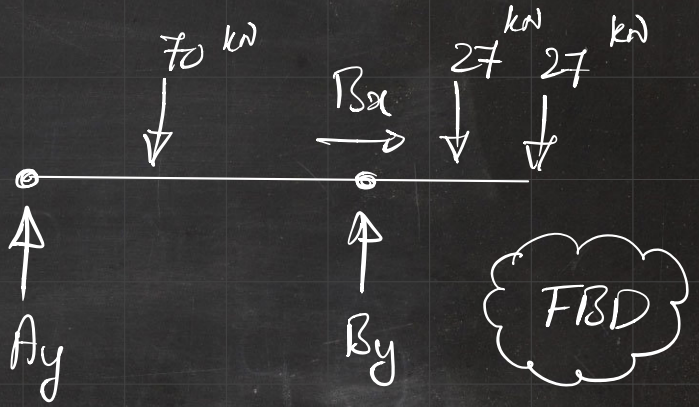
Exercise 1 . [similar to ... P. 167 ... 4.2]

CONSIDER THE BEAM.

NEGLECT WEIGHT.

DETERMINE THE REACTIONS AT THE SUPPORTS A, B.

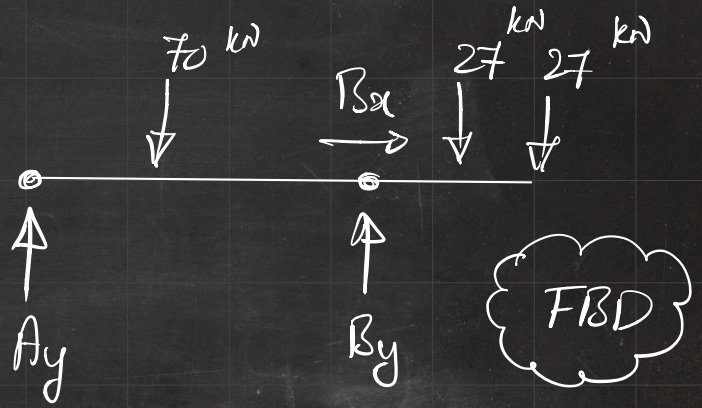



70 kN
Bx
27 kN
27 kN
Ay
By
FBD

$\rightarrow_+ \sum F_x = 0$

$+\uparrow \sum F_y = 0$

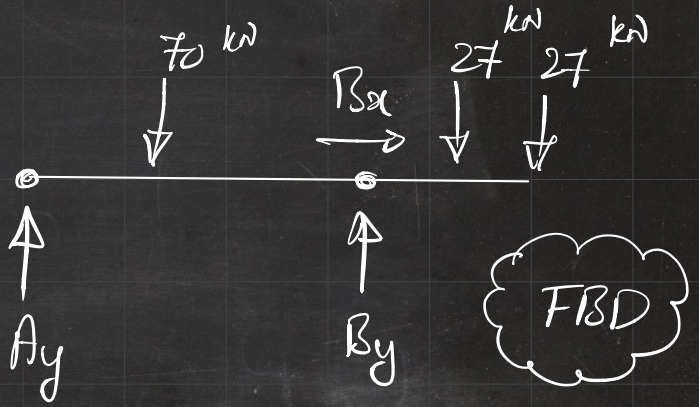
$\circlearrowleft_+ \sum M_B = 0$



$\overset{+}{\rightarrow}_0 \quad \sum F_x = 0 \Rightarrow B_x = 0$

$+\uparrow \quad \sum F_y = 0 \Rightarrow A_y + B_y - 70 - 27 - 27 = 0$

$(+\circlearrowleft) \quad \sum M_B = 0 \Rightarrow -27 \times 0.6 - 27 \times 1.2 - A_y \times 2.7 + 70 \times 1.8 = 0$



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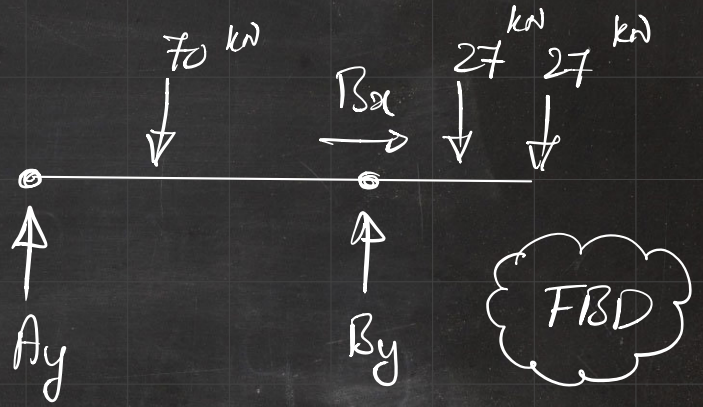
$\Rightarrow A_y = 28.7\ kN \quad , \quad B_y = 95.3\ kN$



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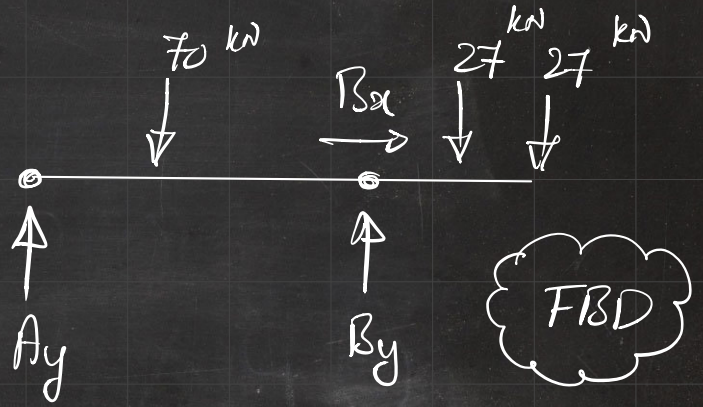


$\Rightarrow A_y = 28.7 \text{ kN}, \quad B_y = 95.3 \text{ kN}$

ALTERNATIVELY →
- $\sum F_x = 0 \rightarrow B_x$ ✓
- $\sum M_A = 0 \rightarrow B_y$ ✓
- $\sum M_B = 0 \rightarrow A_y$ ✓

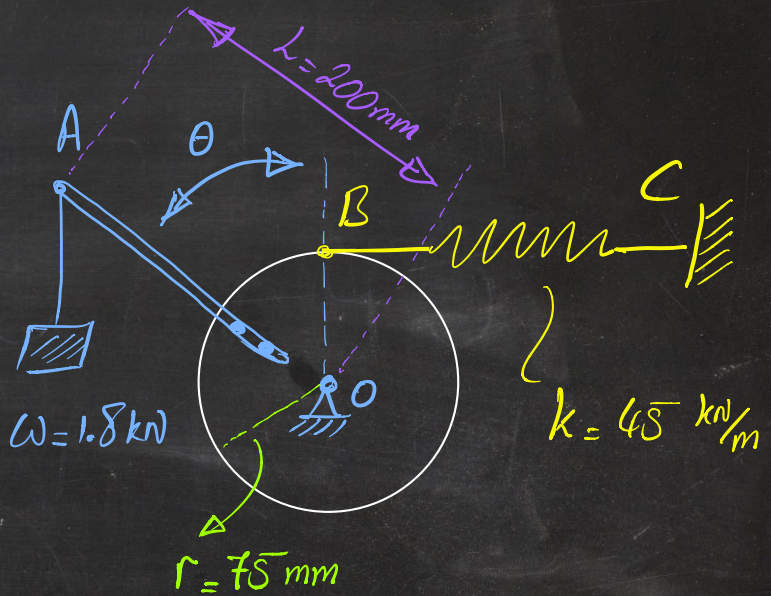
— TO COMPLETE THEM INDIVIDUALLY

Exercise 2 . [similar to ... P. 169 ... 4.5]

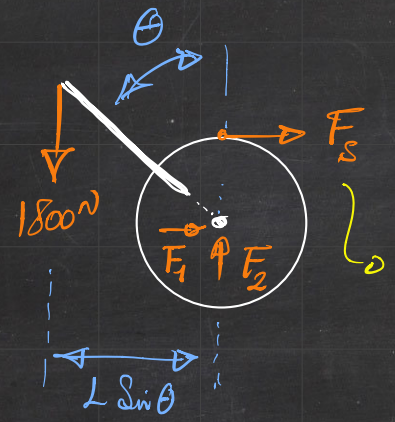
THE WEIGHT W IS ATTACHED TO LEVER AT A.

SPRING IS UNSTRETCHED WHEN $\theta = 0$.

DETERMINE THE POSITION OF EQUILIBRIUM.

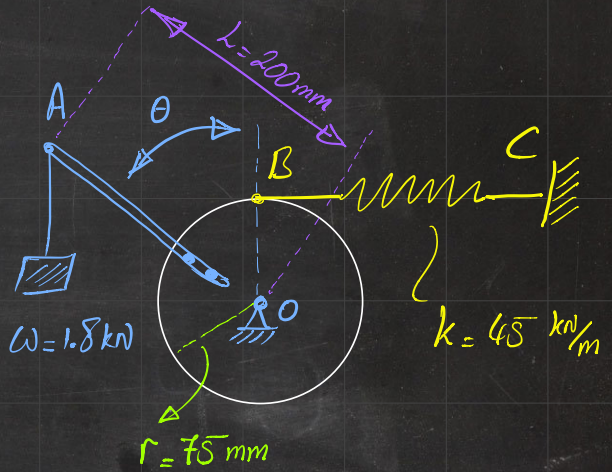


FBD

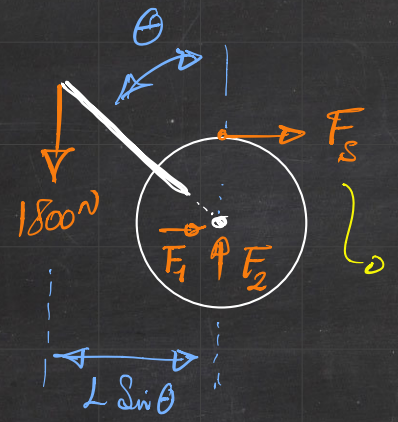


STRETCH

$$F_s = k\,\delta$$
$$= kr\theta$$



FBD



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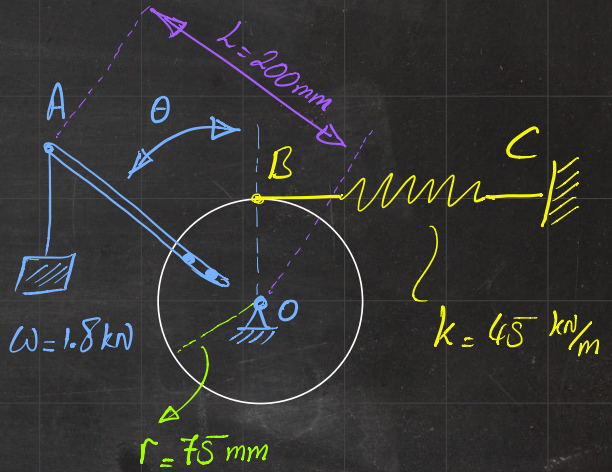
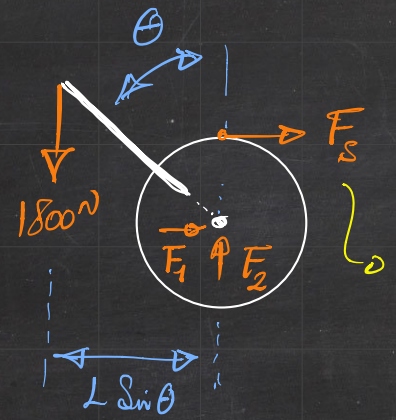


$$\overset{+}{\rightarrow} \sum F_x = 0 \rightarrow F_1 + F_S = 0$$

$$+\uparrow \sum F_y = 0 \rightarrow F_2 - 1800 = 0$$

$$\circlearrowleft + \sum M_O = 0 \rightarrow L \sin\theta \, W - r F_S = 0$$

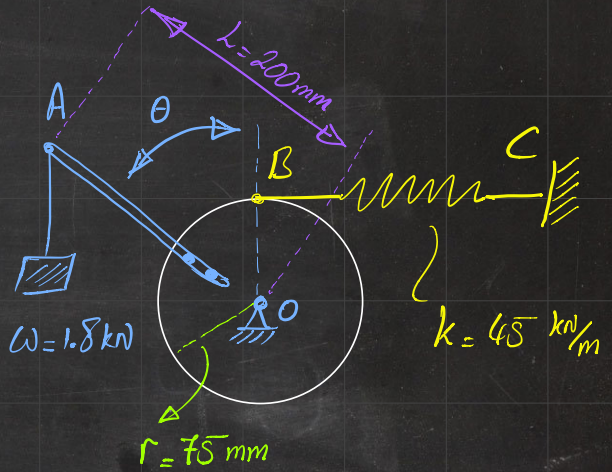
FBD



STRETCH

$F_s = k\,\delta$

$= k r \theta$



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$\circlearrowleft + \sum M_O = 0 \rightarrow L \sin\theta\, \omega - r F_s = 0$

3 Eqns.
3 Unk.
F_1 F_2 θ

$\sin\theta = 0.703\,\theta$

$\Rightarrow \theta = 0^\circ, \theta = 80.3^\circ$

Exercise 3 . [similar to ... P. 203 ... 4.114]

THE BENT ROD ABEF IS SUPPORTED BY BEARINGS AT C AND D AND BY WIRE AH. THE PORTION AB = 250 mm. ASSUME BEARING D DOES NOT EXERT AXIAL THRUST. DETERMINE:

a) TENSION IN WIRE AH,

b) REACTIONS AT C & D.

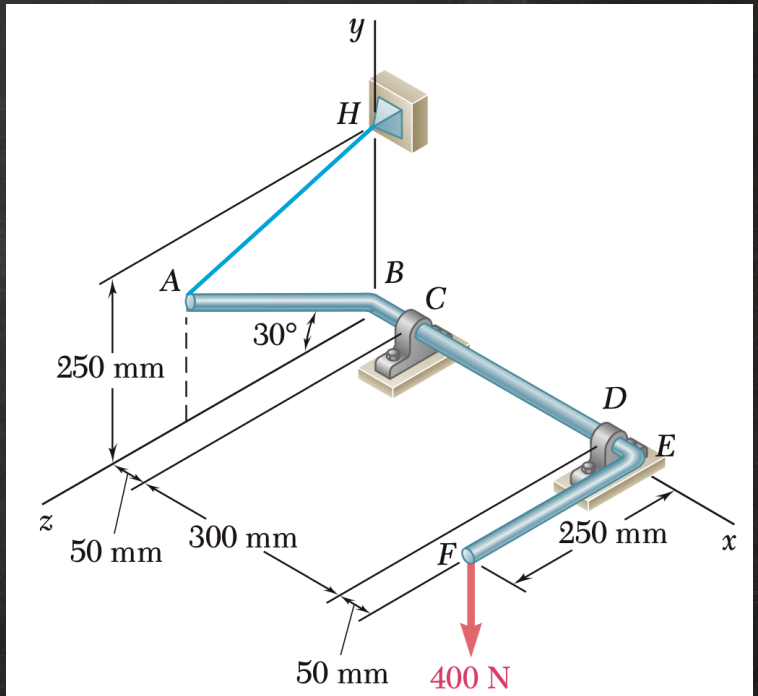
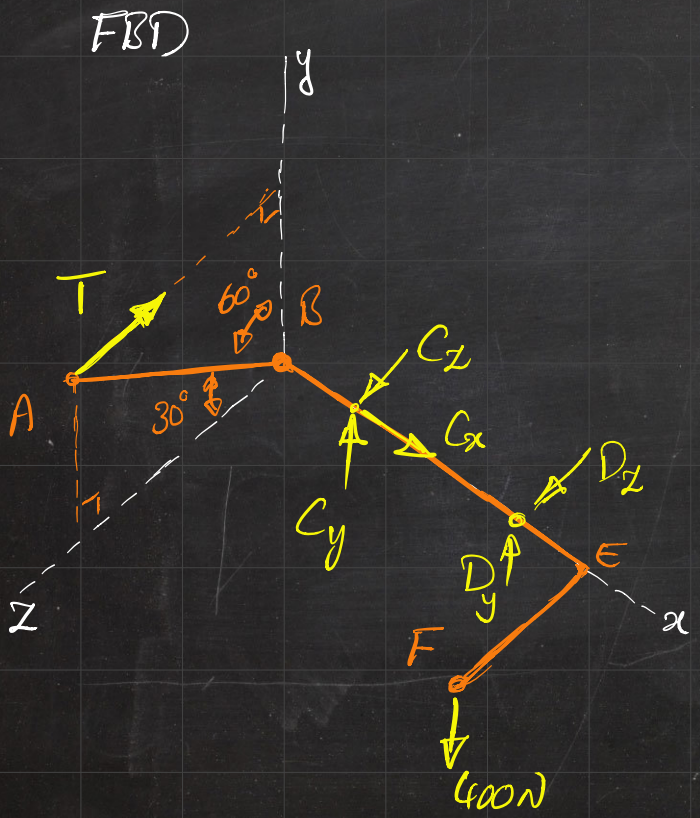


Fig. P4.114


FBD
y
T
60°
B
A
30°
C_z
C_x
C_y
D_z
D_y
E
F
z
x
400N

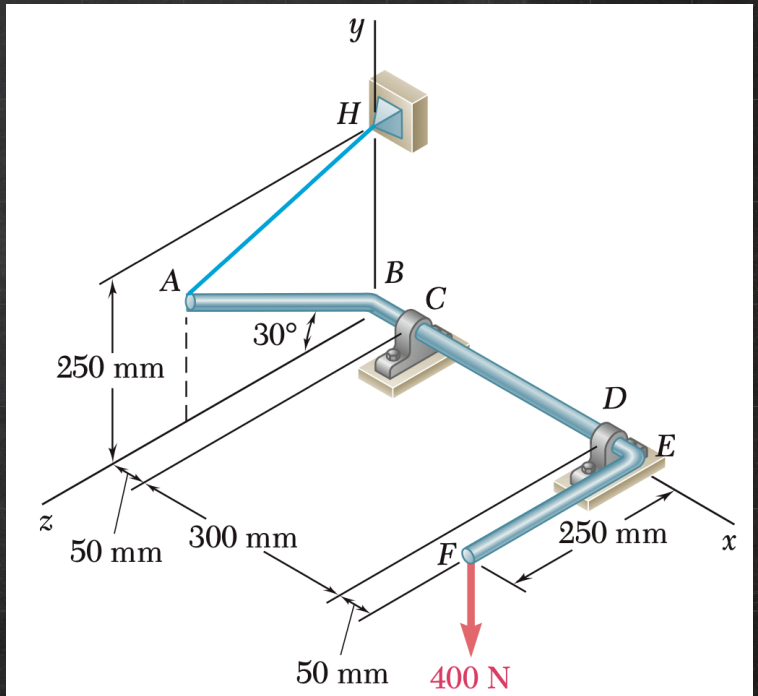

y
H
A
B
C
30°
250 mm
D
E
z
50 mm
300 mm
F
250 mm
x
50 mm
400 N

Fig. P4.114

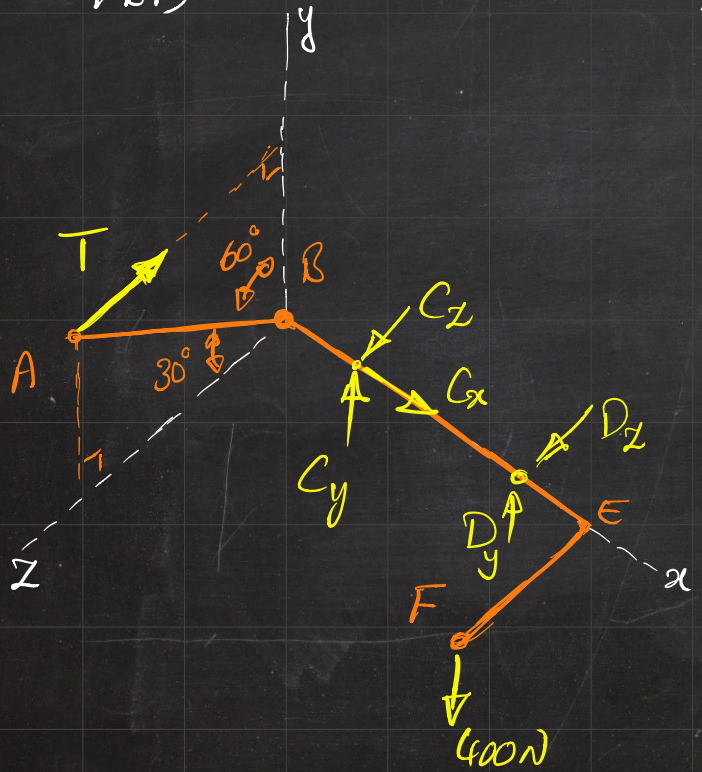
FBD

$$\bar{T} = T\,\lambda_{AH}$$

$$\lambda_{AH} = \frac{\bar{AH}}{|\bar{AH}|} = \frac{125\,\hat{j} - 216.51\,\hat{k}}{250}$$

$$= \frac{1}{2}\,\hat{j} - \frac{\sqrt{3}}{2}\,\hat{k}$$

FBD



$$\vec{T} = T\,\lambda_{AH}$$

$$\lambda_{AH} = \frac{\vec{AH}}{|\vec{AH}|} = \frac{125\,\hat{j} - 216.51\,\hat{k}}{250}$$

$$= \frac{1}{2}\hat{j} - \frac{\sqrt{3}}{2}\hat{k}$$

Alternatively: $\lambda_{AH} = \underbrace{\cos 60}_{\frac{1}{2}}\,\hat{j} - \underbrace{\sin 60}_{\frac{\sqrt{3}}{2}}\,\hat{k}$

FBD

$$\overline{T} = T\lambda_{AH} = \frac{1}{2}T\hat{j} - \frac{\sqrt{3}}{2}T\hat{k}$$

$$\Sigma \overline{F} = \overline{0} \Rightarrow \Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma F_z = 0$$

$$\Sigma \overline{M}_C = \overline{0} \Rightarrow \Sigma M_{Cx} = 0$$

$$\Sigma M_{Cy} = 0$$

$$\Sigma M_{Cz} = 0$$

FBD

$$\overline{T} = T\lambda_{AH} = \frac{1}{2}T\,j - \frac{\sqrt{3}}{2}T\,k$$

$$\Sigma F = 0 \Rightarrow \Sigma F_x = 0 \rightarrow C_x = 0$$

$$\Sigma F_y = 0 \rightarrow 0.5T + C_y + D_y - 400 = 0$$

$$\Sigma F_z = 0 \rightarrow -\frac{\sqrt{3}}{2}T + C_z + D_z = 0$$

$$\Sigma M_C = 0 \Rightarrow \Sigma M_{Cx} = 0$$

$$\Sigma M_{Cy} = 0 \quad \text{... SEE NEXT}$$

$$\Sigma M_{Cz} = 0$$

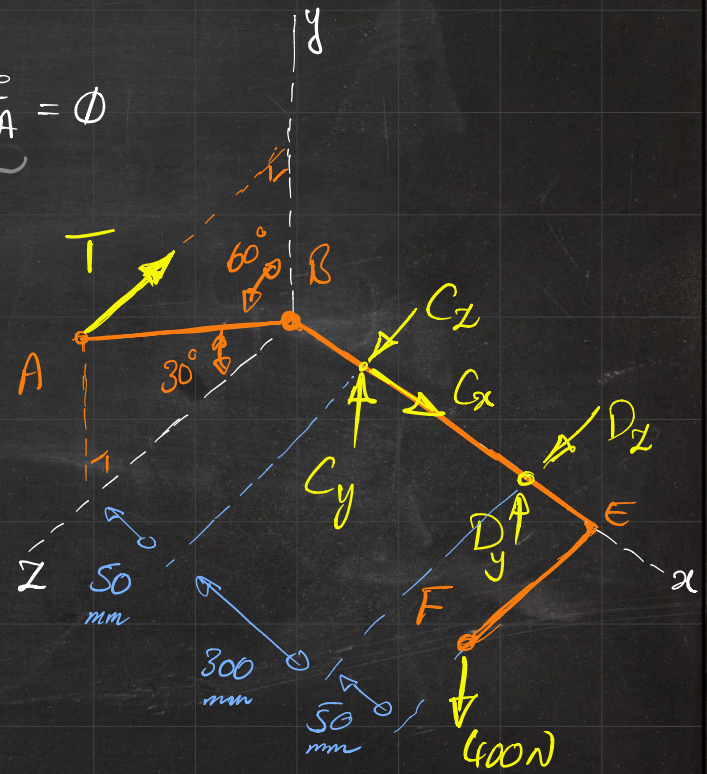
$$\Sigma \mathbb{M}_C = \mathbb{r}_D \times \mathbb{F}_D + \mathbb{r}_F \times \mathbb{F}_F + \mathbb{r}_A \times \mathbb{F}_A = \mathbb{0}$$

$$[300\,\mathbb{i}] \times [D_y\,\mathbb{j} + D_z\,\mathbb{k}]$$

$$+ [350\,\mathbb{i} + 250\,\mathbb{k}] \times [-400\,\mathbb{j}]$$

$$+ [-50\,\mathbb{i} + 125\,\mathbb{j} + 216.5\,\mathbb{k}] \times \left[\tfrac{1}{2}T\,\mathbb{j} - \tfrac{\sqrt{3}}{2}\,\mathbb{k}\right]$$

$$= \mathbb{0}$$



FBD

$$\vec{T} = T\,\lambda_{AH} = \frac{1}{2}T\,\vec{j} - \frac{\sqrt{3}}{2}T\,\vec{k}$$

$$\sum \vec{F} = \vec{0} \Rightarrow \sum F_x = 0 \longrightarrow C_x = 0$$

$$\sum F_y = 0 \longrightarrow 0.5T + C_y + D_y - 400 = 0$$

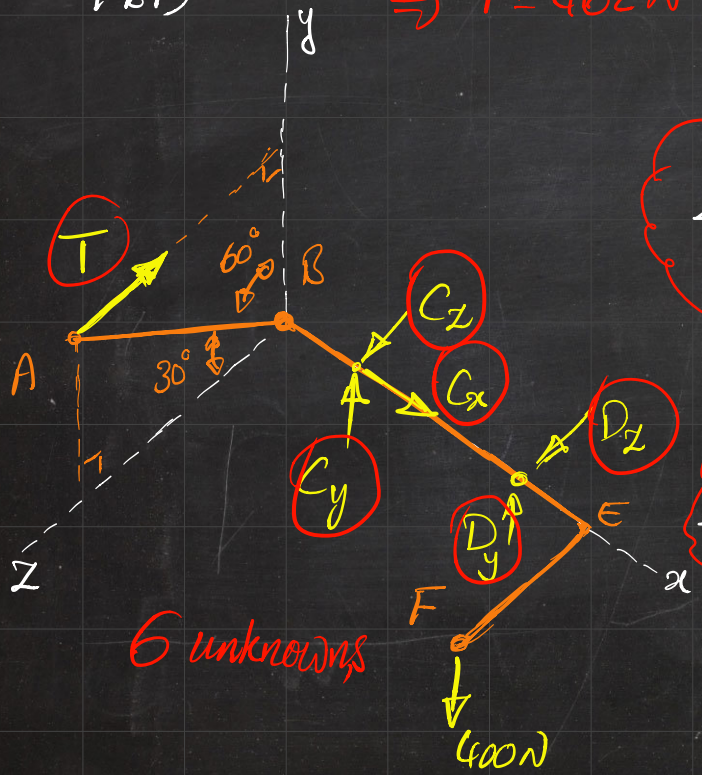
$$\sum F_z = 0 \longrightarrow -\frac{\sqrt{3}}{2}T + C_z + D_z = 0$$

$$\sum \vec{M}_C = \vec{0} \Rightarrow \sum M_{C_x} = 0 \longrightarrow \dots$$

$$\sum M_{C_y} = 0 \longrightarrow \dots$$

$$\sum M_{C_z} = 0 \longrightarrow \dots$$

FBD



6 unknowns

$\Rightarrow T = 462\,N,\ D_z = -66.7\,N,\ D_y = 505\,N,\ C_y = -336\,N$

$C_z = 467\,N$

$$\Sigma F = 0 \Rightarrow \Sigma F_x = 0 \rightarrow C_x = 0$$

$$\Sigma F_y = 0 \rightarrow 0.5T + C_y + D_y - 400 = 0$$

$$\Sigma F_z = 0 \rightarrow -\frac{\sqrt{3}}{2}T + C_z + D_z = 0$$

$$\Sigma M_C = 0 \Rightarrow \Sigma M_{Cx} = 0 \rightarrow \dots$$

$$\Sigma M_{Cy} = 0 \rightarrow \dots$$

$$\Sigma M_{Cz} = 0 \rightarrow \dots$$

6 EQNs.

MECHANICS AND MATERIALS I

Introduction to Statics iii

Equilibrium of Rigid Bodies

Chap. 4

[Beer Johnston et al. 9th edition]

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3

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