MECHANICS AND MATERIALS I

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10

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MECHANICS AND MATERIALS I

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Mechanical properties

Chap. 3

[Hibbeler 9th edition]

MECHANICAL PROPERTIES OF MATERIALS

$$\begin{tikzcd}
\text{Deformation} \arrow[r, leftrightarrow, "?"] \arrow[d, leftrightarrow, "\text{Chap. 2}"'] & \text{Loading} \arrow[d, leftrightarrow, "\text{Chap. 1}"] \\
\text{Strain} \arrow[r, leftrightarrow, "\text{MATERIAL}", "\text{Chap.3}"'] & \text{Stress}
\end{tikzcd}$$

MECHANICAL PROPERTIES OF MATERIALS

Deformation $\overset{?}{\longleftrightarrow}$ Loading

Chap. 2 $\updownarrow$ MATERIAL $\updownarrow$ Chap. 1

Strain $\longleftrightarrow$ Stress

Chap. 3

↳ WE ARE LOOKING FOR A STRESS-STRAIN RELATIONSHIP

MECHANICAL PROPERTIES OF MATERIALS

?

Deformation ⟷ Loading

Chap. 2 ↕ MATERIAL ↕ Chap. 1

Strain ⟷ Stress

Chap. 3

ISOTROPIC MATERIALS

↳ WE ARE LOOKING FOR A STRESS-STRAIN RELATIONSHIP

MECHANICAL PROPERTIES OF MATERIALS

?

Deformation ⟷ Loading

Chap. 2 ↕ MATERIAL ↕ Chap. 1

↓

Strain ⟷ Stress

Chap. 3

↳ WE ARE LOOKING FOR A STRESS-STRAIN RELATIONSHIP

ISOTROPIC MATERIALS

↳ ISOTROPY is uniformity in all directions

greek

ISOS + TROPOS
(equal) (way)

MECHANICAL PROPERTIES OF MATERIALS

?

Deformation ⟷ Loading

Chap. 2 ↕ MATERIAL ↕ Chap. 1

↓

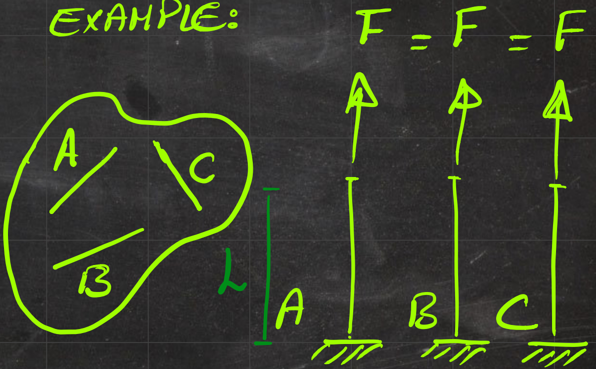
Strain ⟷ Stress

Chap. 3

↳ WE ARE LOOKING FOR A STRESS-STRAIN RELATIONSHIP

ISOTROPIC MATERIALS

EXAMPLE:



$$\delta_A = \delta_B = \delta_C$$

MECHANICAL PROPERTIES OF MATERIALS

?

Deformation ⟷ Loading

Chap. 2 ↕ MATERIAL ↕ Chap. 1

Strain ⟷ Stress

Chap. 3

↳ WE ARE LOOKING FOR A STRESS-STRAIN RELATIONSHIP

* SMALL-STRAIN

* USUALLY ELASTIC

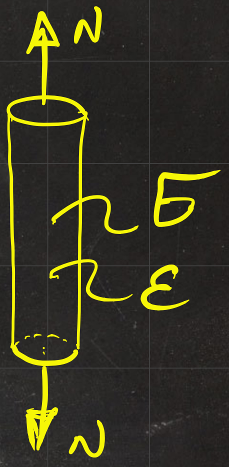
Linear elasticity

Inelastic

e.g. ↳ Plastic (perfect)

σ ← stress

STRESS - STRAIN DIAGRAM



DUCTILE MATERIAL

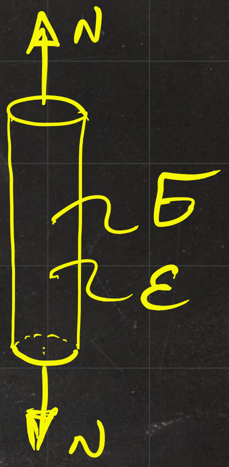
ε ← strain

σ ← stress

↳ $\sigma = \dfrac{N}{A}$ ← CROSS SECTIONAL AREA

LENGTH

$$\varepsilon = \frac{\delta}{L}$$



DUCTILE MATERIAL

ε ← strain

σ ← stress

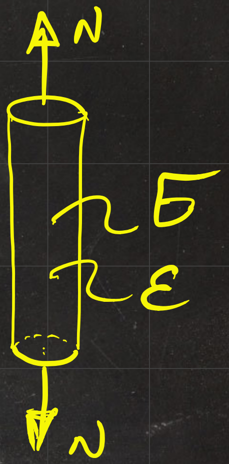
$$\sigma = \frac{N}{A}$$

CROSS SECTIONAL AREA

INITIAL VALUES

LENGTH

$$\varepsilon = \frac{\delta}{L}$$



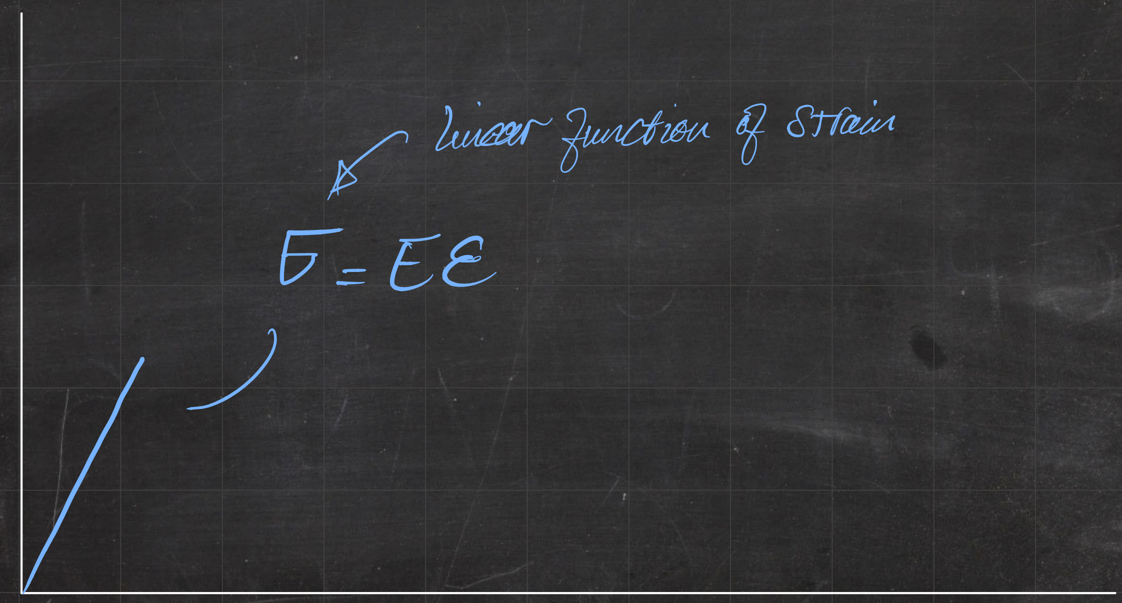
DUCTILE MATERIAL

ε ← strain

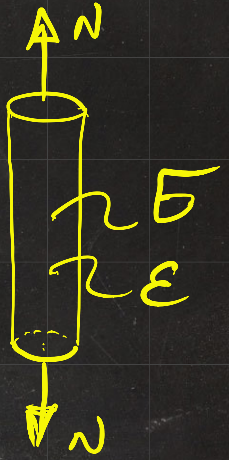
σ ← stress

linear function of strain

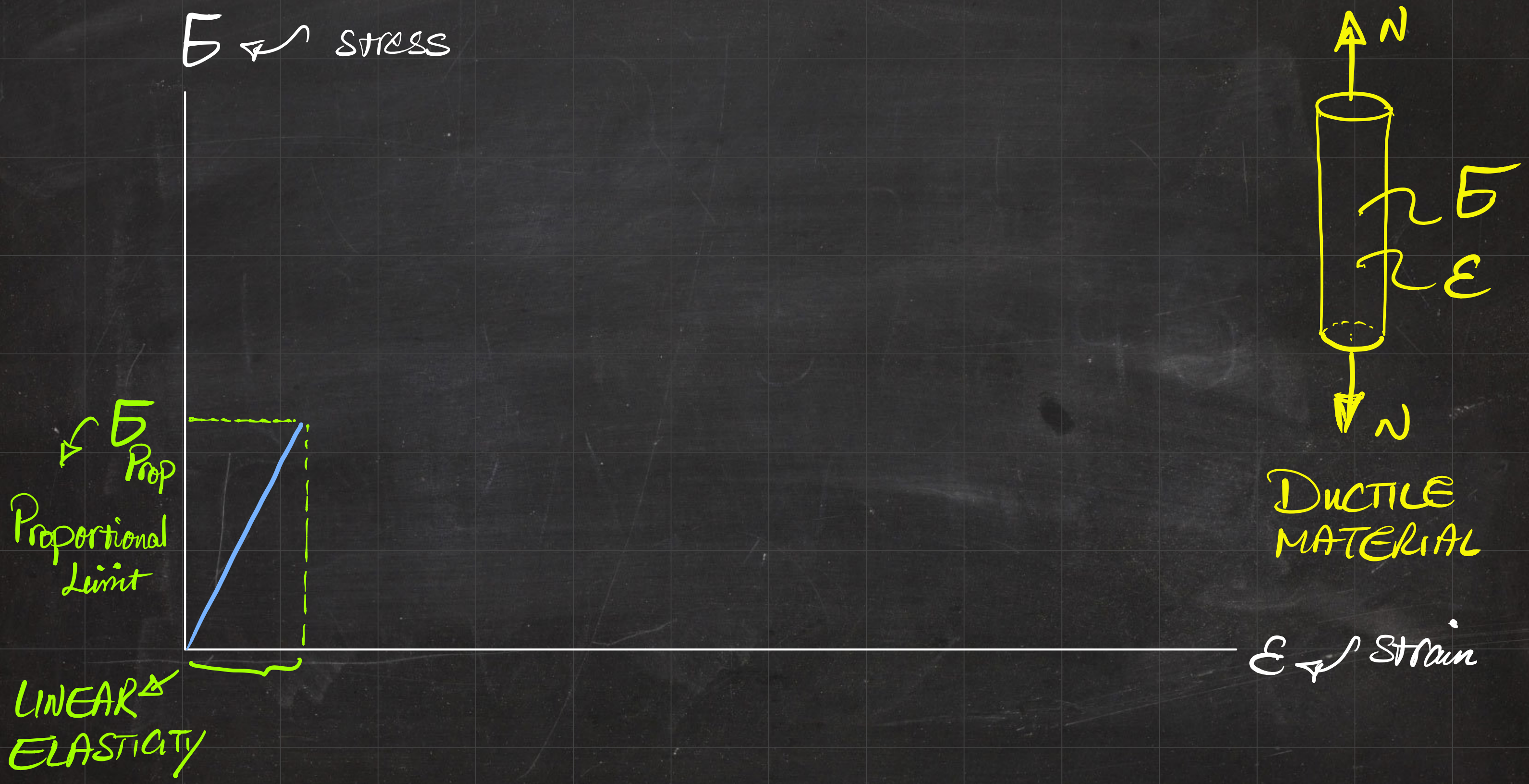
$$\sigma = E\varepsilon$$



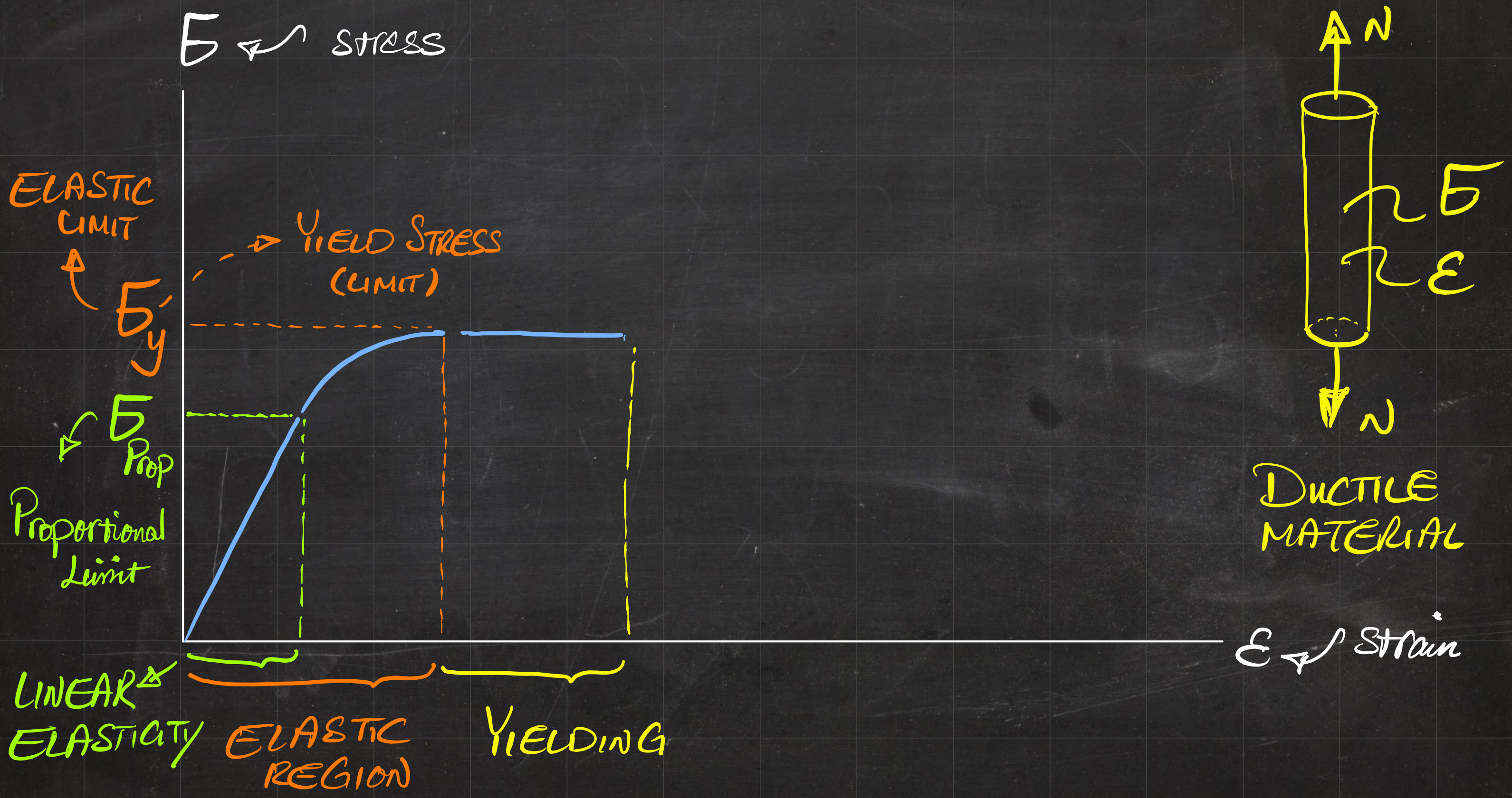
ε ← strain



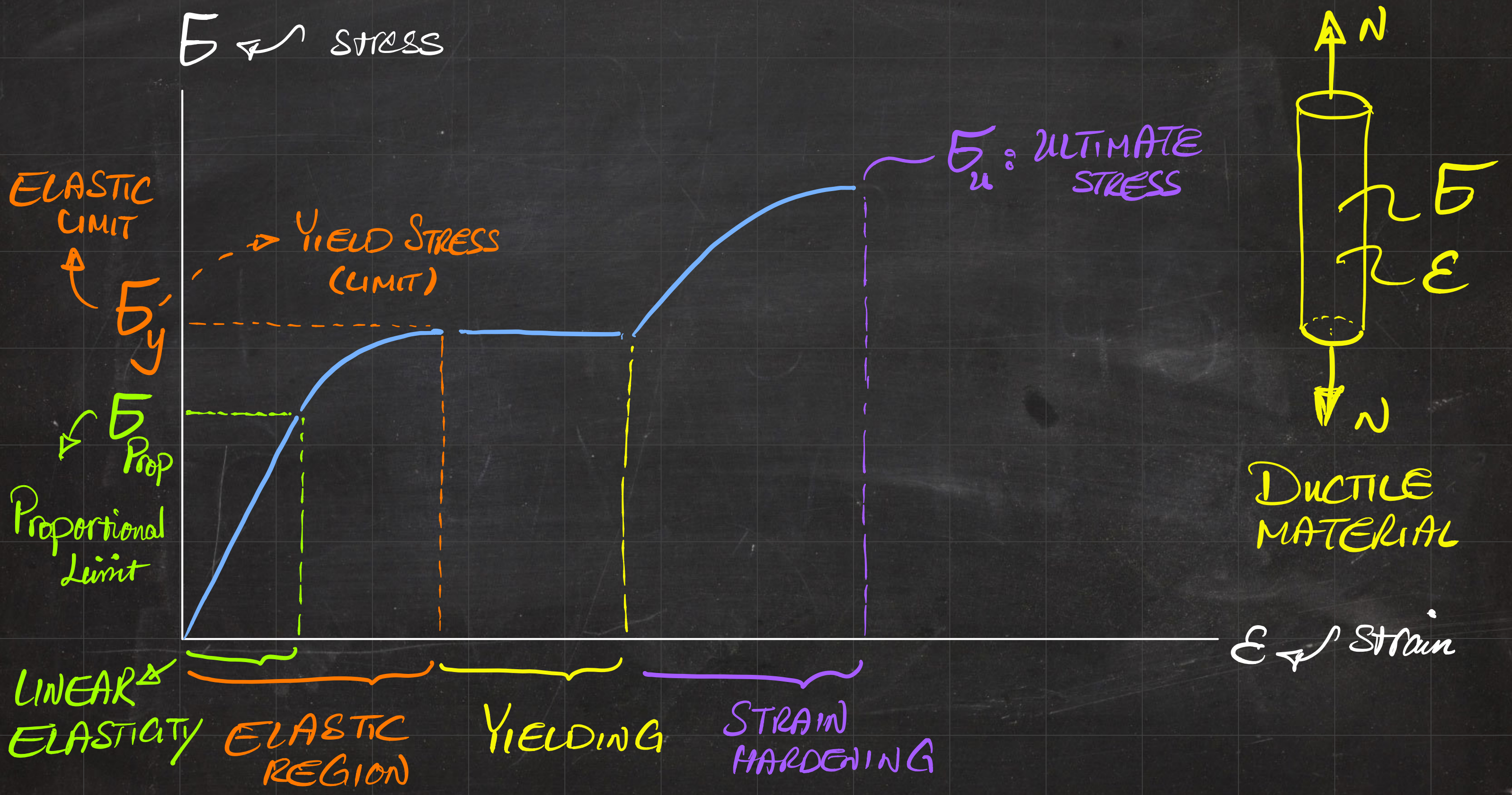
DUCTILE MATERIAL


σ stress
σ Prop
Proportional Limit
LINEAR ELASTICITY
ε strain
N
σ
ε
N
DUCTILE MATERIAL

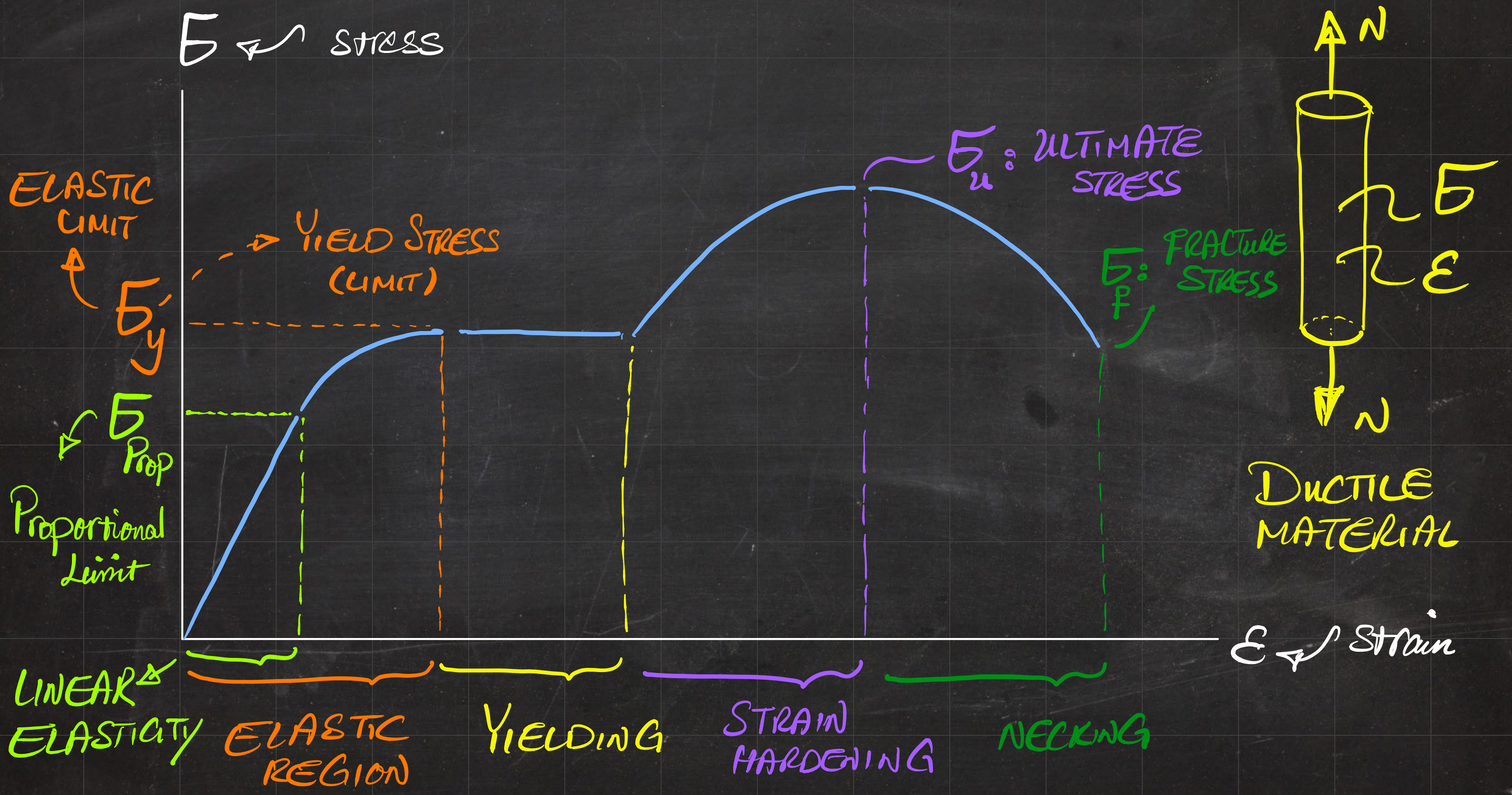

σ STRESS
ELASTIC LIMIT
YIELD STRESS (LIMIT)
σ_y
σ_{Prop}
Proportional Limit
LINEAR ELASTICITY
ELASTIC REGION
ε Strain
N
σ
ε
N
DUCTILE MATERIAL



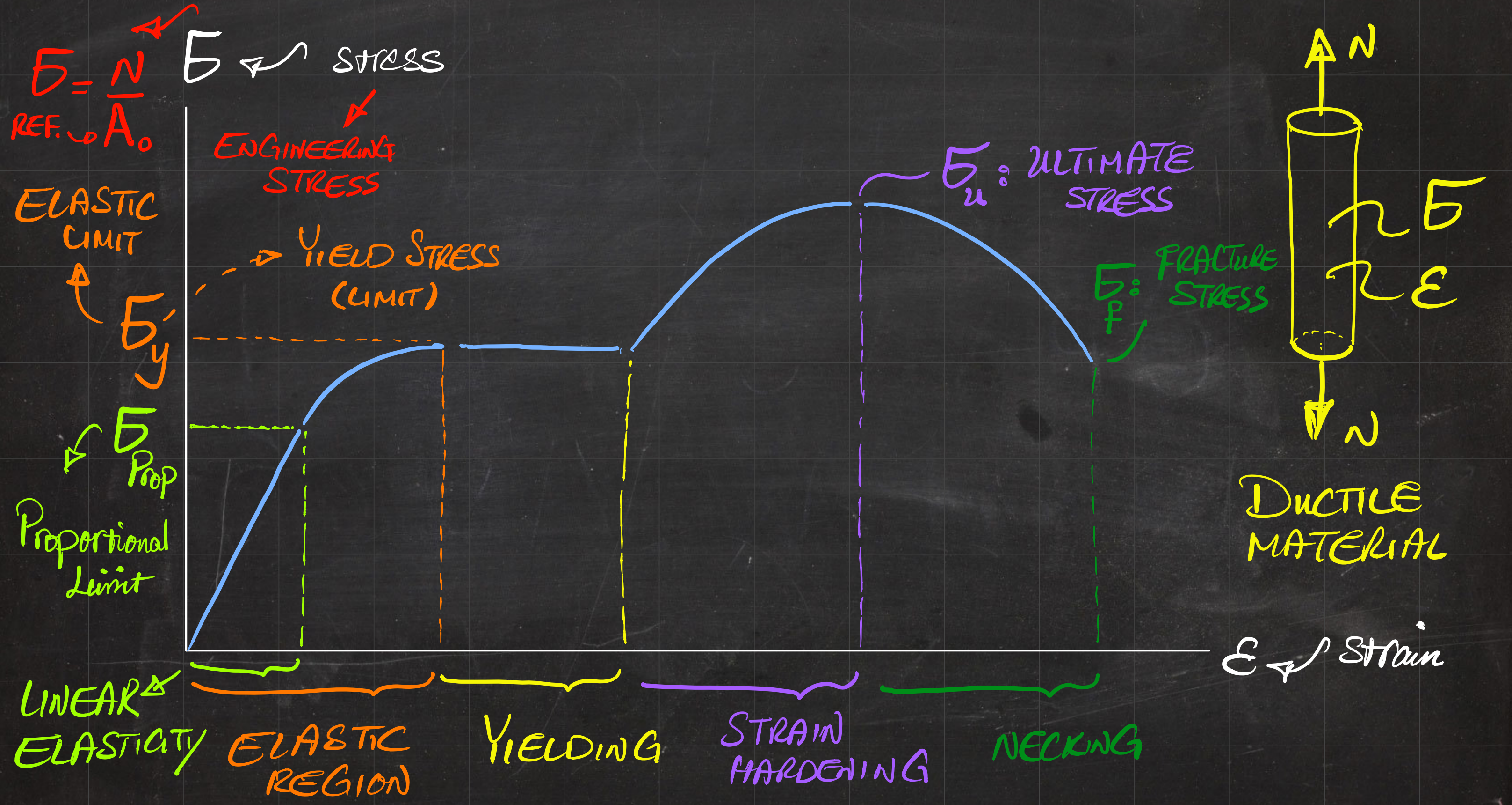
σ stress
ELASTIC LIMIT
YIELD STRESS (LIMIT)
σ_y
σ_{Prop}
Proportional Limit
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
ε strain
N
σ
ε
N
DUCTILE MATERIAL

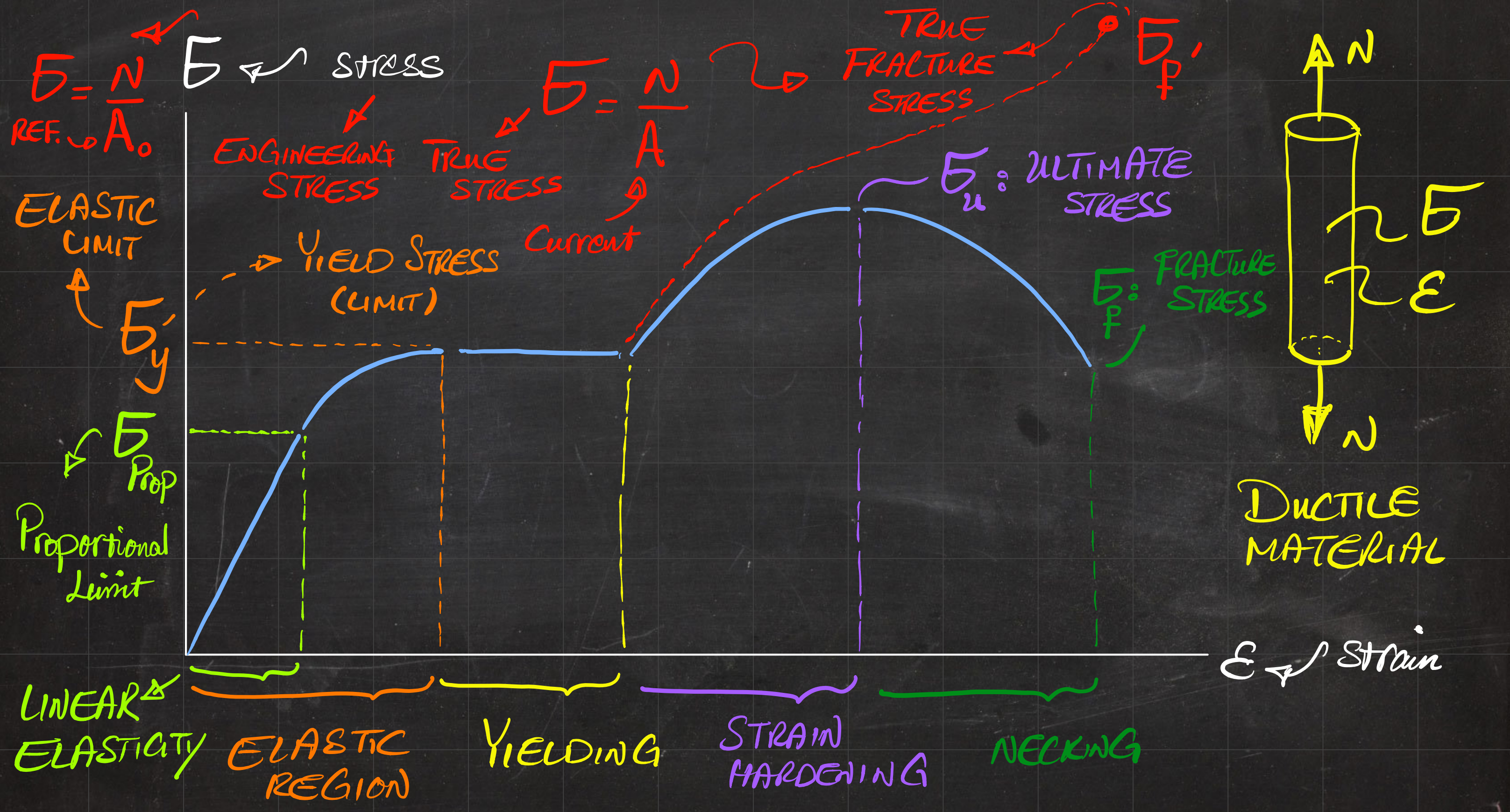


σ ↙ STRESS
ELASTIC LIMIT
YIELD STRESS (LIMIT)
σ_y
σ_{Prop}
Proportional Limit
σ_u : ULTIMATE STRESS
N
σ
ε
N
DUCTILE MATERIAL
ε ↙ Strain
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
STRAIN HARDENING

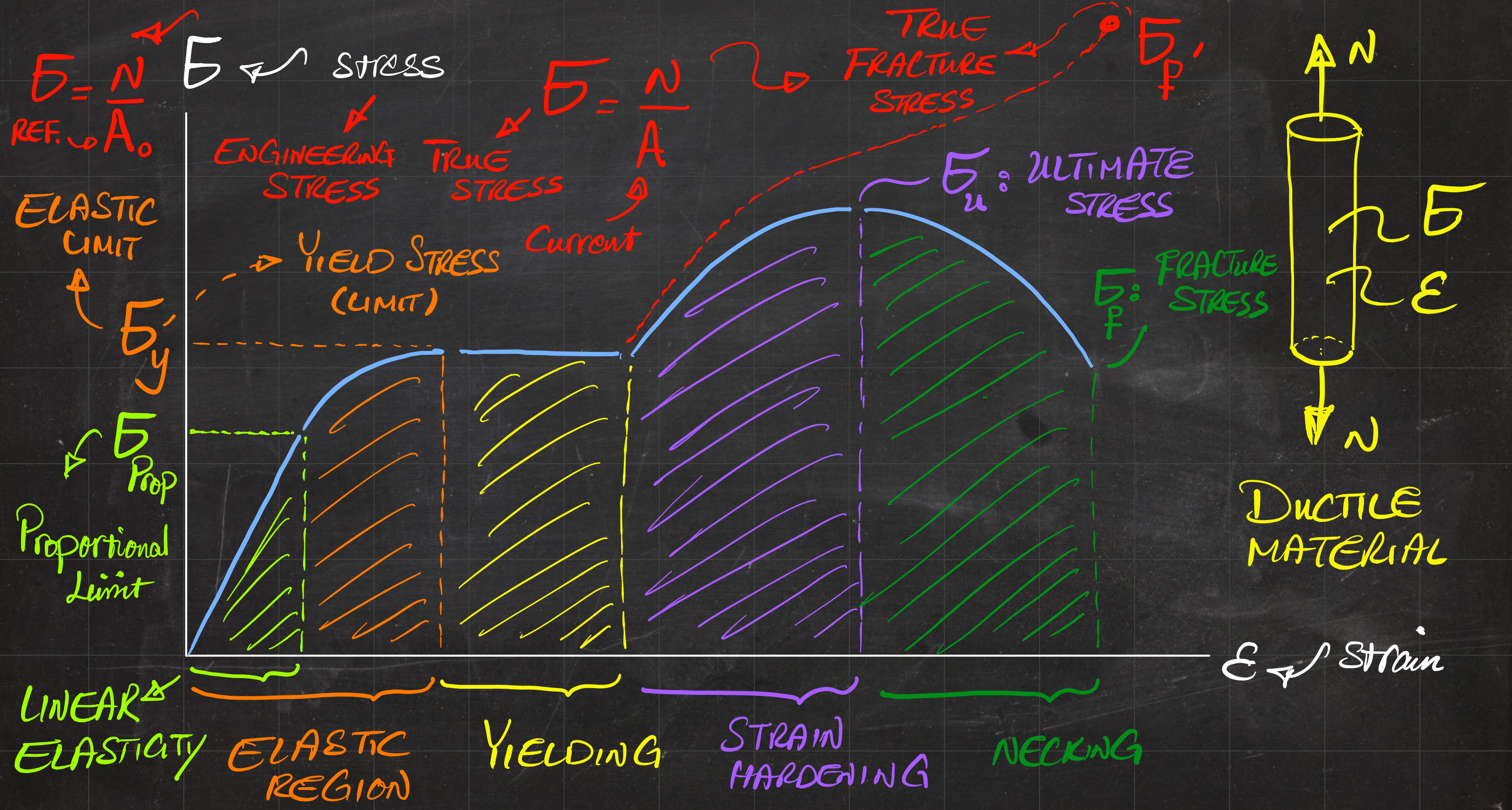


σ stress
ELASTIC LIMIT
YIELD STRESS (LIMIT)
σy
σProp
Proportional Limit
σu : ULTIMATE STRESS
σF : FRACTURE STRESS
N
σ
ε
N
DUCTILE MATERIAL
ε strain
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
STRAIN HARDENING
NECKING


σ = N / A₀
REF.
σ stress
ENGINEERING STRESS
ELASTIC LIMIT
σy
YIELD STRESS (LIMIT)
σProp
Proportional Limit
σu : ULTIMATE STRESS
σF : FRACTURE STRESS
N
σ
ε
N
DUCTILE MATERIAL
ε strain
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
STRAIN HARDENING
NECKING



$\sigma = \frac{N}{A_0}$
REF.
σ STRESS
ENGINEERING STRESS
TRUE STRESS
$\sigma = \frac{N}{A}$
Current
TRUE FRACTURE STRESS
σ_f'
N
ELASTIC LIMIT
σ_y
YIELD STRESS (LIMIT)
σ_u : ULTIMATE STRESS
σ
ε
σ_f : FRACTURE STRESS
σ_{Prop}
Proportional Limit
DUCTILE MATERIAL
ε Strain
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
STRAIN HARDENING
NECKING



σ = N / A₀
REF.
σ
STRESS
ENGINEERING STRESS
TRUE STRESS
σ = N / A
Current
TRUE FRACTURE STRESS
σ_f'
ELASTIC LIMIT
YIELD STRESS (LIMIT)
σ_y
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Proportional Limit
σ_u : ULTIMATE STRESS
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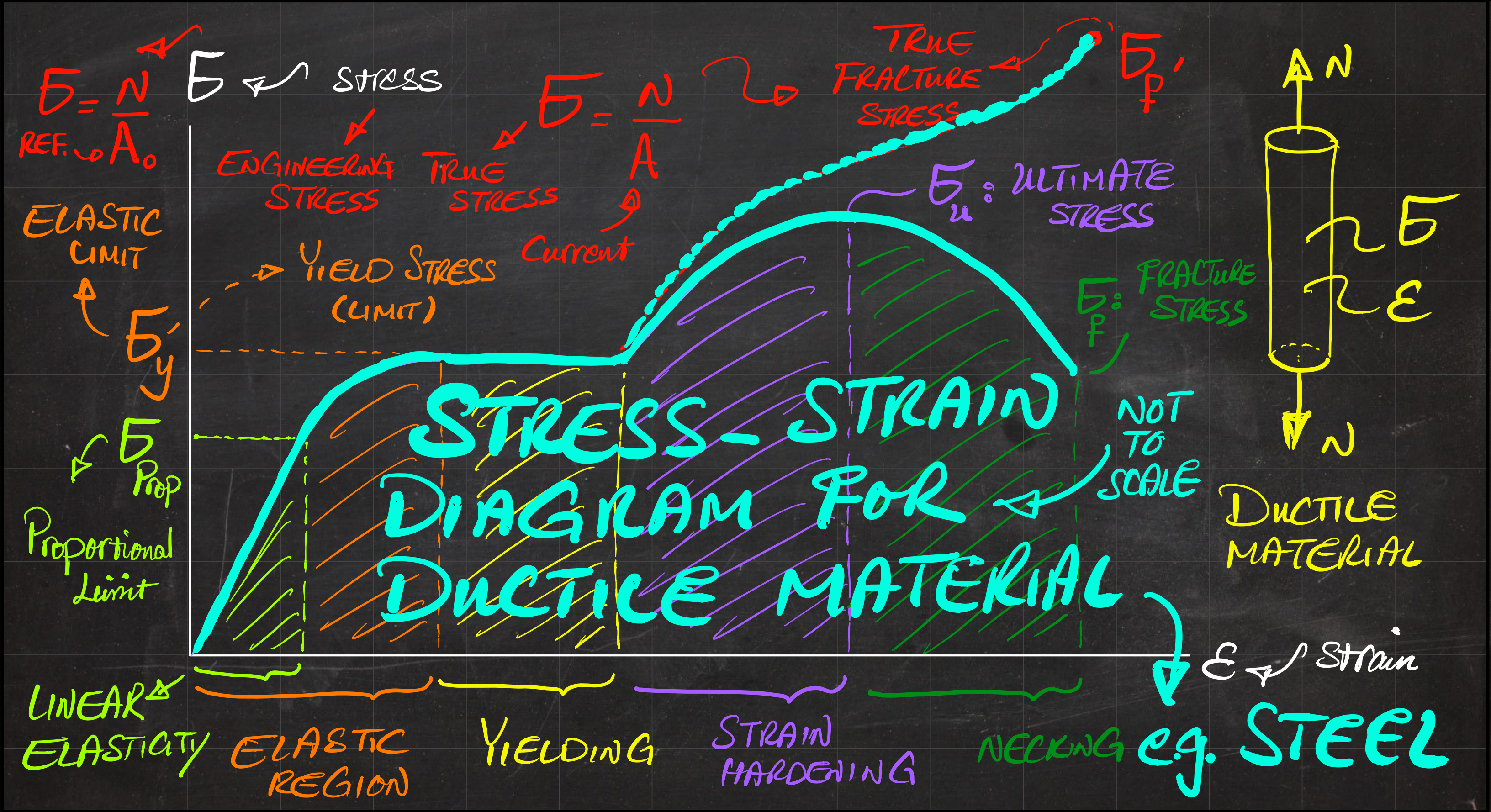
N
N
σ
ε
DUCTILE MATERIAL
ε
Strain
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
STRAIN HARDENING
NECKING


σ = N / A₀
REF. A₀
σ stress
ENGINEERING STRESS
TRUE STRESS
σ = N / A
A Current
TRUE FRACTURE STRESS
σ_f'
σ_u : ULTIMATE STRESS
σ_f : FRACTURE STRESS
ELASTIC LIMIT
YIELD STRESS (LIMIT)
σ_y
σ_prop
Proportional Limit
STRESS-STRAIN DIAGRAM FOR DUCTILE MATERIAL
NOT TO SCALE
N
σ
ε
N
DUCTILE MATERIAL
ε strain
LINEAR ELASTICITY
ELASTIC REGION
YIELDING
STRAIN HARDENING
NECKING
e.g. STEEL

Hooke's Law

STEEL　　　　CAST IRON

Applicable to Ductile Materials and Brittle Materials

(ISOTROPIC) LINEAR ELASTICITY $\Rightarrow$

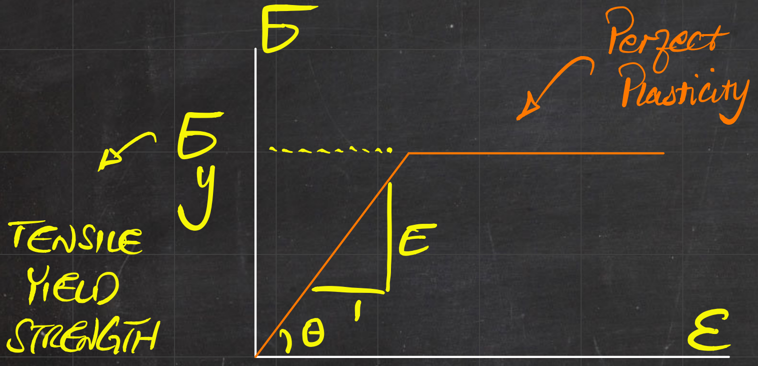
$$\sigma = \underbrace{E}_{\text{Elasticity Modulus / Young's Modulus}} \varepsilon$$

$$\tau = \underbrace{G}_{\text{Shear Modulus}} \gamma$$

STRESS IS PROPORTIONAL TO STRAIN

$\sigma \propto \varepsilon \,, \quad \tau \propto \gamma$

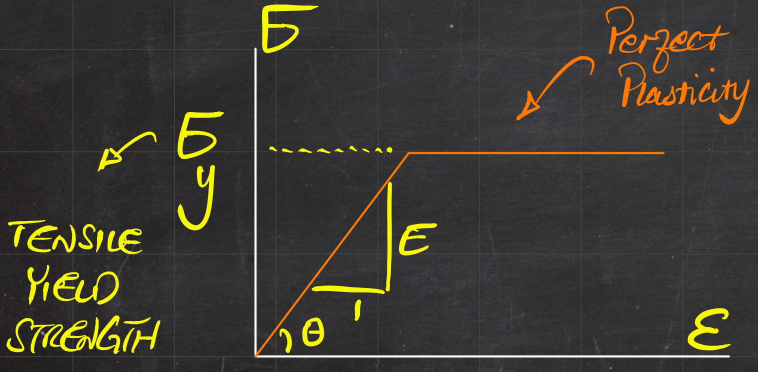
Isotropic Linear Elasticity $\rightarrow$ $\sigma = E\varepsilon$, $\tau = G\gamma$



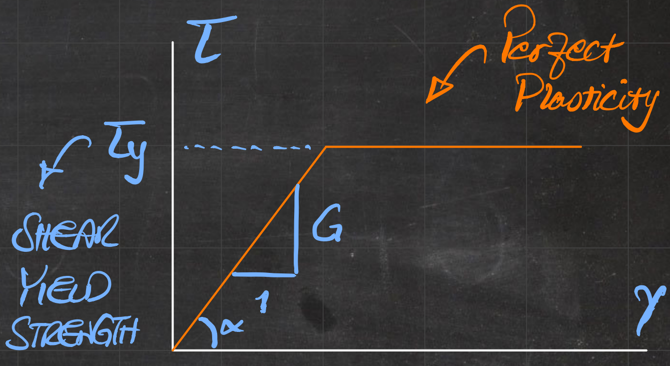
$\hookrightarrow$ $E = \tan\theta$

$E > 0$

Isotropic Linear Elasticity $\rightarrow$ $\sigma = E\varepsilon$, $\tau = G\gamma$



$\hookrightarrow E = \tan\theta$

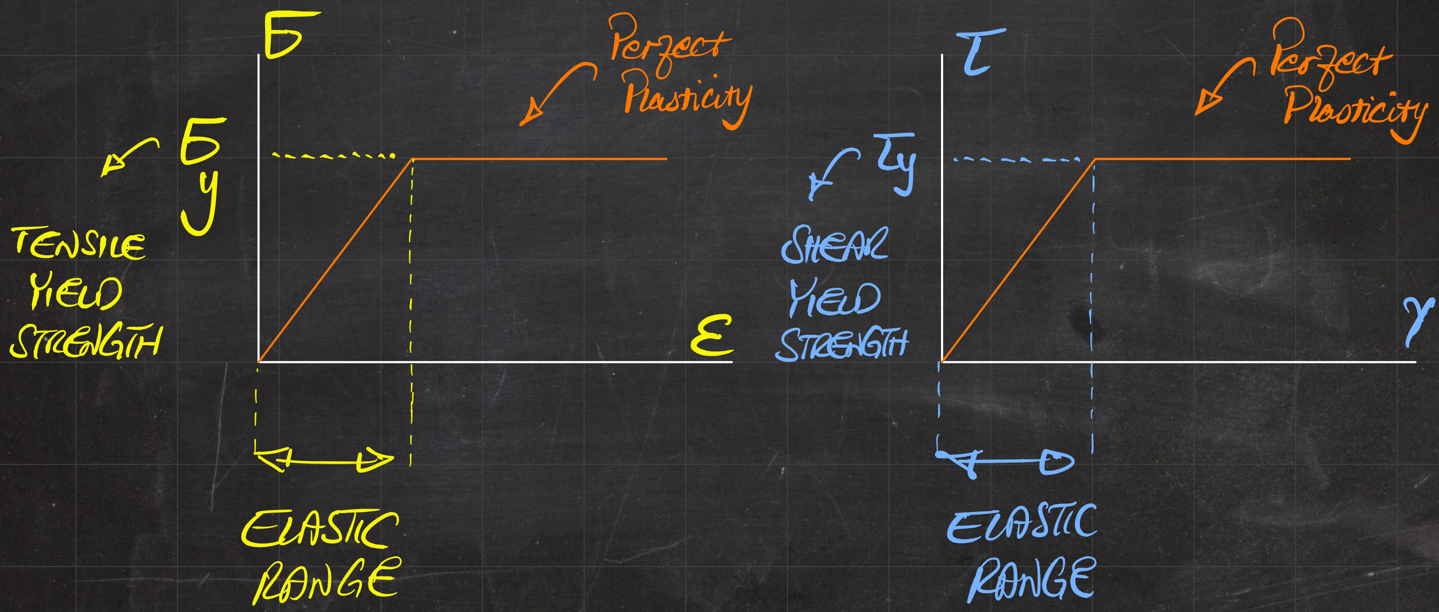


$\hookrightarrow G = \tan\alpha$

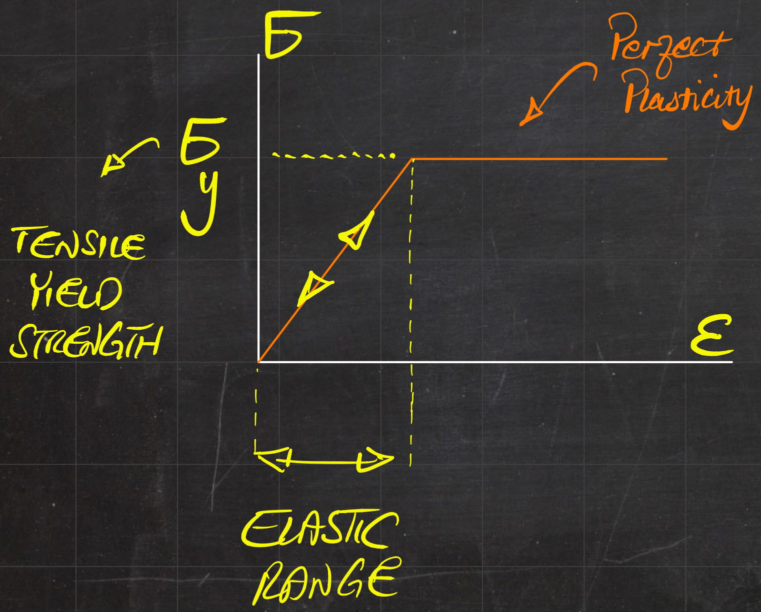
$E > 0$

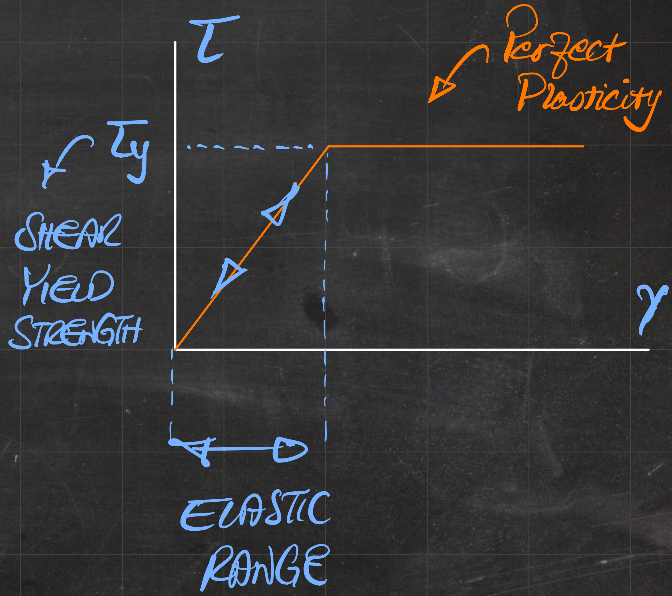
$G > 0$

Isotropic Linear Elasticity $\rightarrow$ $\sigma = E\varepsilon$, $\tau = G\gamma$

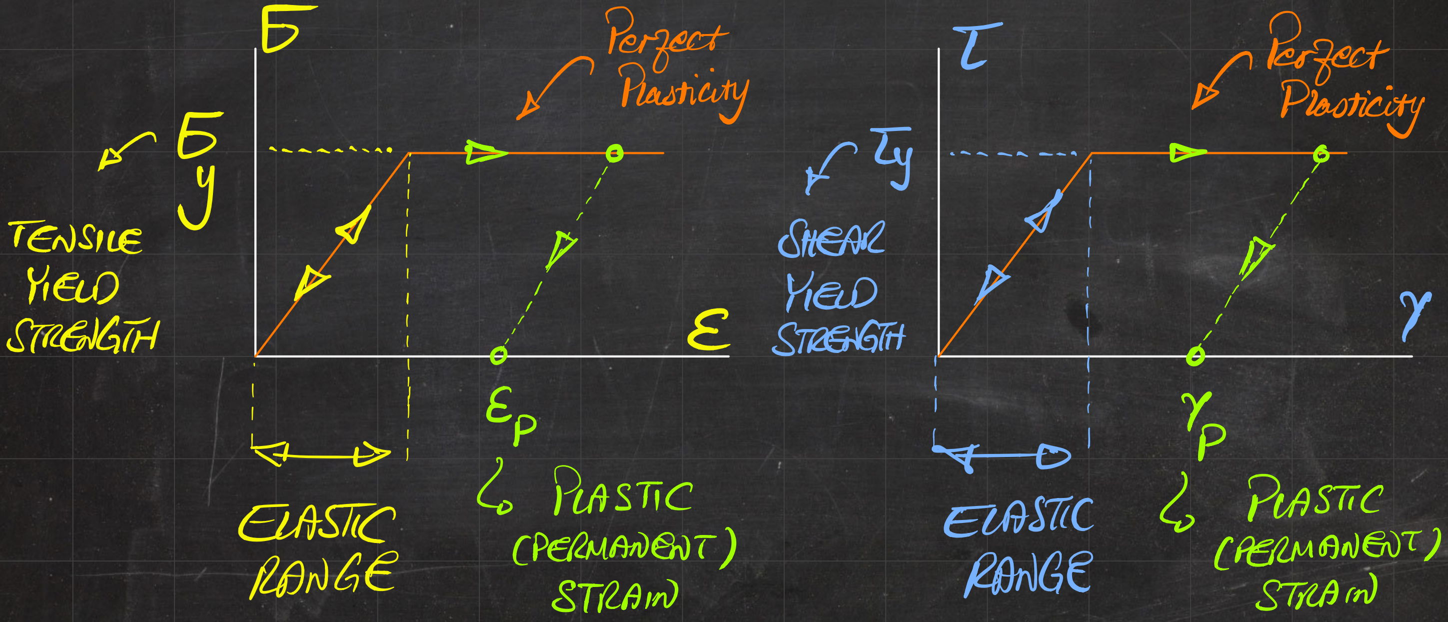


Isotropic Linear Elasticity $\rightarrow$ $\sigma = E\varepsilon$, $\tau = G\gamma$



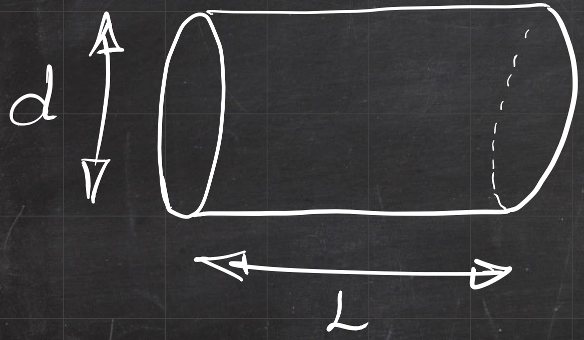


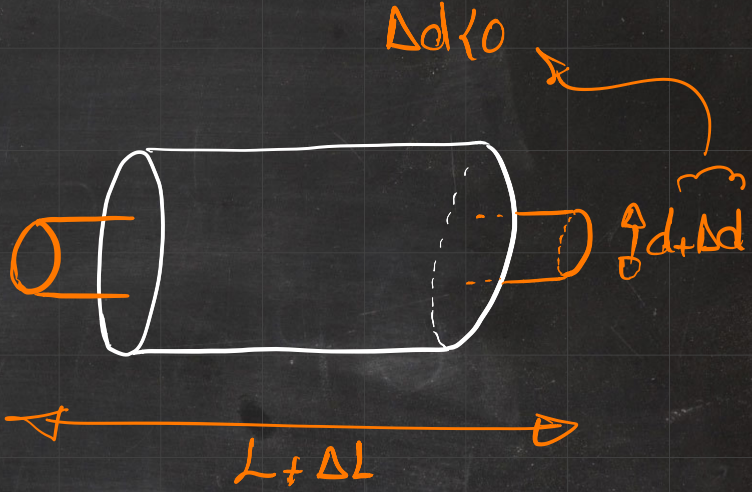
Isotropic Linear Elasticity $\to$ $\sigma = E\varepsilon$, $\tau = G\gamma$



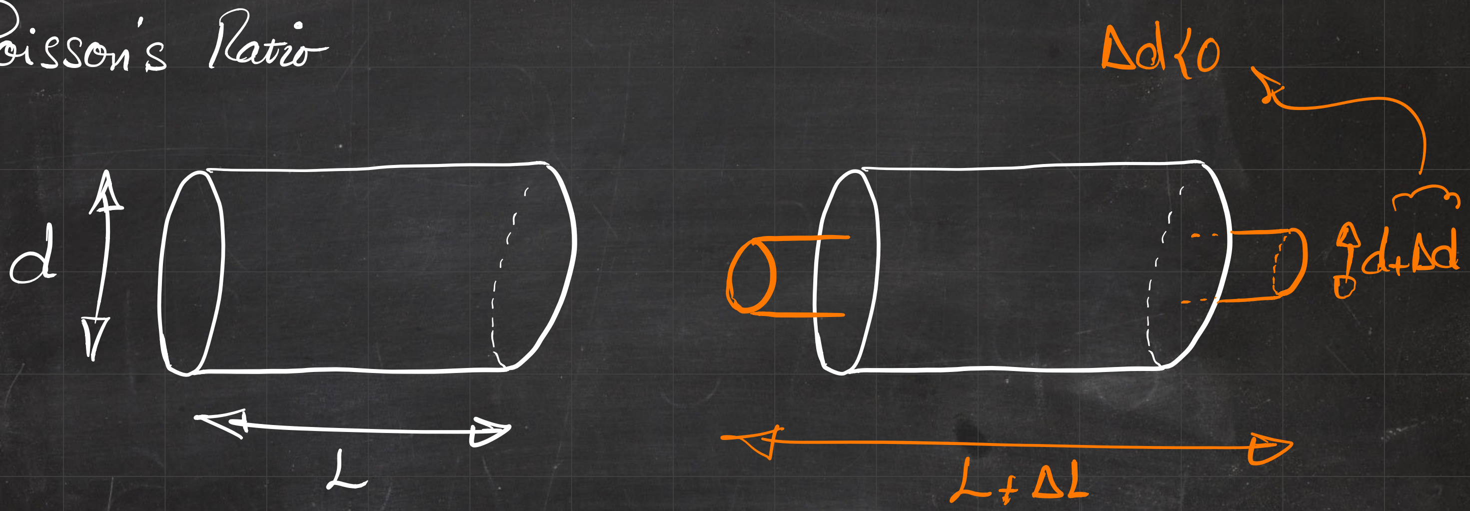
Poisson's Ratio

Poisson's Ratio





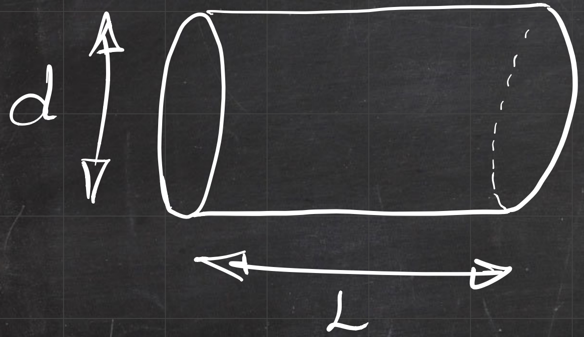
Poisson's Ratio

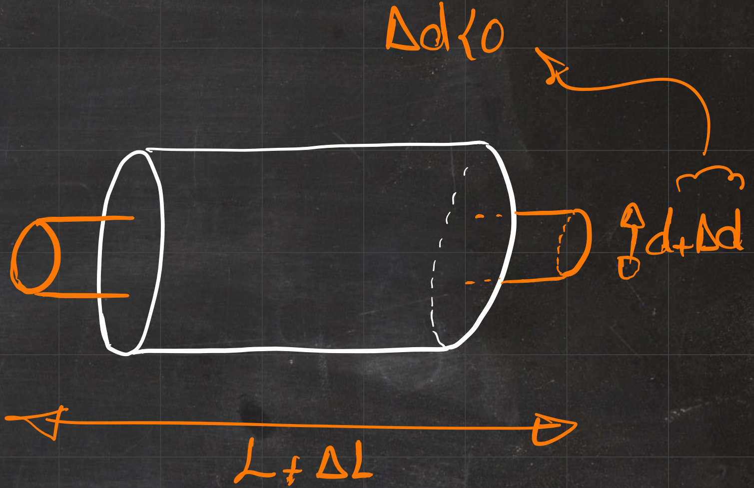


$$\varepsilon_{\text{Longitudinal}} = \frac{\Delta L}{L}$$ → Positive if extension
Negative if compression

$$\varepsilon_{\text{Lateral}} = \frac{\Delta d}{d}$$

Poisson's Ratio





$$\varepsilon_{\text{Longitudinal}} = \frac{\Delta L}{L}$$

→ Positive if extension
Negative if compression

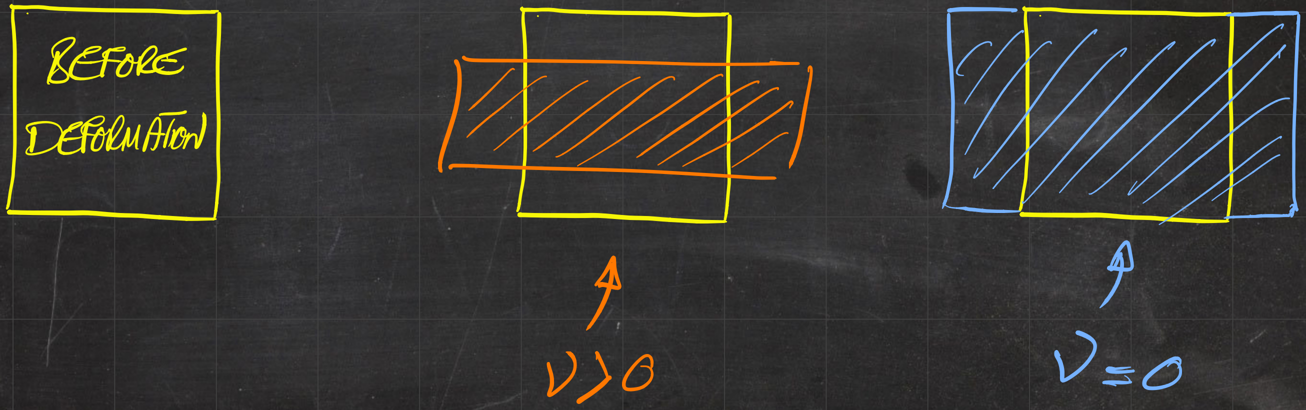
$$\varepsilon_{\text{Lateral}} = \frac{\Delta d}{d}$$

$$\nu = -\frac{\varepsilon_{\text{Lat.}}}{\varepsilon_{\text{Long.}}}$$

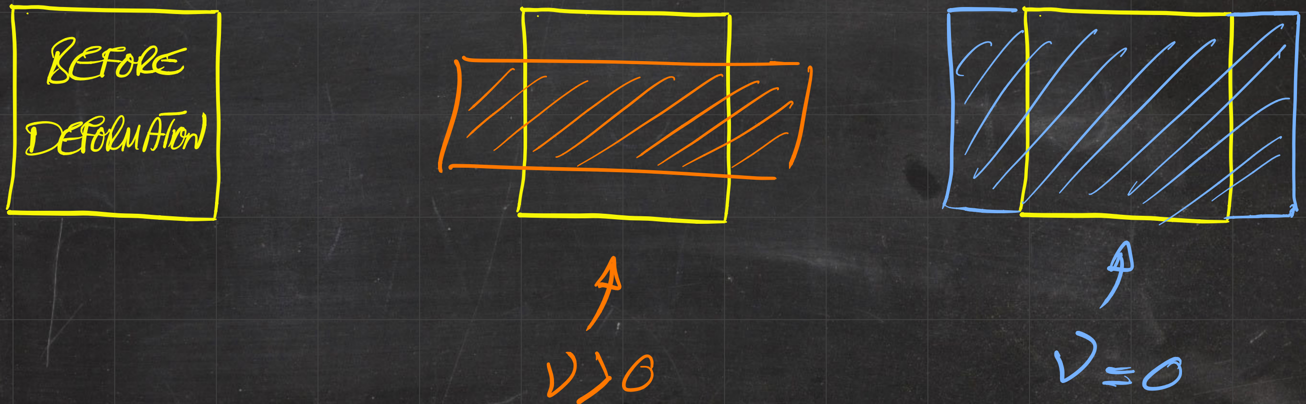
Poisson's Ratio

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

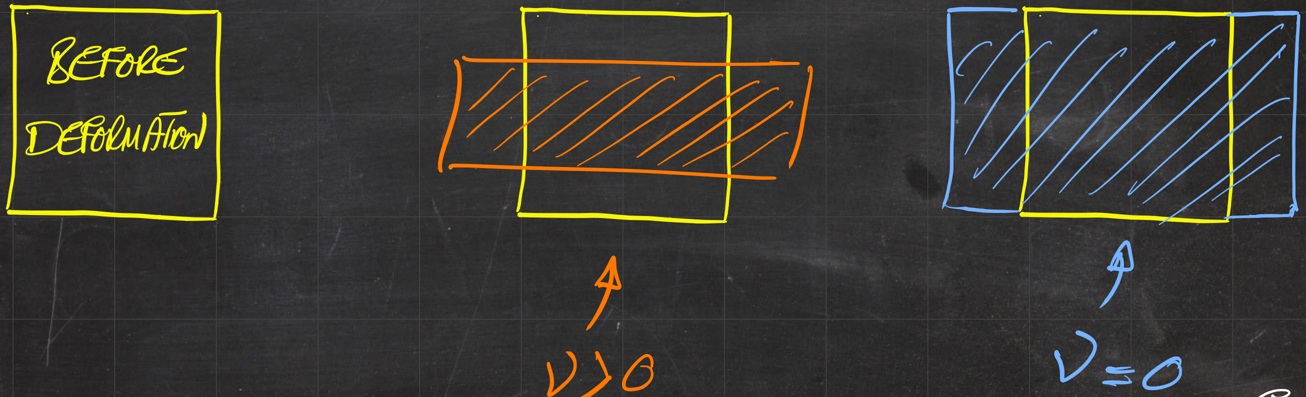


$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \quad \rightsquigarrow \quad \nu \leq 0.5$$



ν is a measure of material incompressibility

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$



ν is a measure of material incompressibility $\Rightarrow \nu = 0.5$ FULLY INCOMPRESSIBLE

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

$$\frac{1}{4} \leq \nu \leq \frac{1}{3} \rightsquigarrow \text{USUAL SOLIDS (IN THIS COURSE)}$$

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \longrightarrow \nu \leq 0.5$$

Cork

$$\frac{1}{4} \leq \nu \leq \frac{1}{3} \longrightarrow$$ USUAL SOLIDS (IN THIS COURSE)

$\nu = 0 \longrightarrow$ Cork $\longrightarrow$ think of bottles sealed

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

Cork

$\frac{1}{4} \leq \nu \leq \frac{1}{3} \rightsquigarrow$ USUAL SOLIDS (IN THIS COURSE)

$\nu = 0 \rightsquigarrow$ Cork $\rightsquigarrow$ think of bottles sealed

$\nu < 0 \rightsquigarrow$ auxetic materials

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

Cork

$$\frac{1}{4} \leq \nu \leq \frac{1}{3}$$ $\rightsquigarrow$ USUAL SOLIDS (IN THIS COURSE)

$\nu = 0$ $\rightsquigarrow$ Cork $\rightsquigarrow$ think of bottles sealed

$\nu < 0$ $\rightsquigarrow$ auxetic materials

$\nu = 0.5$ $\rightsquigarrow$ incompressibility (upper) limit

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

Cork

$\frac{1}{4} \leq \nu \leq \frac{1}{3}$ $\rightsquigarrow$ USUAL SOLIDS (IN THIS COURSE)

$\nu = 0$ $\rightsquigarrow$ Cork $\rightsquigarrow$ think of bottles sealed

$\nu < 0$ $\rightsquigarrow$ auxetic materials

$\nu = 0.5$ $\rightsquigarrow$ incompressibility (upper) limit

$\nu = -1$ $\rightsquigarrow$ thermodynamic (lower) limit

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

Cork

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$\nu = -1$ $\rightsquigarrow$ thermodynamic (lower) limit

$\Rightarrow -1 < \nu \leq 0.5$

$$\nu = -\frac{\varepsilon_{Lat.}}{\varepsilon_{Long.}} \rightsquigarrow \nu \leq 0.5$$

Cork

$\frac{1}{4} \leq \nu \leq \frac{1}{3}$ $\rightsquigarrow$ USUAL SOLIDS (IN THIS COURSE)

$\nu = 0$ $\rightsquigarrow$ Cork $\rightsquigarrow$ think of bottles sealed

$\nu < 0$ $\rightsquigarrow$ auxetic materials

$\nu = 0.5$ $\rightsquigarrow$ incompressibility (upper) limit

$\nu = -1$ $\rightsquigarrow$ thermodynamic (lower) limit

$$\Rightarrow -1 < \nu \leq 0.5$$

$$G = \frac{E}{2(1+\nu)} \Longleftrightarrow \nu = \frac{E}{2G} - 1$$

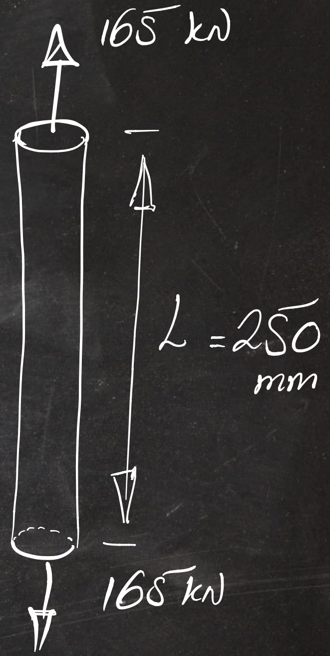
Exercise 1 . [similar to … P. 108 … 3.6]

AN ALUMINUM SPECIMEN SHOWN IN THE FIGURE HAS DIAMETER OF $d = 25$ mm AND LENGTH OF $L_0 = 250$ mm. THE FORCE $F = 165$ kN ELONGATES THE SPECIMEN 1.2 mm.

DETERMINE THE MODULUS OF ELASTICITY.

ALSO, DETERMINE BY HOW MUCH THE FORCE CAUSES THE DIAMETER OF SPECIMEN TO CONTRACT.

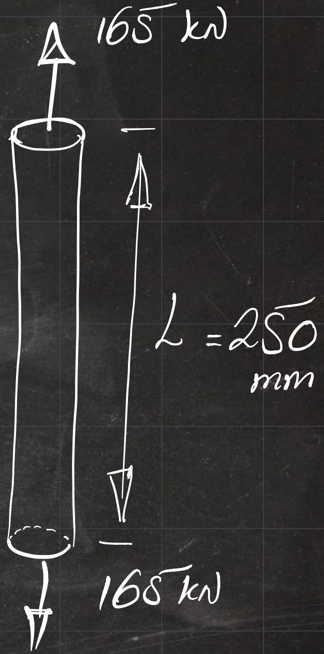
TAKE $G_{Al} = 26$ GPa AND $E_y = 440$ MPa.



$$\sigma = \frac{P}{A} = \frac{165 \times 10^3}{\frac{\pi}{4} \times 0.025^2} = 336.1 \text{ MPa}$$

DIMENSIONLESS

$$\varepsilon = \frac{\Delta L}{L} = \frac{1.2}{250} = 0.0048 \ \text{mm/mm}$$

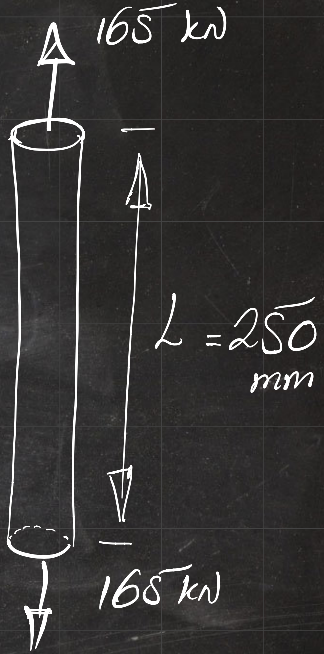


$$\sigma = \frac{P}{A} = \frac{165 \times 10^3}{\frac{\pi}{4} \times 0.025^2} = 336.1 \text{ MPa}$$

DIMENSIONLESS

$$\varepsilon = \frac{\Delta L}{L} = \frac{1.2}{250} = 0.0048 \ \boxed{\text{mm}/\text{mm}}$$

$$\underbrace{\sigma}_{336.1} < \underbrace{\sigma_y}_{440} \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{336.1 \times 10^6}{0.0048} = 70 \text{ GPa}$$



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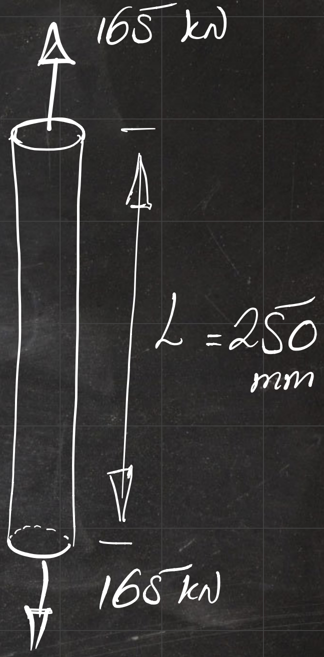
$$\nu = \frac{E}{2G} - 1 \Rightarrow \nu \simeq 0.35$$



$$\sigma = \frac{P}{A} = \frac{165 \times 10^3}{\frac{\pi}{4} \times 0.025^2} = 336.1 \text{ MPa}$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{1.2}{250} = 0.0048 \; \text{mm/mm}$$

DIMENSIONLESS

$$\underbrace{\sigma}_{336.1} < \underbrace{\sigma_y}_{440} \Rightarrow E = \frac{\sigma}{\varepsilon} = \frac{336.1 \times 10^6}{0.0048} = 70 \text{ GPa}$$

$$\nu = \frac{E}{2G} - 1 \Rightarrow \nu \simeq 0.35$$

$$\nu = -\frac{\varepsilon_{lat.}}{\varepsilon_{long.}}$$

$$\Rightarrow \varepsilon_{lat} \simeq -0.0017$$

$$\varepsilon_{lat} = \frac{\Delta d}{d}$$

$$\Rightarrow \Delta d \simeq \overset{(-)}{0.042} \; \text{mm}$$

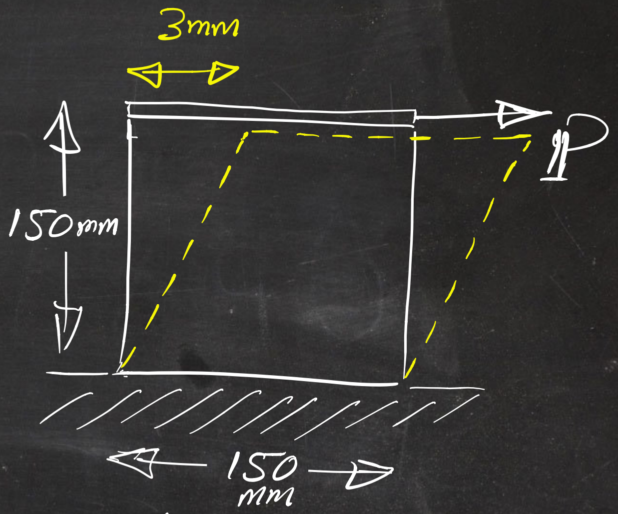
Contraction

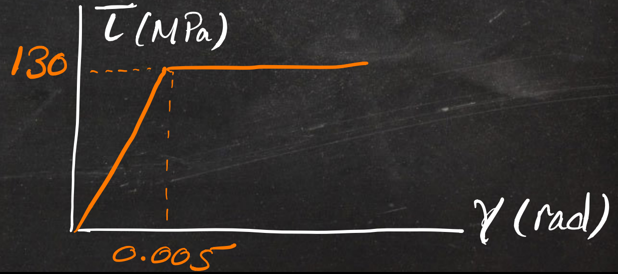
Exercise 2 . [similar to … P. 112 … F3-16]

A 20 mm - WIDE BLOCK IS BONDED TO RIGID PLATES AT ITS TOP AND BOTTOM. WHEN THE FORCE P IS APPLIED, THE BLOCK DEFORMS AS SHOWN IN THE FIGURE.

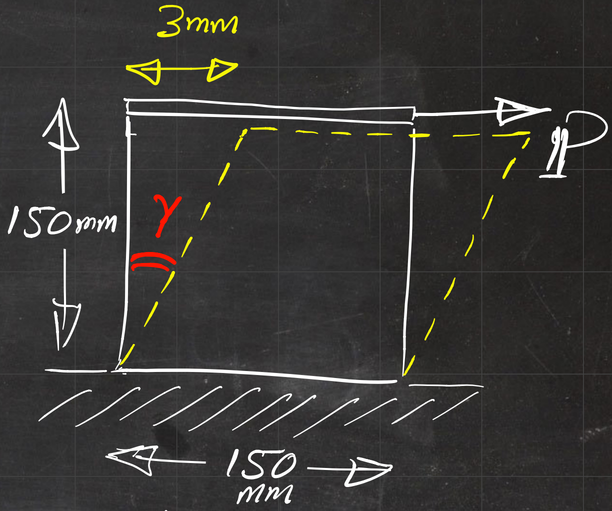
DETERMINE PERMANENT SHEAR STRAIN IN THE BLOCK AFTER P IS RELEASED.

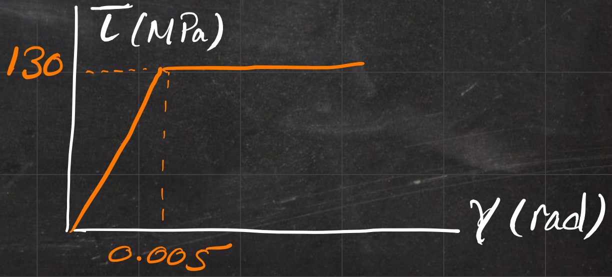
DETERMINE P.





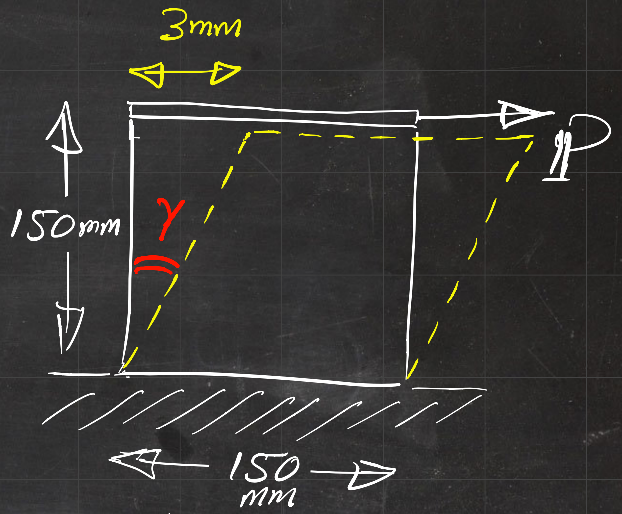
$$\gamma = \operatorname{atan}\frac{3}{150} = 0.02 \text{ rad}$$

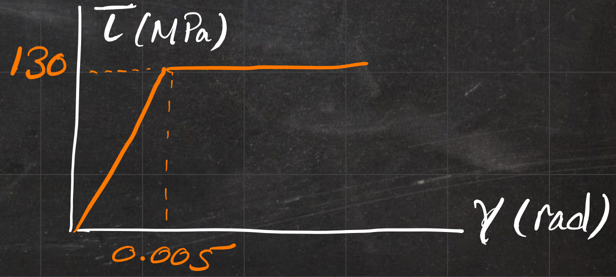




$$\gamma = \operatorname{atan}\frac{3}{150} = 0.02 \text{ rad}$$

$$\gamma_p = \gamma - 0.005 = 0.015 \text{ rad}$$

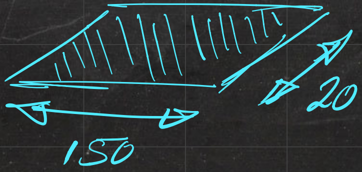


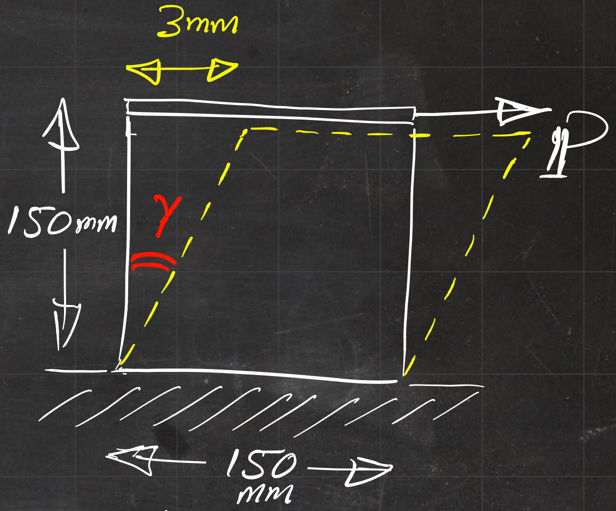


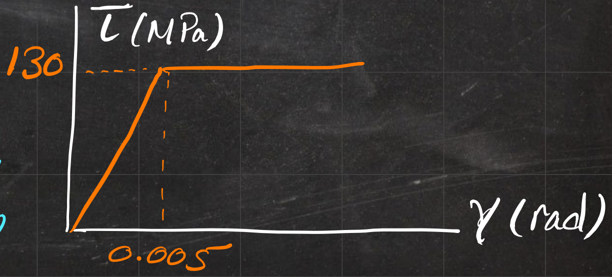
$$\gamma = \operatorname{atan}\frac{3}{150} = 0.02 \text{ rad}$$

$$\gamma_p = \gamma - 0.005 = 0.015 \text{ rad}$$

$$P = \underbrace{\tau}_{130\ \text{MPa}} \times \underbrace{A}_{150\ \text{mm} \times 20\ \text{mm}} = 390 \text{ kN}$$

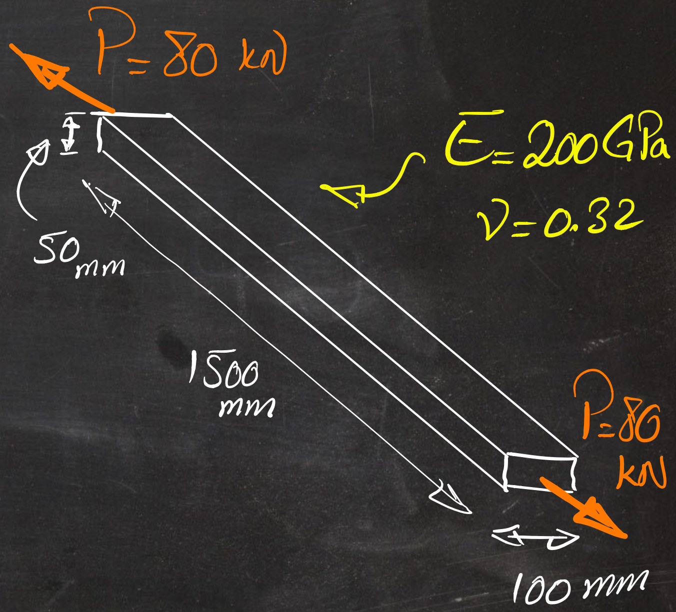




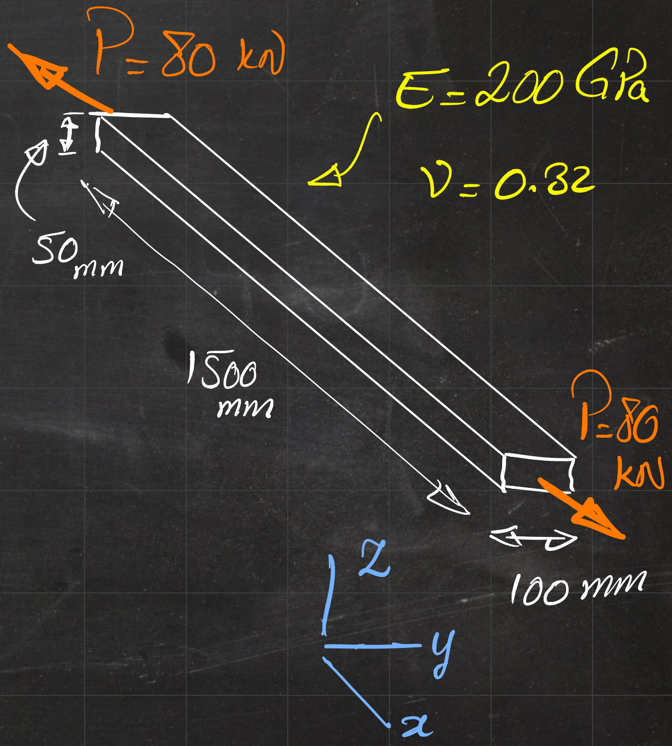


Exercise 3 . [similar to … P. 105 … 3.4]

A BAR MADE OF STEEL HAS THE DIMENSIONS SHOWN IN THE FIGURE. IF AN AXIAL FORCE OF $P = 80\,kN$ IS APPLIED TO THE BAR, DETERMINE THE CHANGE IN ITS LENGTH AND THE CHANGE IN THE DIMENSIONS OF ITS CROSS SECTION. THE BAR BEHAVES ELASTICALLY.

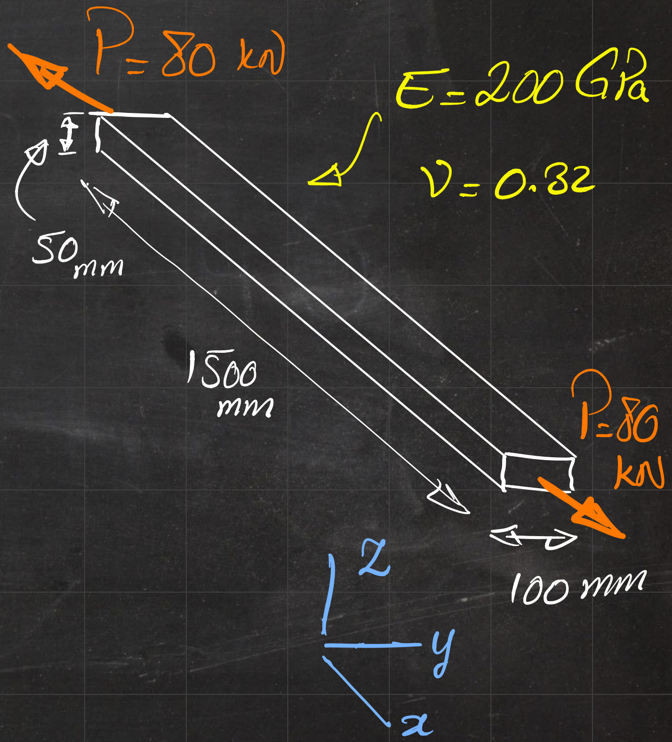


$$\sigma_x = \frac{N}{A} = \frac{80 \times 10^3}{0.1 \times 0.05} = 16\ \text{MPa}$$



$$\sigma_x = \frac{N}{A} = \frac{80 \times 10^3}{0.1 \times 0.05} = 16\ \text{MPa}$$

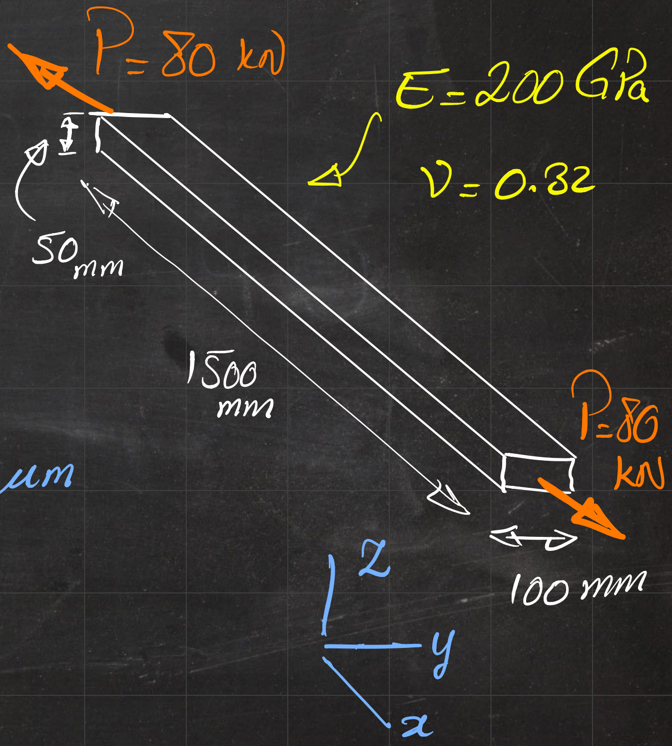
$$\varepsilon_x = \frac{\sigma_x}{E} \Rightarrow \varepsilon_x = 80 \times 10^{-6}$$



$$\sigma_x = \frac{N}{A} = \frac{80 \times 10^{3}}{0.1 \times 0.05} = 16\ \mathrm{MPa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} \Rightarrow \varepsilon_x = 80 \times 10^{-6}$$

$$\Delta L_x = \varepsilon_x L = 80 \times 10^{-6} \times 1.5 = 120\ \mu m$$

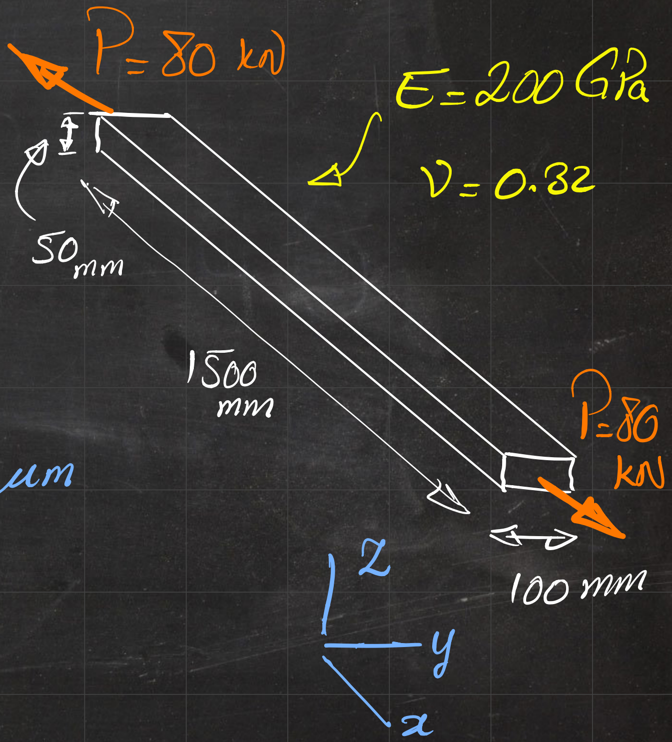


$$\sigma_x = \frac{N}{A} = \frac{80 \times 10^3}{0.1 \times 0.05} = 16\ \mathrm{MPa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} \Rightarrow \varepsilon_x = 80 \times 10^{-6}$$

$$\Delta L_x = \varepsilon_x L = 80 \times 10^{-6} \times 1.5 = 120\ \mu m$$

$$\varepsilon_y = \varepsilon_z = -\nu \varepsilon_x = -25.6 \times 10^{-6}$$



$$\sigma_x = \frac{N}{A} = \frac{80\times10^3}{0.1\times0.05} = 16\ \text{MPa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} \Rightarrow \varepsilon_x = 80\times10^{-6}$$

$$\Delta L_x = \varepsilon_x L = 80\times10^{-6}\times1.5 = 120\ \mu m$$

$$\varepsilon_y = \varepsilon_z = -\nu\,\varepsilon_x = -25.6\times10^{-6}$$

100 mm

$$\Delta L_y = \varepsilon_y L_y$$

$$\leadsto -2.56\ \mu m$$

$$\sigma_x = \frac{N}{A} = \frac{80 \times 10^3}{0.1 \times 0.05} = 16\ \mathrm{MPa}$$

$$\varepsilon_x = \frac{\sigma_x}{E} \Rightarrow \varepsilon_x = 80 \times 10^{-6}$$

$$\Delta L_x = \varepsilon_x L = 80 \times 10^{-6} \times 1.5 = 120\ \mu m$$

$$\varepsilon_y = \varepsilon_z = -\nu \varepsilon_x = -25.6 \times 10^{-6}$$

$$\Delta L_y = \varepsilon_y L_y$$

100 mm

$\hookrightarrow -2.56\ \mu m$

$$\Delta L_z = \varepsilon_z L_z$$

50 mm

$\hookrightarrow -1.28\ \mu m$

MECHANICS AND MATERIALS I

Mechanical properties

Chap. 3

[Hibbeler 9th edition]

MECHANICS AND MATERIALS I

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