MECHANICS AND MATERIALS I

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Buckling of Columns

Sections ... 13.1 – 13.3

Chap. 13

[Hibbeler 9th edition]

Chapter 13

The columns of this water tank are braced at their length in order to reduce their chance of buckling.

Buckling of Columns

CHAPTER OBJECTIVES

- In this chapter, we will discuss the behavior of columns and indicate some of the methods used for their design. The chapter begins with a general discussion of buckling, followed by a determination of the axial load needed to buckle a so-called ideal column. Afterwards, a more realistic analysis is considered, which accounts for any bending of the column. Also, inelastic buckling of a column is presented as a special topic. At the end of the chapter we will discuss some of the methods used to design both concentrically and eccentrically loaded columns made of common engineering materials.

13.1 Critical Load

Whenever a member is designed, it is necessary that it satisfy specific strength, deflection, and stability requirements. In the preceding chapters, we have discussed some of the methods used to determine a member's strength and deflection, while assuming that the member was always in stable equilibrium. Some members, however, may be subjected to compressive loadings, and if these members are long and slender the loading may be large enough to cause the member to deflect laterally or sidesway. To be specific, long slender members subjected to an axial compressive force are called ***columns***, and the lateral deflection that occurs is called ***buckling***. Quite often the buckling of a column can lead to a sudden and dramatic failure of a structure or mechanism, and as a result, special attention must be given to the design of columns so that they can safely support their intended loadings without buckling.

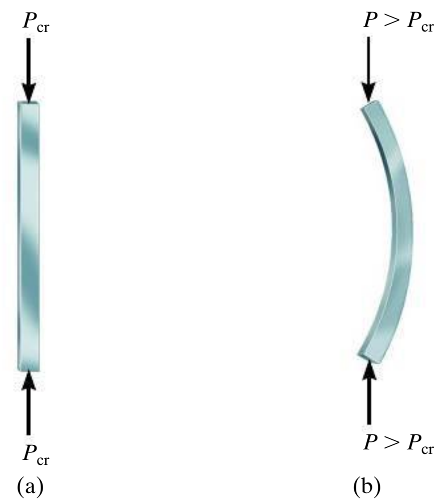

Fig. 13–1

The maximum axial load that a column can support when it is on the *verge* of buckling is called the ***critical load***, $P_{cr,}$ Fig. 13–1*a*. Any additional loading will cause the column to buckle and therefore deflect laterally as shown in Fig. 13–1*b*. In order to better understand the nature of this instability, consider a two-bar mechanism consisting of weightless bars that are rigid and pin connected as shown in Fig. 13–2*a*. When the bars are in the vertical position, the spring, having a stiffness k, is unstretched, and a *small* vertical force **P** is applied at the top of one of the bars. We can upset this equilibrium position by displacing the pin at A by a small amount Δ, Fig. 13–2*b*. As shown on the free-body diagram of the pin when the bars are displaced, Fig. 13–2*c*, the spring will produce a restoring force $F = k\Delta$, while the applied load **P** develops two horizontal components, $P_x = P\tan\theta$, which tend to push the pin (and the bars) further out of equilibrium. Since θ is small, $\Delta \approx \theta(L/2)$ and $\tan\theta \approx \theta$. Thus the *restoring* spring force becomes $F = k\theta L/2$, and the *disturbing* force is $2P_x = 2P\theta$.

If the restoring force is greater than the disturbing force, that is, $k\theta L/2 > 2P\theta$, then, noticing that θ cancels out, we can solve for P, which gives

$$P < \frac{kL}{4} \qquad \text{stable equilibrium}$$

This is a condition for *stable equilibrium* since the force developed by the spring would be adequate to restore the bars back to their vertical position. However, if $kL\theta/2 < 2P\theta$, or

$$P > \frac{kL}{4} \qquad \text{unstable equilibrium}$$

then the mechanism would be in *unstable equilibrium*. In other words, if this load **P** is applied, and a slight displacement occurs at A, the mechanism will tend to move out of equilibrium and not be restored to its original position.

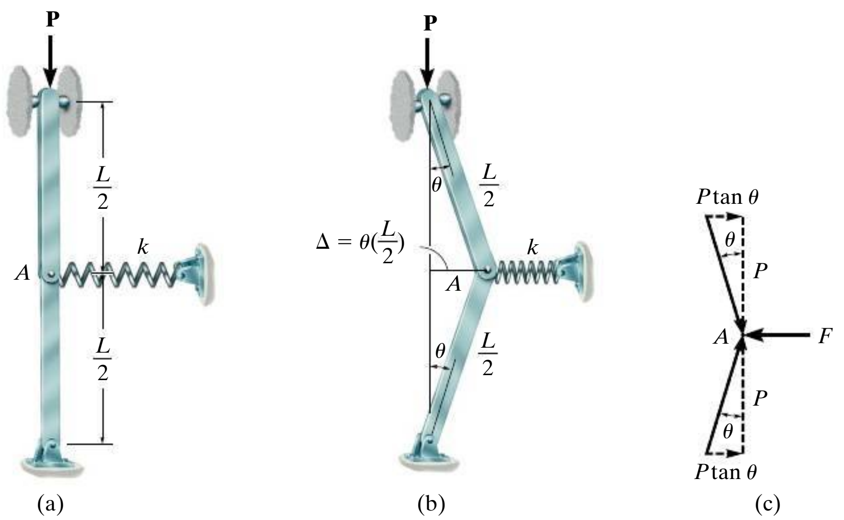

Fig. 13–2

The intermediate value of P, which requires $kL\theta/2 = 2P\theta$, is the *critical load*. Here

$$P_{cr} = \frac{kL}{4} \qquad \text{neutral equilibrium}$$

This loading represents a case of the mechanism being in *neutral equilibrium*. Since P_{cr} is *independent* of the (small) displacement θ of the bars, any slight disturbance given to the mechanism will not cause it to move further out of equilibrium, nor will it be restored to its original position. Instead, the bars will *remain* in the deflected position.

These three different states of equilibrium are represented graphically in Fig. 13–3. The transition point where the load is equal to the critical value $P = P_{cr}$ is called the *bifurcation point*. At this point the mechanism will be in equilibrium for any *small value* of θ, measured either to the right or to the left of the vertical. Physically, P_{cr} represents the load for which the mechanism is on the verge of buckling. It is quite reasonable to determine this value by assuming *small displacements* as done here; however, it should be understood that P_{cr} may *not* be the largest value of P that the mechanism can support. Indeed, if a larger load is placed on the bars, then the mechanism may have to undergo a further deflection before the spring is compressed or elongated enough to hold the mechanism in equilibrium.

Like the two-bar mechanism just discussed, the critical buckling loads on columns supported in various ways can be obtained, and the method used to do this will be explained in the next section. Although in engineering design the critical load may be considered to be the largest load the column can support, realize that, like the two-bar mechanism in the deflected or buckled position, a column may actually support an

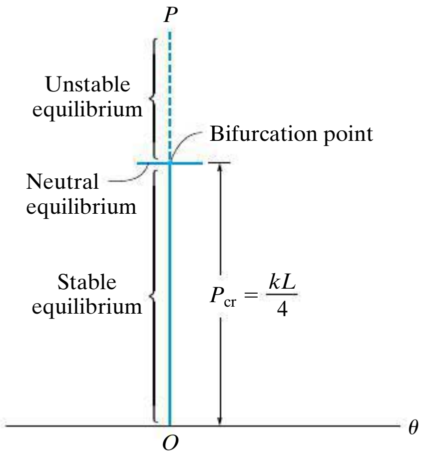

Fig. 13–3

even greater load than P_{cr}. Unfortunately, however, this loading may require the column to undergo a *large* deflection, which is generally not tolerated in engineering structures or machines. For example, it may take only a few newtons of force to buckle a meterstick, but the additional load it may support can be applied only after the stick undergoes a relatively large lateral deflection.

13.2 Ideal Column with Pin Supports

The dramatic failure of this off-shore oil platform was caused by the horizontal forces of hurricane winds, which led to buckling of its supporting columns.

In this section we will determine the critical buckling load for a column that is pin supported as shown in Fig. 13–4*a*. The column to be considered is an ***ideal column***, meaning one that is perfectly straight before loading, is made of homogeneous material, and upon which the load is applied through the centroid of the cross section. It is further assumed that the material behaves in a linear-elastic manner and that the column buckles or bends in a single plane. In reality, the conditions of column straightness and load application are never accomplished; however, the analysis to be performed on an "ideal column" is similar to that used to analyze initially crooked columns or those having an eccentric load application. These more realistic cases will be discussed later in this chapter.

Since an ideal column is straight, theoretically the axial load P could be increased until failure occurs by either fracture or yielding of the material. However, when the critical load P_{cr} is reached, the column will be on the verge of becoming unstable, so that a small lateral force F, Fig. 13–4*b*, will cause the column to remain in the deflected position when F is removed, Fig. 13–4*c*. Any slight reduction in the axial load P from P_{cr} will allow the column to straighten out, and any slight increase in P, beyond P_{cr}, will cause further increases in lateral deflection.

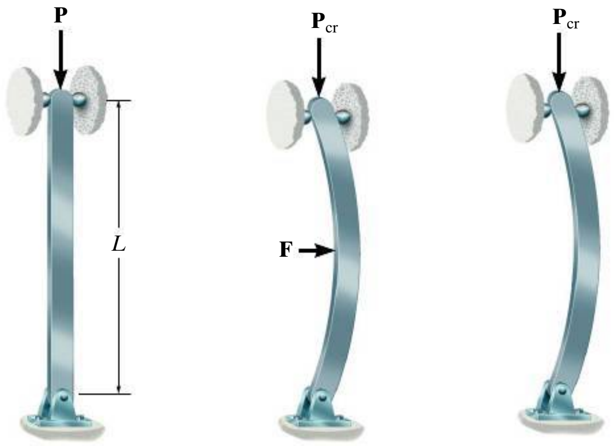

Fig. 13–4

Whether or not a column will remain stable or become unstable when subjected to an axial load will depend on its ability to restore itself, which is based on its resistance to bending. Hence, in order to determine the critical load and the buckled shape of the column, we will apply Eq. 12–10, which relates the internal moment in the column to its deflected shape, i.e.,

$$EI\frac{d^2v}{dx^2} = M \tag{13–1}$$

Recall that this equation assumes that the slope of the elastic curve is small and that deflections occur only by bending. When the column is in its deflected position, Fig. 13–5*a*, the internal bending moment can be determined by using the method of sections. The free-body diagram of a segment in the deflected position is shown in Fig. 13–5*b*. Here both the deflection v and the internal moment M are shown in the *positive direction* according to the sign convention used to establish Eq. 13–1. Moment equilibrium requires $M = -Pv$. Thus Eq. 13–1 becomes

$$EI\frac{d^2v}{dx^2} = -Pv$$

$$\frac{d^2v}{dx^2} + \left(\frac{P}{EI}\right)v = 0 \tag{13–2}$$

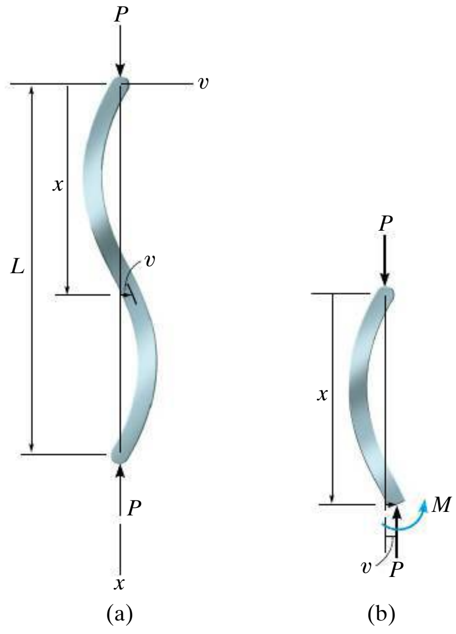

Fig. 13–5

This is a homogeneous, second-order, linear differential equation with constant coefficients. It can be shown by using the methods of differential equations, or by direct substitution into Eq. 13–2, that the general solution is

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}\,x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}\,x\right) \tag{13–3}$$

The two constants of integration are determined from the boundary conditions at the ends of the column. Since $v = 0$ at $x = 0$, then $C_2 = 0$. And since $v = 0$ at $x = L$, then

$$C_1 \sin\left(\sqrt{\frac{P}{EI}}\,L\right) = 0$$

This equation is satisfied if $C_1 = 0$; however, then $v = 0$, which is a *trivial solution* that requires the column to always remain straight, even though the load may cause the column to become unstable. The other possibility is for

$$\sin\left(\sqrt{\frac{P}{EI}}\,L\right) = 0$$

which is satisfied if

$$\sqrt{\frac{P}{EI}}\,L = n\pi$$

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or

$$P = \frac{n^2\pi^2 EI}{L^2} \quad n = 1, 2, 3, \ldots \tag{13–4}$$

The *smallest value* of P is obtained when $n = 1$, so the *critical load* for the column is therefore*

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

This load is sometimes referred to as the *Euler load*, named after the Swiss mathematician Leonhard Euler, who originally solved this problem in 1757. The corresponding buckled shape is defined by the equation

$$v = C_1 \sin\frac{\pi x}{L}$$

Here the constant C_1 represents the maximum deflection, v_{max}, which occurs at the midpoint of the column, Fig. 13–5*c*. Specific values for C_1 cannot be obtained, since the exact deflected form for the column is unknown once it has buckled. It has been assumed, however, that this deflection is small.

Note that the critical load is independent of the strength of the material; rather it only depends on the column's dimensions (I and L) and the material's stiffness or modulus of elasticity E. For this reason, as far as elastic buckling is concerned, columns made, for example, of high-strength steel offer no advantage over those made of lower-strength steel, since the modulus of elasticity for both is approximately the same. Also note that the load-carrying capacity of a column will increase as the moment of inertia of the cross section increases. Thus, efficient columns are designed so that most of the column's cross-sectional area is located as far away as possible from the principal centroidal axes for the section. This is why hollow sections such as tubes are more economical than solid sections. Furthermore, wide-flange sections, and columns that are "built up" from channels, angles, plates, etc., are better than sections that are solid and rectangular.

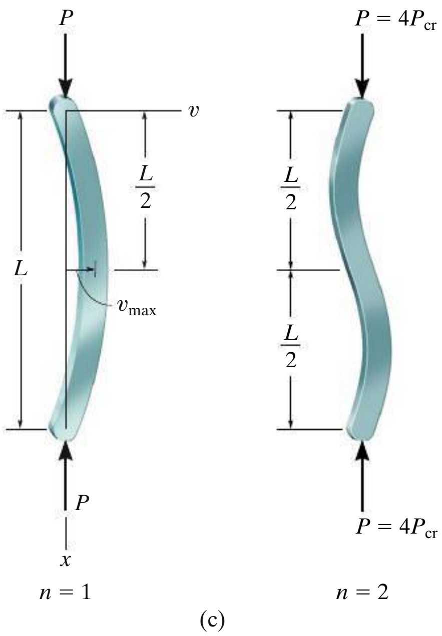

Fig. 13–5 (cont.)

*n represents the number of curves in the deflected shape of the column. For example, if $n = 2$, then *two* curves will appear, Fig. 13–5*c*. Here the critical load is 4 P_{cr} just prior to buckling, which practically speaking will not exist.

It is also important to realize that a column will buckle about the principal axis of the cross section having the ***least moment of inertia*** (the weakest axis) provided it is supported the same way about each axis. For example, a column having a rectangular cross section, like a meter stick, as shown in Fig. 13–6, will buckle about the *a–a* axis, not the *b–b* axis. As a result, engineers usually try to achieve a balance, keeping the moments of inertia the same in all directions. Geometrically, then, circular tubes would make excellent columns. Also, square tubes or those shapes having $I_x \approx I_y$ are often selected for columns.

Fig. 13–6

Summarizing the above discussion, the buckling equation for a pin-supported long slender column can be rewritten, and the terms defined as follows:

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}} \qquad (13\text{–}5)$$

where

P_{cr} = critical or maximum axial load on the column just before it begins to buckle. This load must *not* cause the stress in the column to exceed the proportional limit

E = modulus of elasticity for the material

I = *least* moment of inertia for the column's cross-sectional area

L = unsupported length of the column, whose ends are pinned

For purposes of design, the above equation can also be written in a more useful form by expressing $I = Ar^2$, where A is the cross-sectional area and r is the ***radius of gyration*** of the cross-sectional area. Thus,

$$P_{cr} = \frac{\pi^2 E(Ar^2)}{L^2}$$

$$\left(\frac{P}{A}\right)_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

or

$$\boxed{\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}} \qquad (13\text{–}6)$$

Failure of this crane boom was caused by the localized buckling of one of its tubular struts.

Here

σ_{cr} = critical stress, which is an average normal stress in the column just before the column buckles. This stress is an *elastic stress* and therefore $\sigma_{cr} \leq \sigma_Y$

E = modulus of elasticity for the material

L = unsupported length of the column, whose ends are pinned

r = *smallest* radius of gyration of the column, determined from $r = \sqrt{I/A}$, where I is the *least* moment of inertia of the column's cross-sectional area A

The geometric ratio L/r in Eq. 13–6 is known as the ***slenderness ratio***. It is a measure of the column's flexibility, and as will be discussed later, it serves to classify columns as long, intermediate, or short.

It is possible to graph Eq. 13–6 using axes that represent the critical stress versus the slenderness ratio. Examples of this graph for columns made of a typical structural steel and aluminum alloy are shown in Fig. 13–7. Note that the curves are hyperbolic and are valid only for critical stresses below the material's yield point (proportional limit), since the material must behave elastically. For the steel the yield stress is $(\sigma_Y)_{st} = 36$ ksi $[E_{st} = 29(10^3)$ ksi$]$, and for the aluminum it is $(\sigma_Y)_{al} = 27$ ksi $[E_{al} = 10(10^3)$ ksi$]$. Substituting $\sigma_{cr} = \sigma_Y$ into Eq. 13–6, the *smallest* allowable slenderness ratios for the steel and aluminum columns are therefore $(L/r)_{st} = 89$ and $(L/r)_{al} = 60.5$, respectively. Thus, for a steel column, if $(L/r)_{st} \geq 89$, Euler's formula can be used to determine the critical load since the stress in the column remains elastic. On the other hand, if $(L/r)_{st} < 89$, the column's stress will exceed the yield point before buckling can occur, and therefore the Euler formula is not valid in this case.

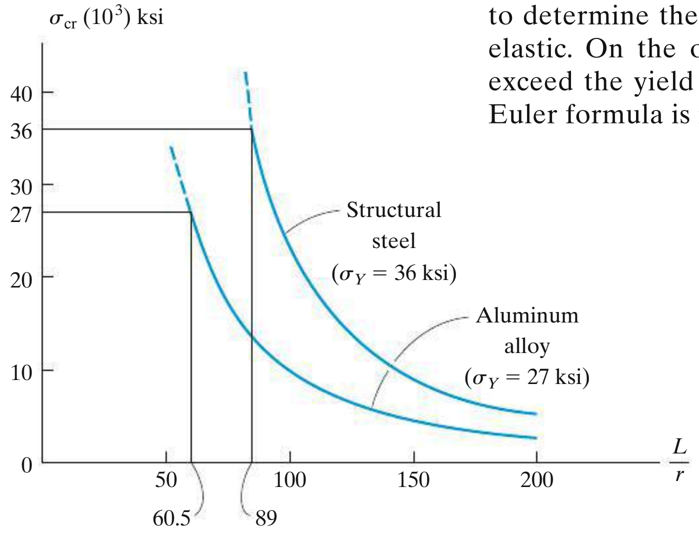

Fig. 13–7

Important Points

- *Columns* are long slender members that are subjected to axial compressive loads.
- The *critical load* is the maximum axial load that a column can support when it is on the verge of buckling. This loading represents a case of *neutral equilibrium*.
- An *ideal column* is initially perfectly straight, made of homogeneous material, and the load is applied through the centroid of the cross section.
- A pin-connected column will buckle about the principal axis of the cross section having the *least* moment of inertia.
- The *slenderness ratio* is L/r, where r is the smallest radius of gyration of the cross section. Buckling will occur about the axis where this ratio gives the greatest value.

EXAMPLE 13.1

The A992 steel W8 $\times$ 31 member shown in Fig. 13–8 is to be used as a pin-connected column. Determine the largest axial load it can support before it either begins to buckle or the steel yields.

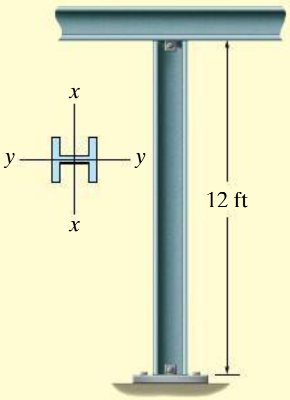

Fig. 13–8

SOLUTION

From the table in Appendix B, the column's cross-sectional area and moments of inertia are $A = 9.13\text{ in}^2$, $I_x = 110\text{ in}^4$, and $I_y = 37.1\text{ in}^4$. By inspection, buckling will occur about the y–y axis. Why? Applying Eq. 13–5, we have

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2[29(10^3)\text{ kip/in}^2](37.1\text{ in}^4)}{[12\text{ ft}(12\text{ in./ft})]^2} = 512\text{ kip}$$

When fully loaded, the average compressive stress in the column is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{512\text{ kip}}{9.13\text{ in}^2} = 56.1\text{ ksi}$$

Since this stress exceeds the yield stress (50 ksi), the load P is determined from simple compression:

$$50\text{ ksi} = \frac{P}{9.13\text{ in}^2}; \qquad P = 456\text{ kip} \qquad \textit{Ans.}$$

In actual practice, a factor of safety would be placed on this loading.

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13.3 Columns Having Various Types of Supports

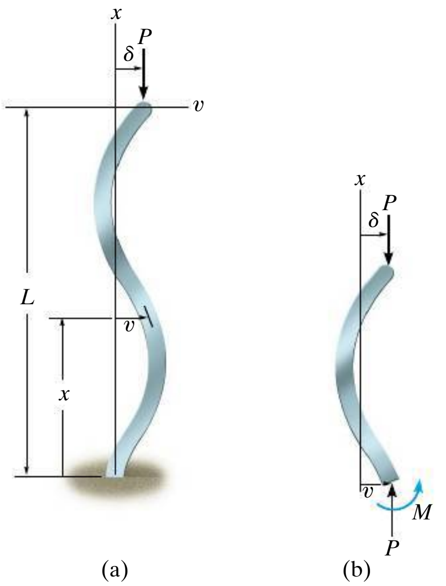


Fig. 13–9

The tubular columns used to support this water tank have been braced at three locations along their length to prevent them from buckling.

The Euler load was derived for a column that is pin connected or free to rotate at its ends. Oftentimes, however, columns may be supported in some other way. For example, consider the case of a column fixed at its base and free at the top, Fig. 13–9*a*. As the column buckles the load displaces δ and at x the displacement is v. From the free-body diagram in Fig. 13–9*b*, the internal moment at the arbitrary section is $M = P(\delta - v)$. Consequently, the differential equation for the deflection curve is

$$EI\frac{d^2v}{dx^2} = P(\delta - v)$$

$$\frac{d^2v}{dx^2} + \frac{P}{EI}v = \frac{P}{EI}\delta \tag{13–7}$$

Unlike Eq. 13–2, this equation is nonhomogeneous because of the nonzero term on the right side. The solution consists of both a complementary and a particular solution, namely,

$$v = C_1 \sin\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \cos\left(\sqrt{\frac{P}{EI}}x\right) + \delta$$

The constants are determined from the boundary conditions. At $x = 0$, $v = 0$, so that $C_2 = -\delta$. Also,

$$\frac{dv}{dx} = C_1\sqrt{\frac{P}{EI}}\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2\sqrt{\frac{P}{EI}}\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

At $x = 0$, $dv/dx = 0$, so that $C_1 = 0$. The deflection curve is therefore

$$v = \delta\left[1 - \cos\left(\sqrt{\frac{P}{EI}}x\right)\right] \tag{13–8}$$

Since the deflection at the top of the column is δ, that is, at $x = L$, $v = \delta$, we require

$$\delta \cos\left(\sqrt{\frac{P}{EI}}L\right) = 0$$

The trivial solution $\delta = 0$ indicates that no buckling occurs, regardless of the load P. Instead,

$$\cos\left(\sqrt{\frac{P}{EI}}L\right) = 0 \qquad \text{or} \qquad \sqrt{\frac{P}{EI}}L = \frac{n\pi}{2}, \quad n = 1, 3, 5 \ldots$$

The smallest critical load occurs when $n = 1$, so that

$$P_{cr} = \frac{\pi^2 EI}{4L^2} \tag{13–9}$$

By comparison with Eq. 13–5, it is seen that a column fixed supported at its base and free at its top will support only one-fourth the critical load that can be applied to a column pin supported at both ends.

Other types of supported columns are analyzed in much the same way and will not be covered in detail here.* Instead, we will tabulate the results for the most common types of column support and show how to apply these results by writing Euler's formula in a general form.

Effective Length. As stated previously, the Euler formula, Eq. 13–5, was developed for the case of a column having ends that are pinned or free to rotate. In other words, L in the equation represents the unsupported distance between the points of zero moment. This formula can be used to determine the critical load on columns having other types of support provided "L" represents the distance between the zero-moment points. This distance is called the column's ***effective length***, L_e. Obviously, for a pin-ended column $L_e = L$, Fig. 13–10*a*. For the fixed- and free-ended column, the deflection curve, Eq. 13–8, was found to be one-half that of a column that is pin connected and has a length of $2L$, Fig. 13–10*b*. Thus the effective length between the points of zero moment is $L_e = 2L$. Examples for two other columns with different end supports are also shown in Fig. 13–10. The column fixed at its ends, Fig. 13–10*c*, has inflection points or points of zero moment $L/4$ from each support. The effective length is therefore represented by the middle half of its length, that is, $L_e = 0.5L$. Lastly, the pin- and fixed-ended column, Fig. 13–10*d*, has an inflection point at approximately $0.7L$ from its pinned end, so that $L_e = 0.7L$.

Rather than specifying the column's effective length, many design codes provide column formulas that employ a dimensionless coefficient K called the ***effective-length factor***. This factor is defined from

$$L_e = KL \tag{13–10}$$

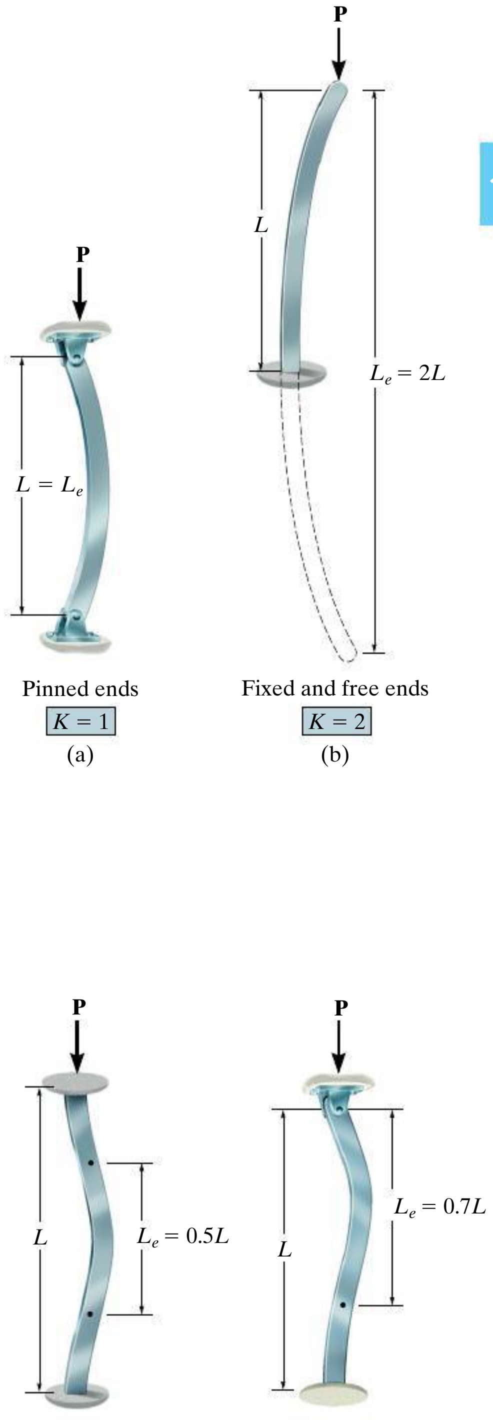
Specific values of K are also given in Fig. 13–10. Based on this generality, we can therefore write Euler's formula as

$$\boxed{P_{\mathrm{cr}} = \frac{\pi^2 EI}{(KL)^2}} \tag{13–11}$$

or

$$\boxed{\sigma_{\mathrm{cr}} = \frac{\pi^2 E}{(KL/r)^2}} \tag{13–12}$$

Here (KL/r) is the column's ***effective-slenderness ratio***. For example, if the column is fixed at its base and free at its end, we have $K = 2$, and therefore Eq. 13–11 gives the same result as Eq. 13–9.



Pinned ends $K = 1$ (a)

Fixed and free ends $K = 2$ (b)

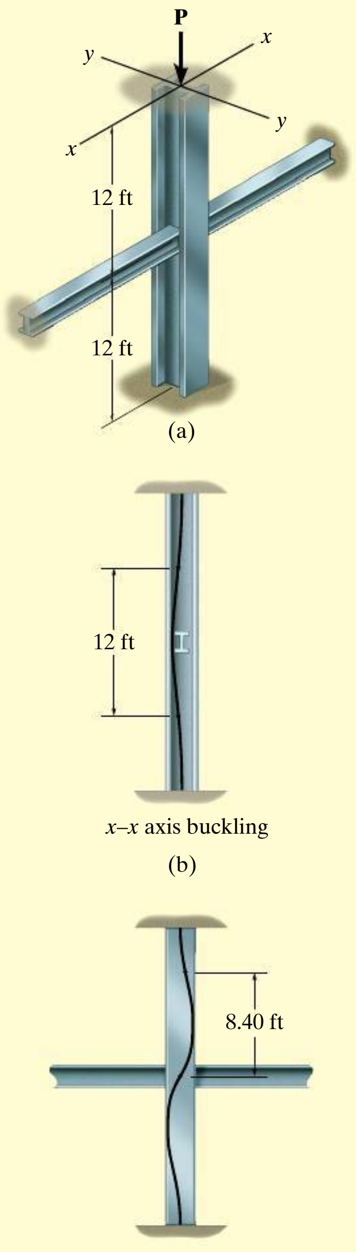
Fixed ends $K = 0.5$ (c)

Pinned and fixed ends $K = 0.7$ (d)

Fig. 13–10

*See Problems 13–43, 13–44, and 13–45.

EXAMPLE 13.2



(a)

x–x axis buckling

(b)

y–y axis buckling

(c)

Fig. 13–11

A W6 $\times$ 15 steel column is 24 ft long and is fixed at its ends as shown in Fig. 13–11*a*. Its load-carrying capacity is increased by bracing it about the y–y (weak) axis using struts that are assumed to be pin connected to its midheight. Determine the load it can support so that the column does not buckle nor the material exceed the yield stress. Take $E_{st} = 29(10^3)$ ksi and $\sigma_Y = 60$ ksi.

SOLUTION

The buckling behavior of the column will be *different* about the x–x and y–y axes due to the bracing. The buckled shape for each of these cases is shown in Figs. 13–11*b* and 13–11*c*. From Fig. 13–11*b*, the effective length for buckling about the x–x axis is $(KL)_x = 0.5(24\text{ ft}) = 12\text{ ft} = 144\text{ in.}$, and from Fig. 13–11*c*, for buckling about the y–y axis, $(KL)_y = 0.7(24\text{ ft}/2) = 8.40\text{ ft} = 100.8\text{ in.}$ The moments of inertia for a W6 $\times$ 15 are found from the table in Appendix B. We have $I_x = 29.1\text{ in}^4$, $I_y = 9.32\text{ in}^4$.

Applying Eq. 13–11,

$$(P_{cr})_x = \frac{\pi^2 EI_x}{(KL)_x^2} = \frac{\pi^2[29(10^3)\text{ ksi}]29.1\text{ in}^4}{(144\text{ in.})^2} = 401.7\text{ kip} \qquad (1)$$

$$(P_{cr})_y = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2[29(10^3)\text{ ksi}]9.32\text{ in}^4}{(100.8\text{ in.})^2} = 262.5\text{ kip} \qquad (2)$$

By comparison, buckling will occur about the y–y axis.

The area of the cross section is 4.43 in^2, so the average compressive stress in the column is

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{262.5\text{ kip}}{4.43\text{ in}^2} = 59.3\text{ ksi}$$

Since this stress is less than the yield stress, buckling will occur before the material yields. Thus,

$$P_{cr} = 263\text{ kip} \qquad \textit{Ans.}$$

NOTE: From Eq. 13–12 it can be seen that buckling will always occur about the column axis having the *largest* slenderness ratio, since a large slenderness ratio will give a small critical stress. Thus, using the data for the radius of gyration from the table in Appendix B, we have

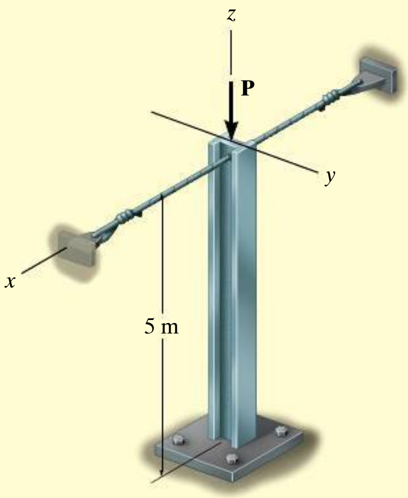
$$\left(\frac{KL}{r}\right)_x = \frac{144\text{ in.}}{2.56\text{ in.}} = 56.2$$

$$\left(\frac{KL}{r}\right)_y = \frac{100.8\text{ in.}}{1.46\text{ in.}} = 69.0$$

Hence, y–y axis buckling will occur, which is the same conclusion reached by comparing Eqs. 1 and 2.

EXAMPLE 13.3

The aluminum column is braced at its top by cables so as to prevent movement at the top along the x axis, Fig. 13–12a. If it is assumed to be fixed at its base, determine the largest allowable load P that can be applied. Use a factor of safety for buckling of F.S. = 3.0. Take $E_{al} = 70$ GPa, $\sigma_Y = 215$ MPa, $A = 7.5(10^{-3})$ m^2, $I_x = 61.3(10^{-6})$ m^4, $I_y = 23.2(10^{-6})$ m^4.



(a)

Fig. 13–12

SOLUTION

Buckling about the x and y axes is shown in Figs. 13–12b and 13–12c, respectively. Using Fig. 13–10a, for x–x axis buckling, $K = 2$, so $(KL)_x = 2(5\text{ m}) = 10$ m. Also, for y–y axis buckling, $K = 0.7$, so $(KL)_y = 0.7(5\text{ m}) = 3.5$ m.

Applying Eq. 13–11, the critical loads for each case are

$$(P_{cr})_x = \frac{\pi^2 EI_x}{(KL)_x^2} = \frac{\pi^2[70(10^9)\text{ N/m}^2](61.3(10^{-6})\text{ m}^4)}{(10\text{ m})^2}$$

$$= 424\text{ kN}$$

$$(P_{cr})_y = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2[70(10^9)\text{ N/m}^2](23.2(10^{-6})\text{ m}^4)}{(3.5\text{ m})^2}$$

$$= 1.31\text{ MN}$$

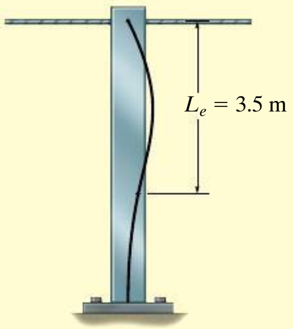
By comparison, as P is increased the column will buckle about the x–x axis. The allowable load is therefore

$$P_{allow} = \frac{P_{cr}}{\text{F.S.}} = \frac{424\text{ kN}}{3.0} = 141\text{ kN} \qquad \textit{Ans.}$$

Since

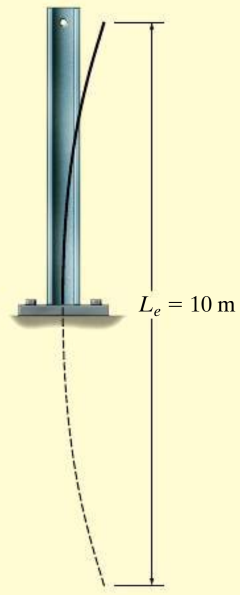
$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{424\text{ kN}}{7.5(10^{-3})\text{ m}^2} = 56.5\text{ MPa} < 215\text{ MPa}$$

Euler's equation can be applied.



y–y axis buckling

(c)



x–x axis buckling

(b)

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Buckling of Columns

Sections ... 13.1 – 13.3

Chap. 13

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