MECHANICS AND MATERIALS I

17

Ali Javili

Bilkent University

MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

Bending ii

Sections … 6.2

Chap. 6

[Hibbeler 9th edition]

Ali Javili

Bilkent University

Jump Condition

↳ Due to Concentrated Force
or Concentrated Moment

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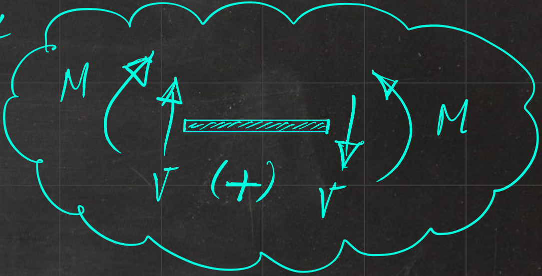
RECALL (



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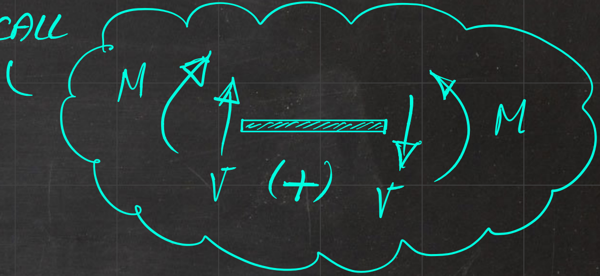

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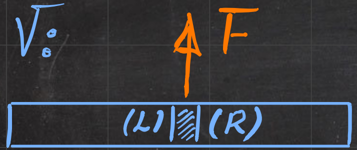

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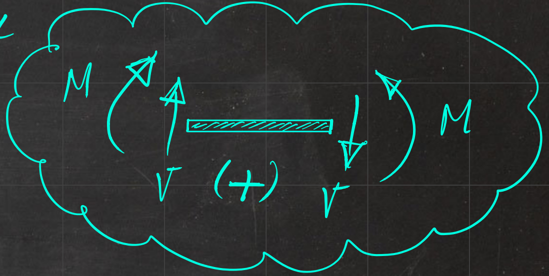

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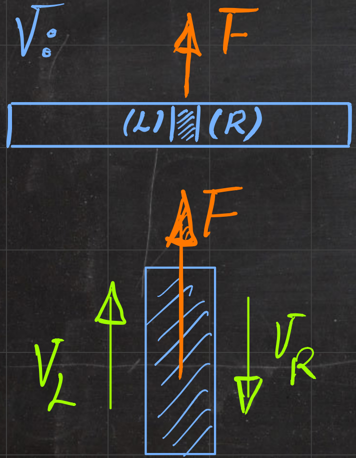

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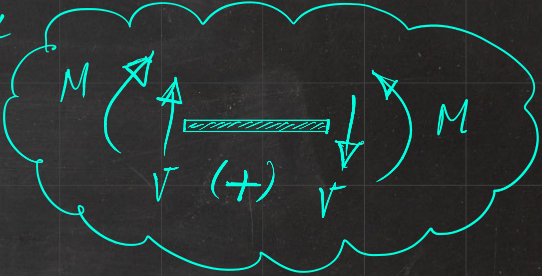

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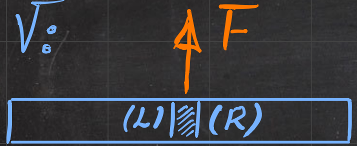

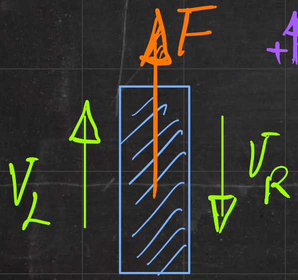

$+\uparrow \sum F_y = 0$

$\Rightarrow V_L + F - V_R = 0$

$\Rightarrow V_R = V_L + F$

Jump Condition

↳ Due to Concentrated Force
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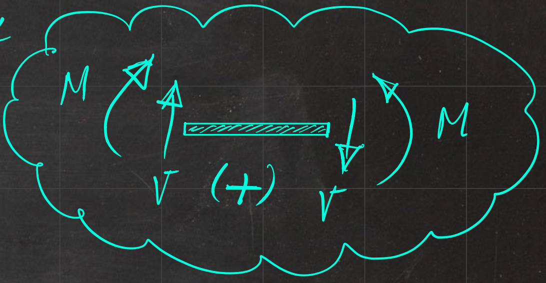
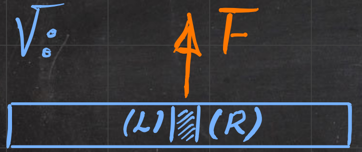
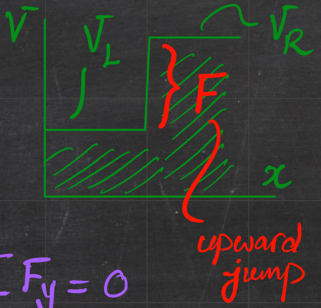
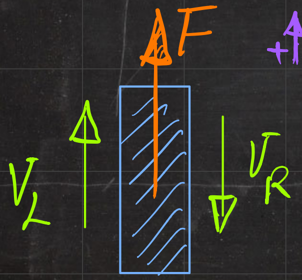
RECALL









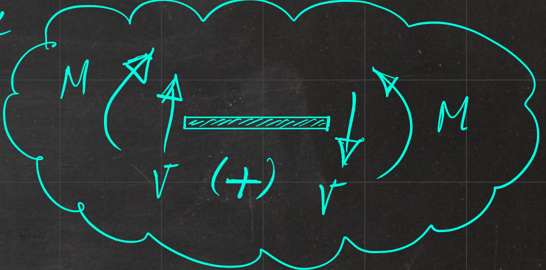
$$+\uparrow \sum F_y = 0$$

$$\Rightarrow V_L + F - V_R = 0$$

$$\Rightarrow V_R = V_L + F$$

Jump Condition

RECALL



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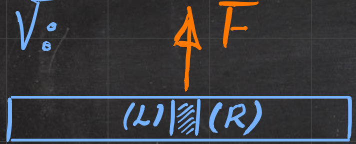
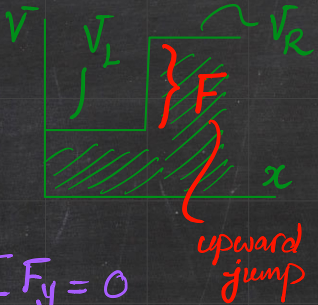
V:





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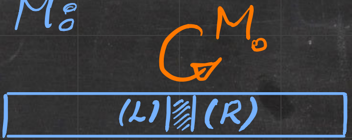

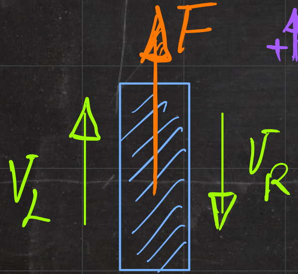

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Jump Condition

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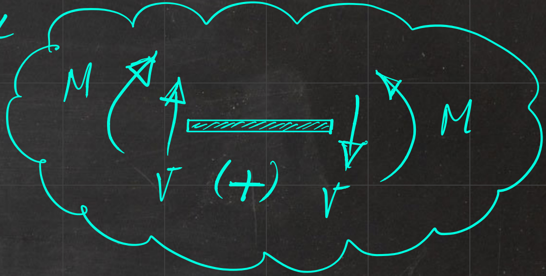

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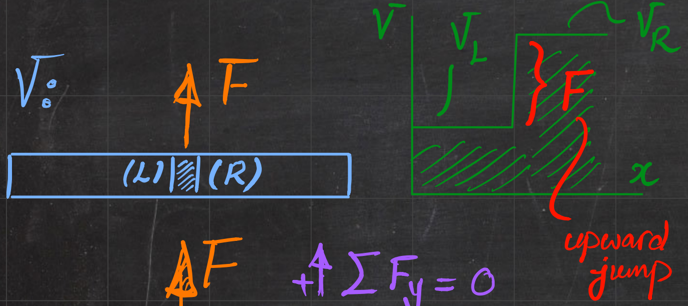
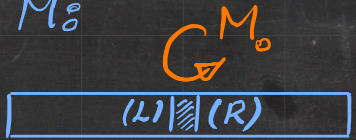
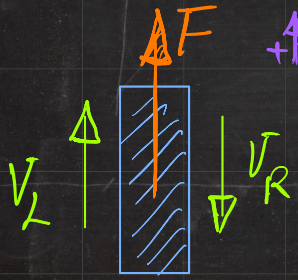


M_o:





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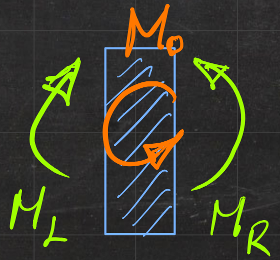

Jump Condition

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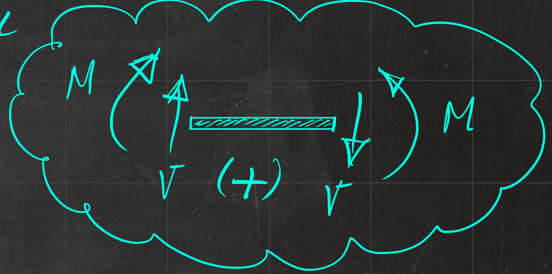
RECALL



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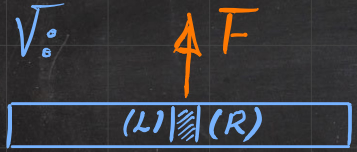

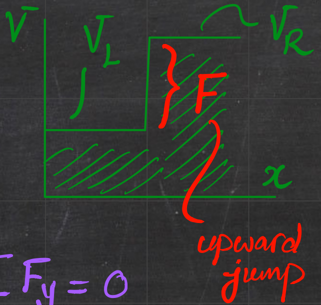

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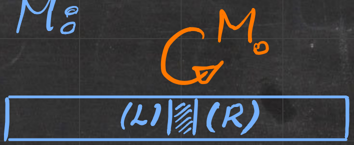
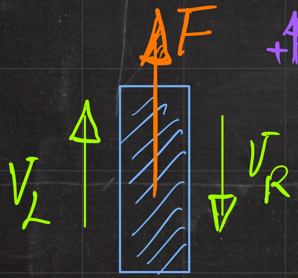
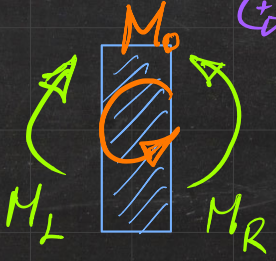




$+\uparrow \sum F_y = 0$

$\Rightarrow V_L + F - V_R = 0$

$\Rightarrow V_R = V_L + F$



$\circlearrowleft + \sum M = 0$

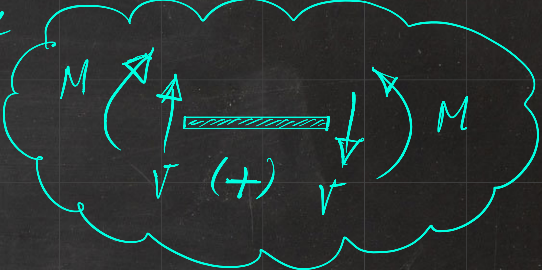
$\Rightarrow M_R + M_o - M_L = 0$

$\Rightarrow M_R = M_L - M_o$

Jump Condition

↳ Due to Concentrated Force or Concentrated Moment

Recall



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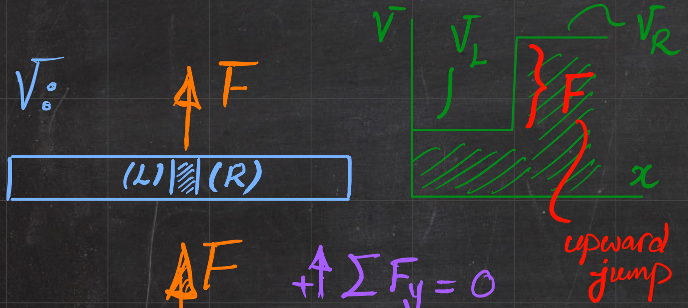
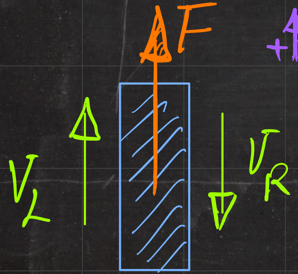


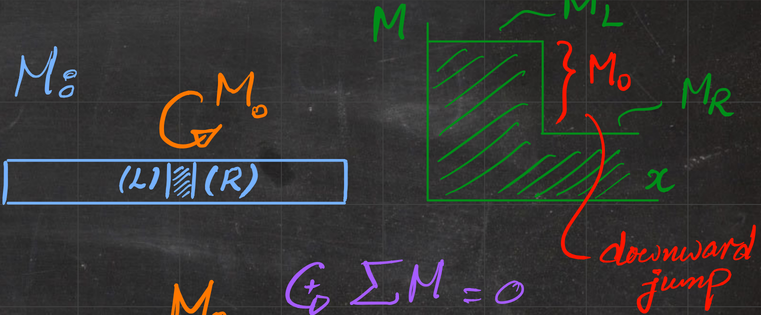


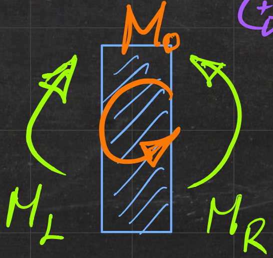
$+\uparrow \sum F_y = 0$

$\Rightarrow V_L + F - V_R = 0$

$\Rightarrow V_R = V_L + F$

M:





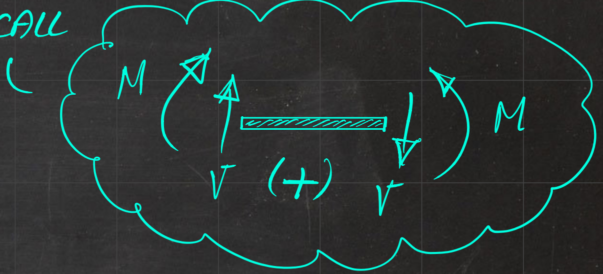
$\circlearrowleft^{+} \sum M = 0$

$\Rightarrow M_R + M_0 - M_L = 0$

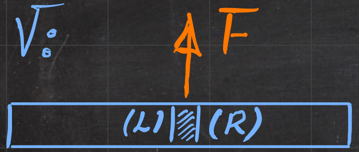
$\Rightarrow M_R = M_L - M_0$

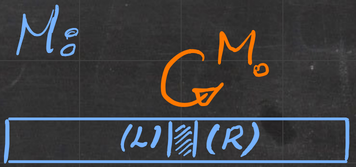
Jump Condition

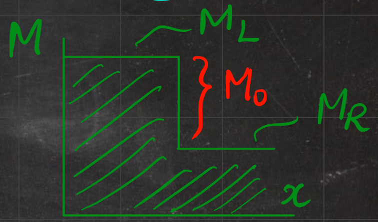
RECALL

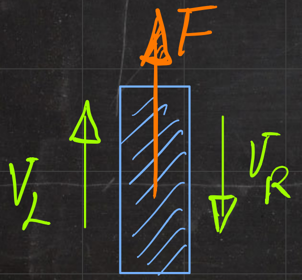


↳ Due to Concentrated Force
or Concentrated Moment



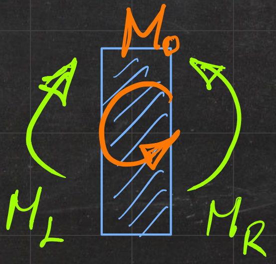






⇓

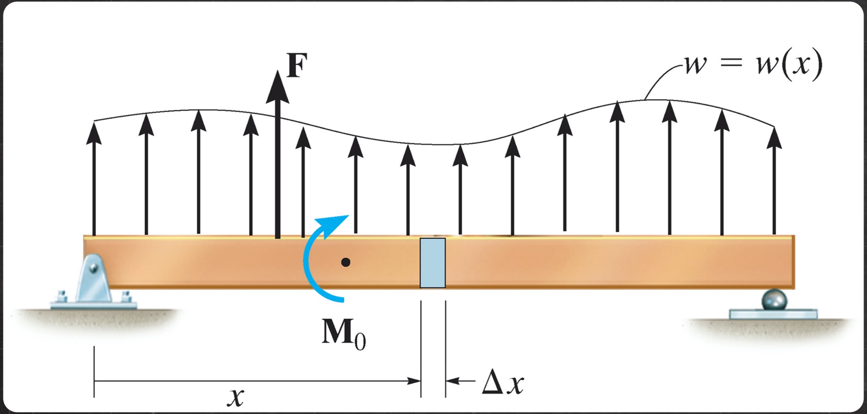
Jump of shear force occurs due to concentrated force



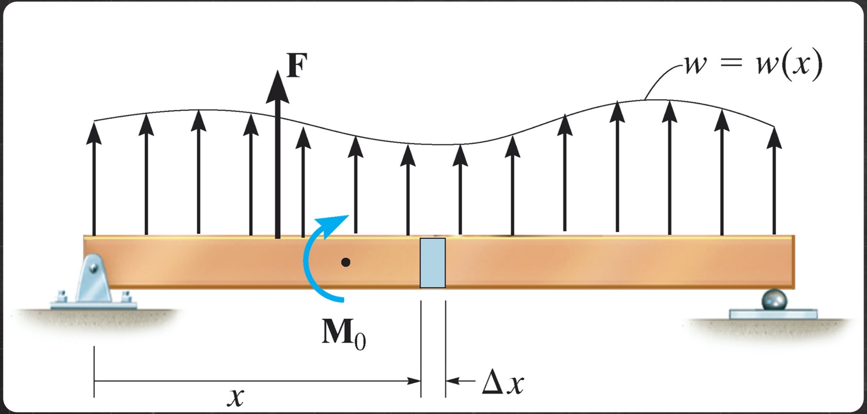
⇓

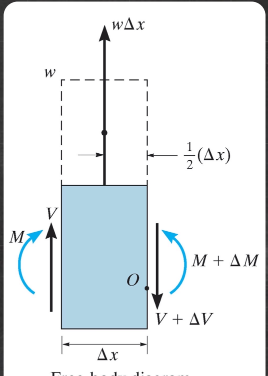
Jump of bending moment occurs due to concentrated moment

Distributed Load



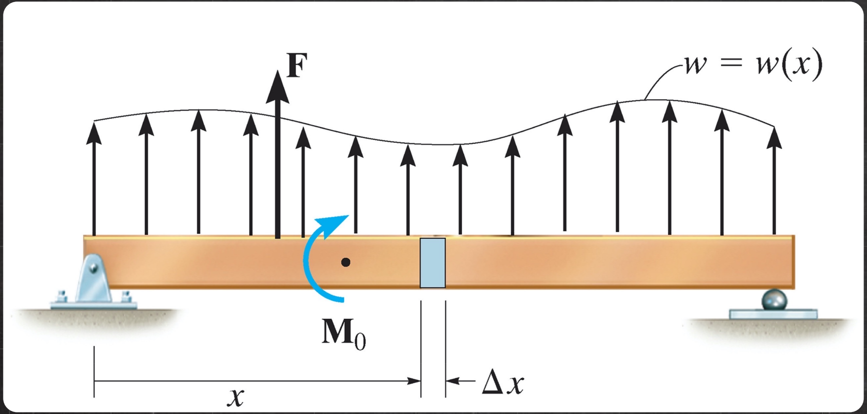
Distributed Load

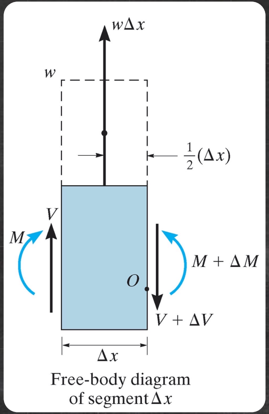




Free-body diagram of segment Δx

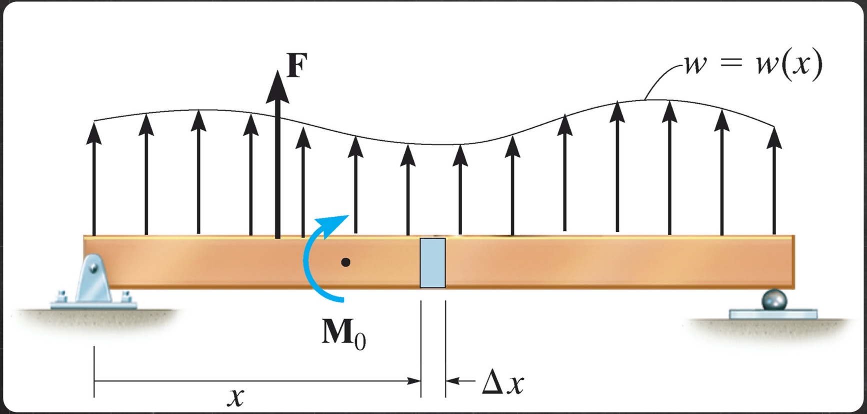
Distributed Load

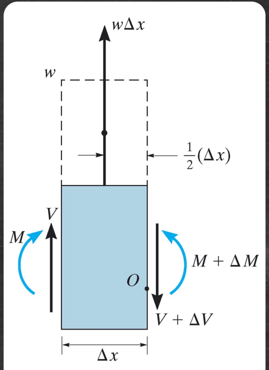




$$+\uparrow \Sigma F_y = 0 \Rightarrow V + w\Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = w\,\Delta x$$

Distributed Load



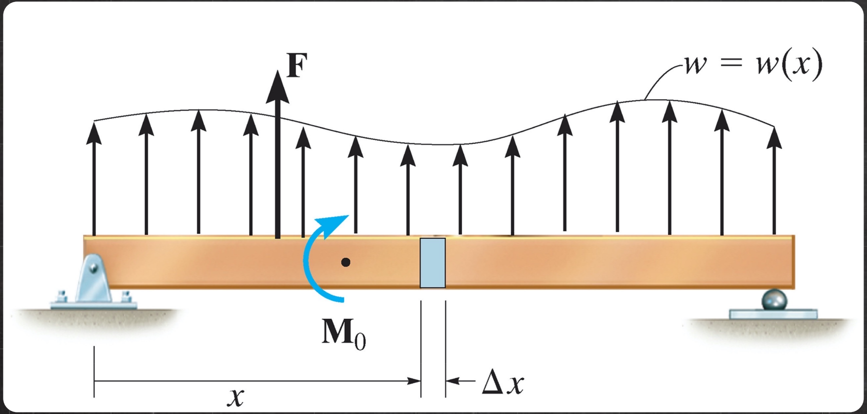


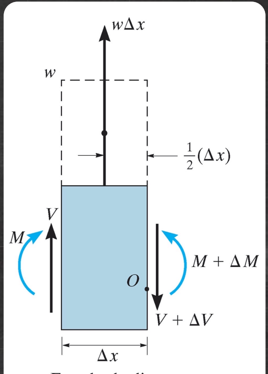
Free-body diagram of segment Δx

$$+\uparrow \sum F_y = 0 \Rightarrow V + w\Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = w\,\Delta x$$

$$\circlearrowleft + \sum M_O = 0 \Rightarrow -M - V\Delta x - w\Delta x\,\frac{\Delta x}{2} + M + \Delta M = 0 \Rightarrow \Delta M = V\Delta x + \frac{1}{2} w \Delta x^2$$

Distributed Load





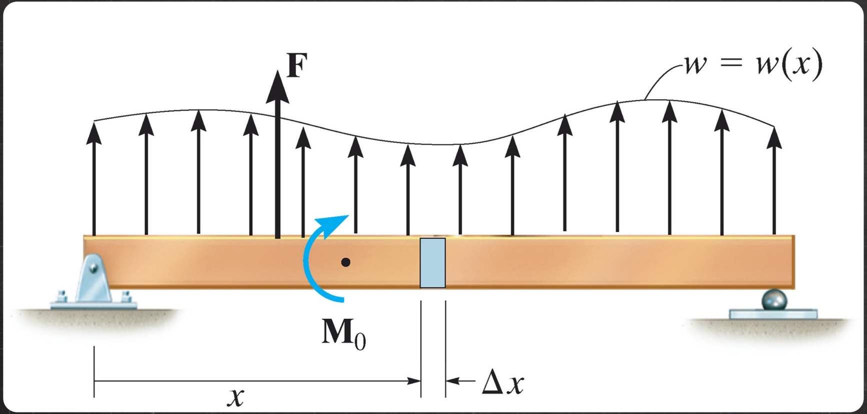
Free-body diagram of segment Δx

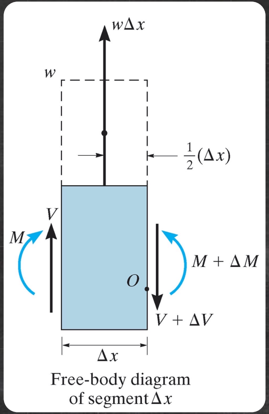
$$+\uparrow \Sigma F_y = 0 \Rightarrow V + w\Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = w\,\Delta x$$

$$\Delta x \to 0$$

$$\circlearrowleft + \Sigma M_O = 0 \Rightarrow -M - V\Delta x - w\Delta x \frac{\Delta x}{2} + M + \Delta M = 0 \Rightarrow \Delta M = V\Delta x + \frac{1}{2} w \Delta x^2$$

Distributed Load





Free-body diagram of segment Δx

$$\frac{dV}{dx} = \omega(x)$$

$$\frac{dM}{dx} = V(x)$$

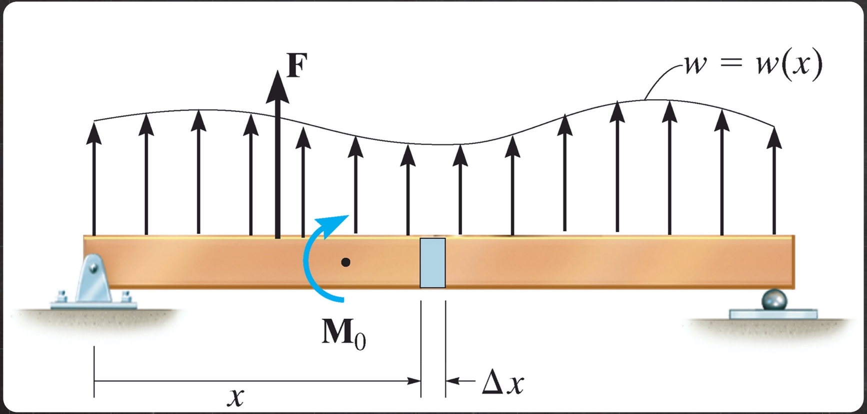
$$\Uparrow$$

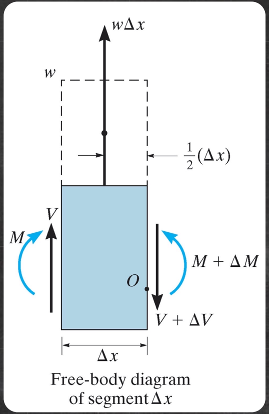
$$\Delta x \to 0$$

$$+\uparrow \Sigma F_y = 0 \Rightarrow V + \omega \Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = \omega \Delta x$$

$$\circlearrowleft + \Sigma M_O = 0 \Rightarrow -M - V\Delta x - \omega \Delta x \frac{\Delta x}{2} + M + \Delta M = 0 \Rightarrow \Delta M = V \Delta x + \frac{1}{2} \omega \Delta x^2$$

DISTRIBUTED LOAD





Free-body diagram of segment Δx

SLOPE OF SHEAR DIAGRAM

$$\frac{dV}{dx} = w(x)$$

$$\frac{dM}{dx} = V(x)$$

SLOPE OF MOMENT DIAGRAM

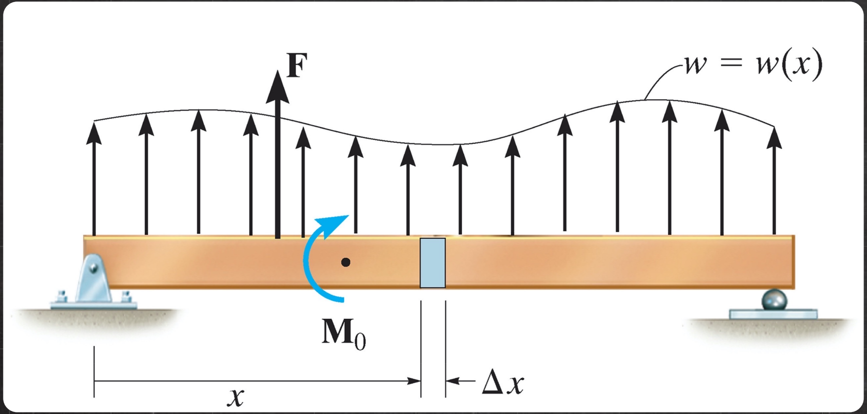
$\Uparrow$

$\Delta x \to 0$

$$+\uparrow \Sigma F_y = 0 \Rightarrow V + w\Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = w\,\Delta x$$

$$\circlearrowleft + \Sigma M_O = 0 \Rightarrow -M - V\Delta x - w\Delta x \frac{\Delta x}{2} + M + \Delta M = 0 \Rightarrow \Delta M = V\Delta x + \frac{1}{2} w \Delta x^2$$

DISTRIBUTED LOAD



SLOPE OF SHEAR DIAGRAM

$$\Delta V = \int \omega \, dx \Leftarrow \frac{dV}{dx} = \omega(x)$$

$$\Delta M = \int V \, dx \Leftarrow \frac{dM}{dx} = V(x)$$

SLOPE OF MOMENT DIAGRAM

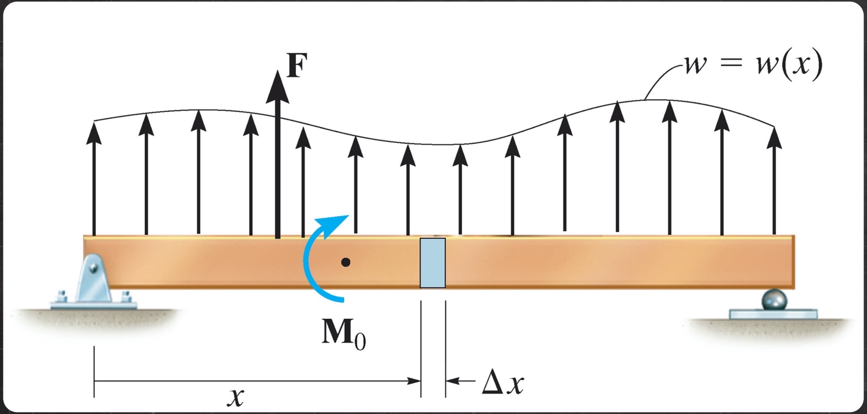
$$\Uparrow$$

$$\Delta x \to 0$$

$$+\uparrow \sum F_y = 0 \Rightarrow V + \omega \Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = \omega \Delta x$$

$$\circlearrowleft + \sum M_0 = 0 \Rightarrow -M - V\Delta x - \omega \Delta x \frac{\Delta x}{2} + M + \Delta M = 0 \Rightarrow \Delta M = V\Delta x + \frac{1}{2}\omega \Delta x^2$$

DISTRIBUTED LOAD



AREA UNDER DISTRIBUTED LOAD

SLOPE OF SHEAR DIAGRAM

$$\Delta V = \int \omega \, dx \Leftarrow \frac{dV}{dx} = \omega(x)$$

$$\Delta M = \int V \, dx \Leftarrow \frac{dM}{dx} = V(x)$$

AREA UNDER SHEAR DIAGRAM

SLOPE OF MOMENT DIAGRAM

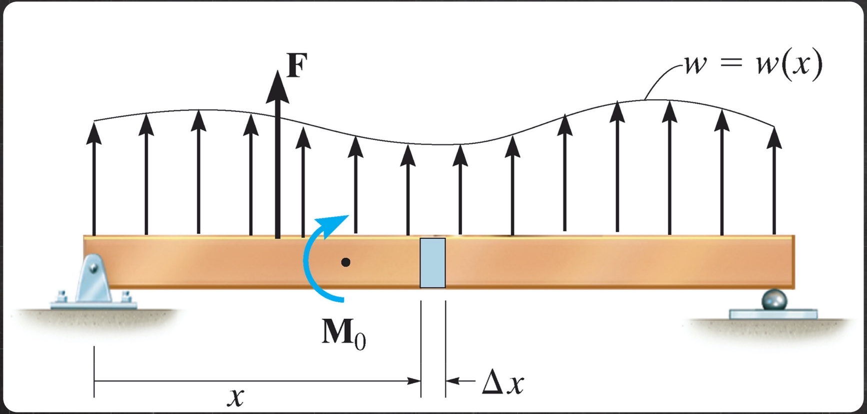
$\Uparrow$

$\Delta x \to 0$

$$+\uparrow \sum F_y = 0 \Rightarrow V + \omega \Delta x - (V + \Delta V) = 0 \Rightarrow \Delta V = \omega \Delta x$$

$$\circlearrowleft + \sum M_0 = 0 \Rightarrow -M - V\Delta x - \omega \Delta x \frac{\Delta x}{2} + M + \Delta M = 0 \Rightarrow \Delta M = V\Delta x + \frac{1}{2}\omega \Delta x^2$$

DISTRIBUTED LOAD



AREA UNDER DISTRIBUTED LOAD

$$\Delta V = \int w \, dx \Leftarrow \frac{dV}{dx} = w(x)$$

SLOPE OF SHEAR DIAGRAM

$$\Delta M = \int V \, dx \Leftarrow \frac{dM}{dx} = V(x)$$

AREA UNDER SHEAR DIAGRAM

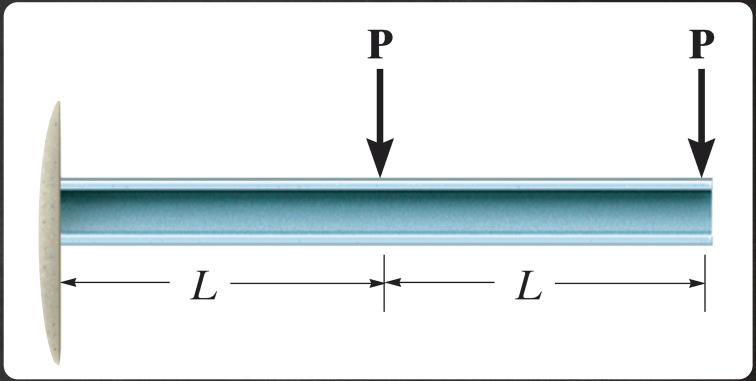
SLOPE OF MOMENT DIAGRAM

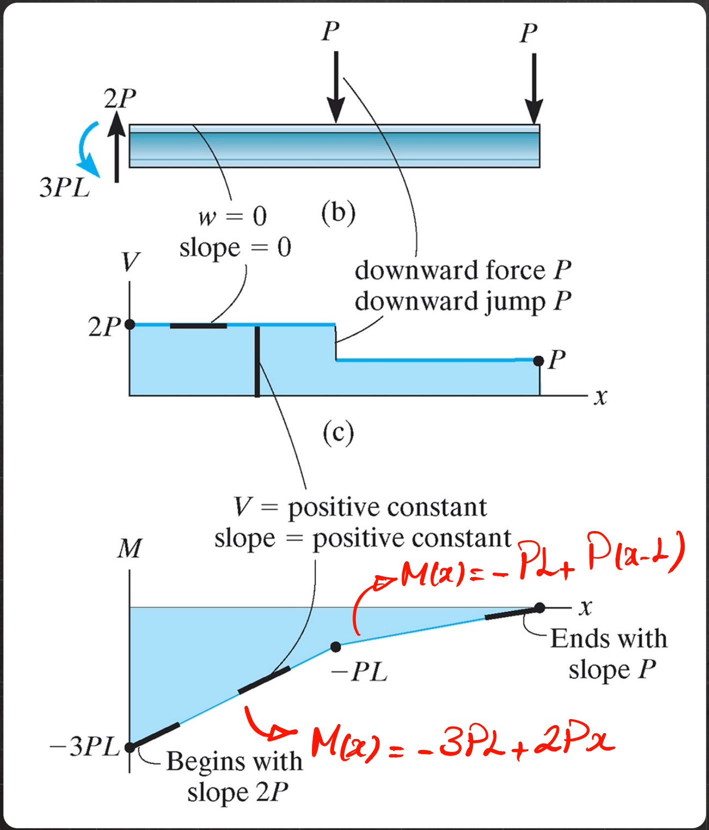
GRAPHICAL METHOD FOR

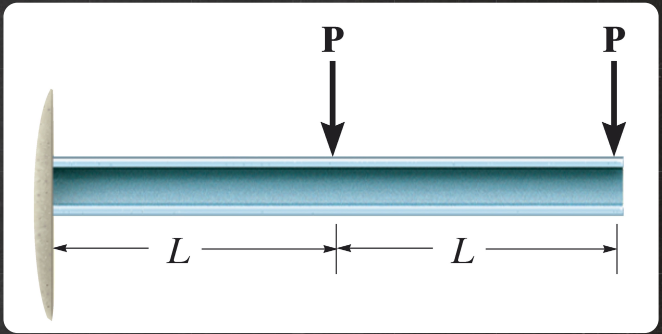
CONSTRUCTING SHEAR AND MOMENT DIAGRAM

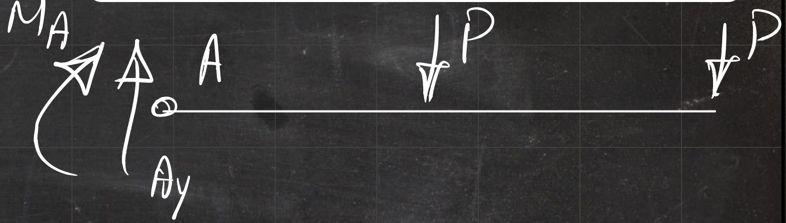
Exercise 1 . [similar to ... P. 270 ... 6.5]

DRAW SHEAR AND MOMENT DIAGRAM FOR THE BEAM SHOWN IN THE FIGURE.









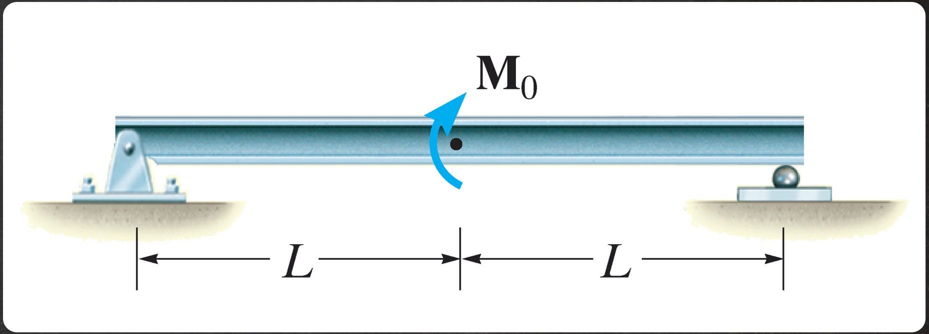
$+\uparrow \sum F_y = 0 \Rightarrow A_y - P - P = 0 \Rightarrow A_y = 2P$

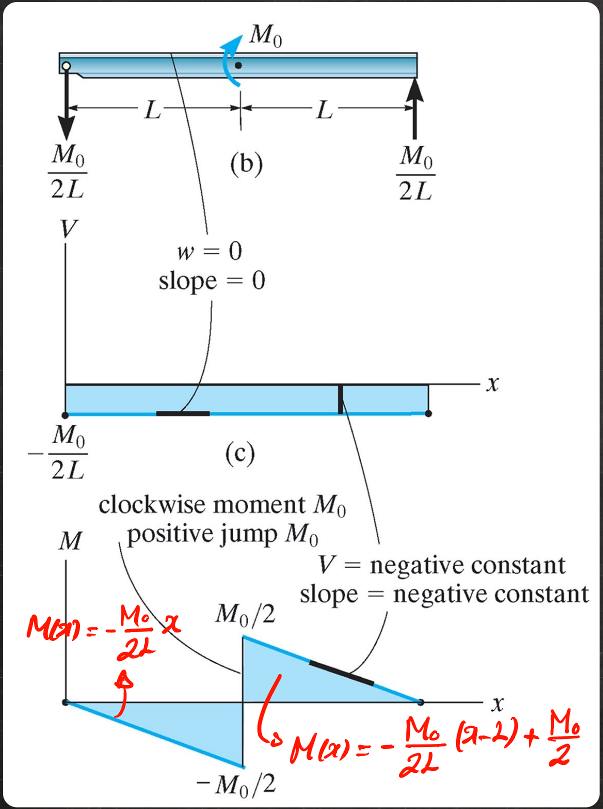
$\circlearrowleft + \sum M_A = 0 \Rightarrow -M - PL - P \times 2L = 0$

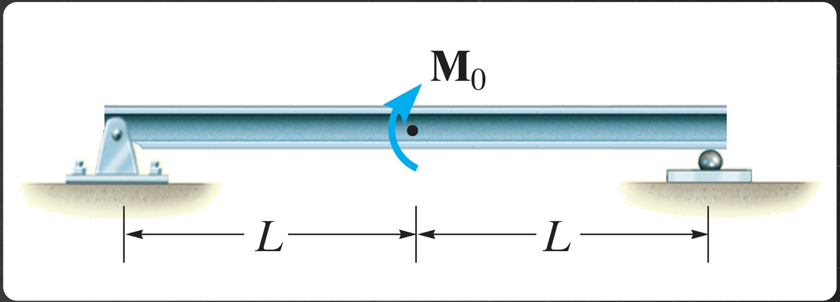
$\Rightarrow M = -3PL$

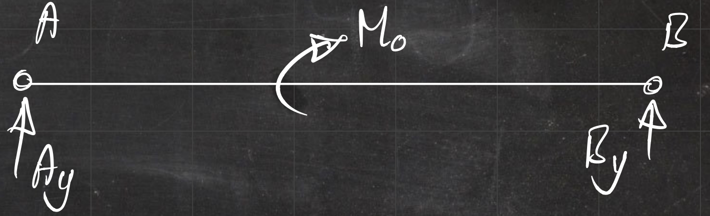
Exercise 2 . [similar to ... P. 271 ... 6.6]

DRAW SHEAR AND MOMENT DIAGRAM FOR THE BEAM SHOWN IN THE FIGURE.









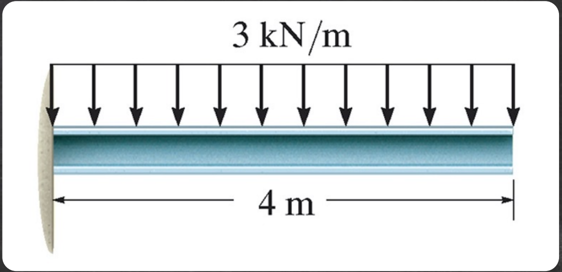
$$+\uparrow \sum F_y = 0 \Rightarrow A_y + B_y = 0$$

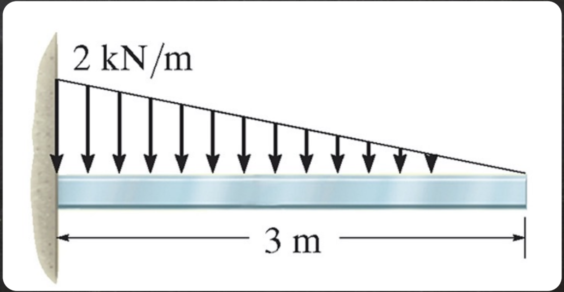
$$\circlearrowleft + \sum M_A = 0 \Rightarrow B_y \times 2L - M_0 = 0 \Rightarrow B_y = \frac{M_0}{2L}$$

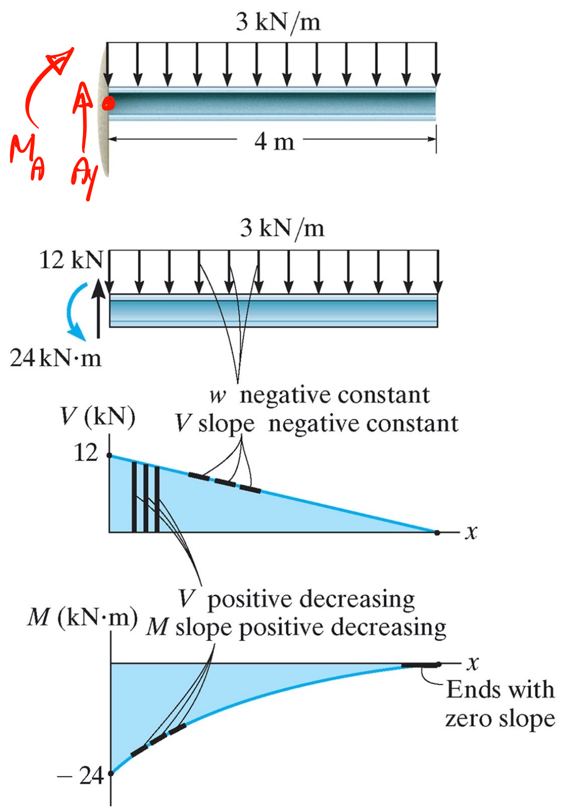
$$A_y = -\frac{M_0}{2L}$$

Exercise 3 . [similar to ... P. 272 ... 6.7]

DRAW SHEAR AND MOMENT DIAGRAM FOR THE BEAM SHOWN IN THE FIGURE.





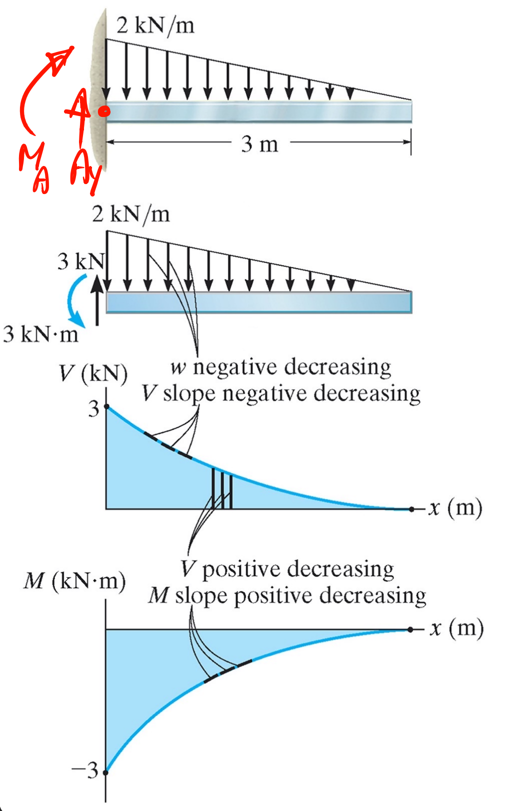


$V(0) = +A_y$

$M(0) = -M_A$

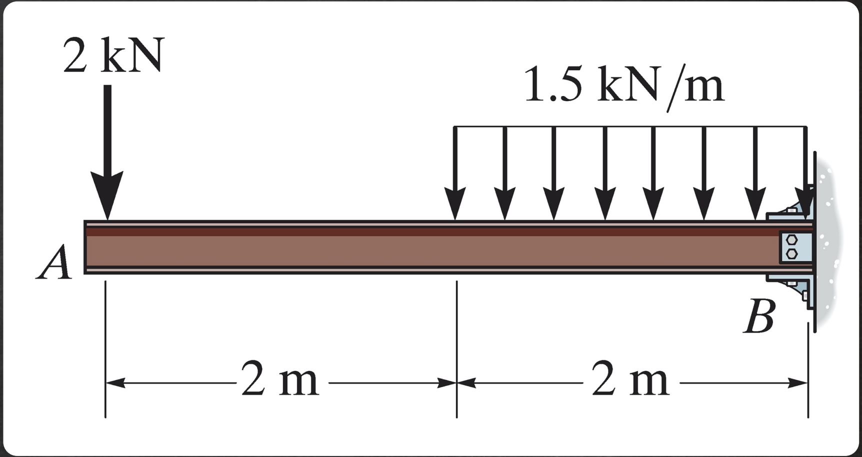
$V(L) = 0$

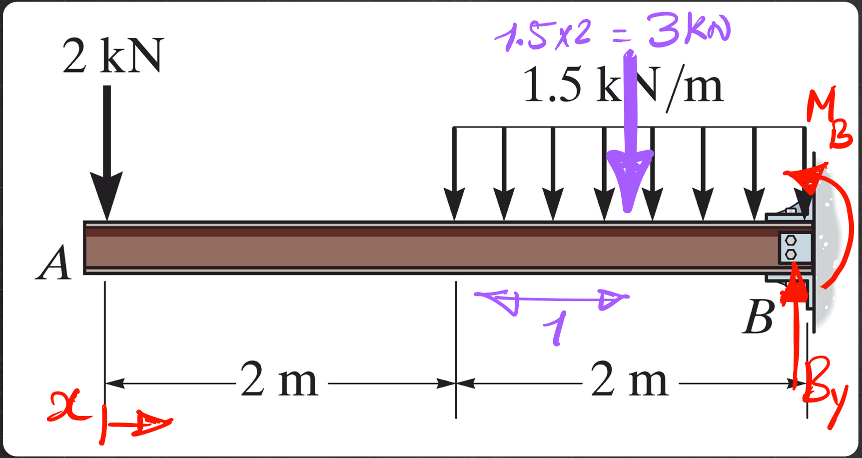
$M(L) = 0$

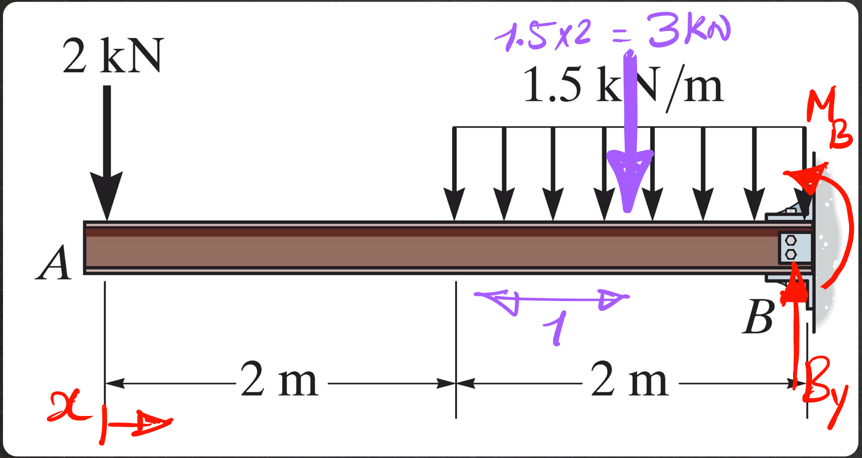


Exercise 4 . [similar to … P. 273 … 6.8]

DRAW SHEAR AND MOMENT DIAGRAM FOR THE BEAM SHOWN IN THE FIGURE.



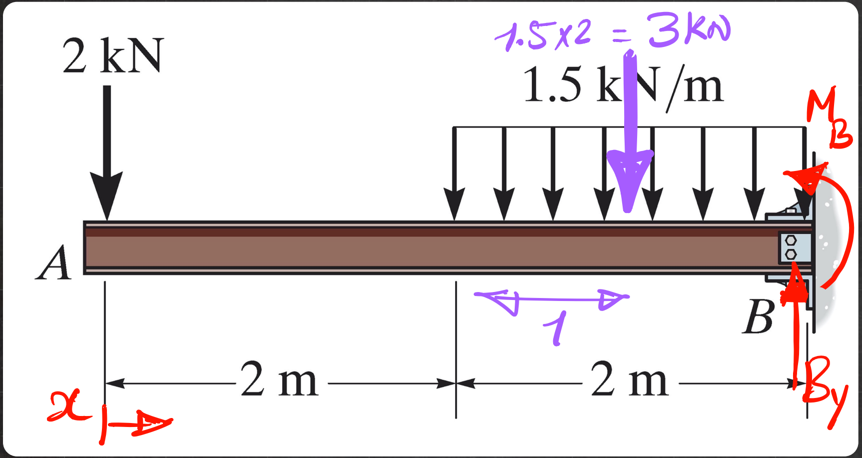

1.5x2 = 3kN
2 kN
1.5 kN/m
M_B
A
B
1
B_y
2 m
2 m
x



$+\uparrow \sum F_y = 0 \Rightarrow B_y - 2 - 3 = 0 \Rightarrow B_y = 5 \text{ kN}$

$\circlearrowleft + \sum M_A = 0 \Rightarrow M_B - 3\times 3 + 4B_y = 0$

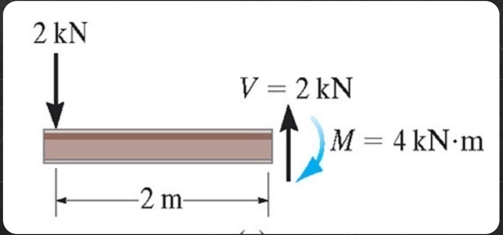
$\Rightarrow M_B = -11 \text{ kN.m}$

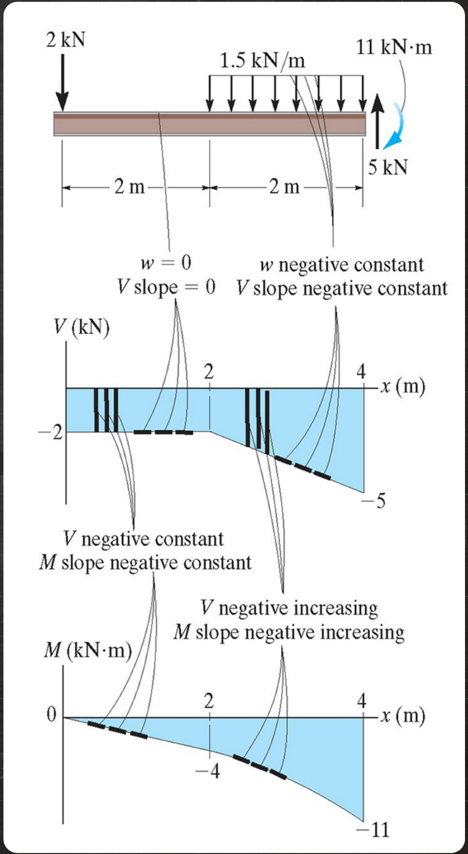


$+\uparrow \Sigma F_y = 0 \Rightarrow B_y - 2 - 3 = 0 \Rightarrow B_y = 5 \text{ kN}$

$\circlearrowleft + \Sigma M_A = 0 \Rightarrow M_B - 3\times 3 + 4B_y = 0$

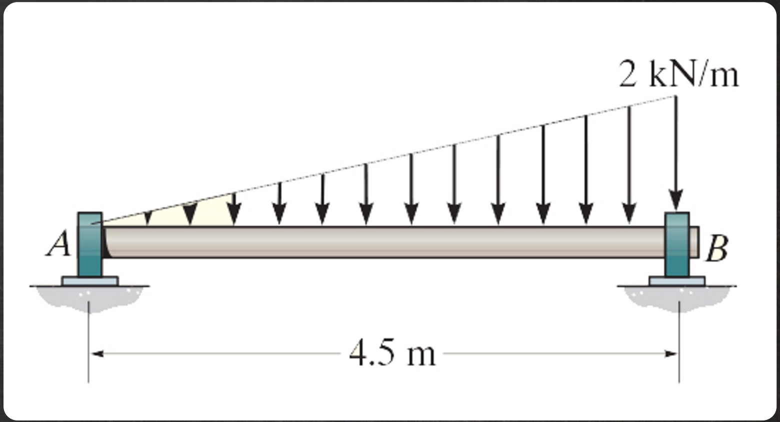
$\Rightarrow M_B = -11 \text{ kN.m}$





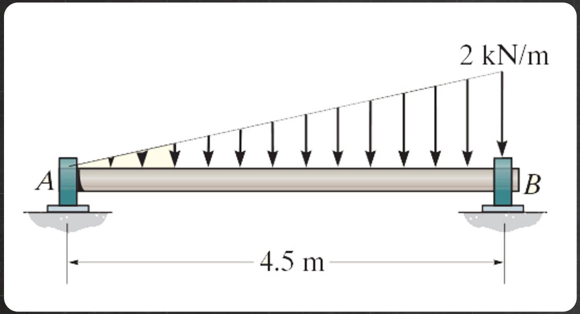
Exercise 5 . [similar to … P. 275 … 6.10]

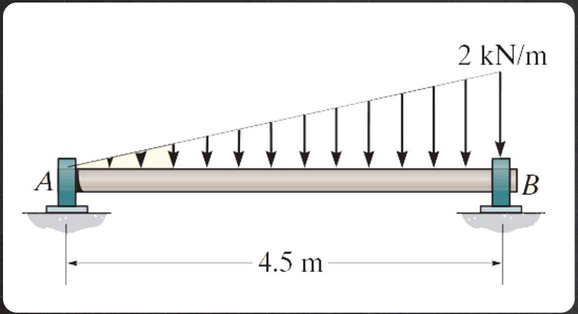
DRAW SHEAR AND MOMENT DIAGRAM FOR THE BEAM SHOWN IN THE FIGURE.

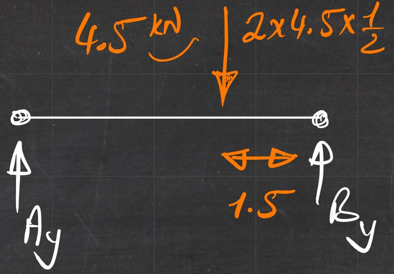


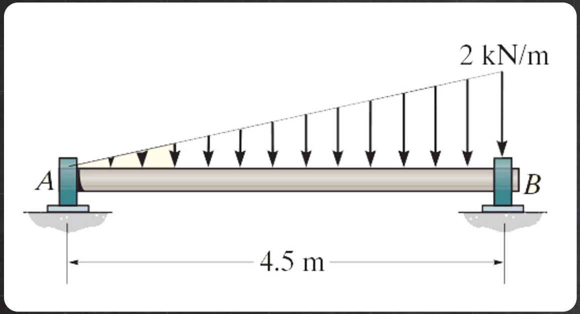
↳ A & B : BEARINGS (NO MOMENT)

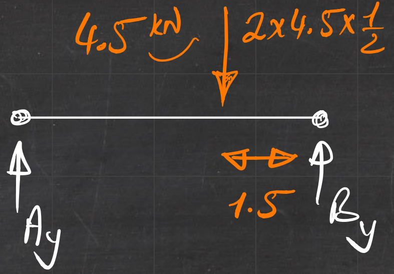
- A: THRUST BEARING
- B: JOURNAL BEARING


2 kN/m
A
B
4.5 m


2 kN/m
A
B
4.5 m


4.5 kN
2x4.5x1/2
1.5
Ay
By



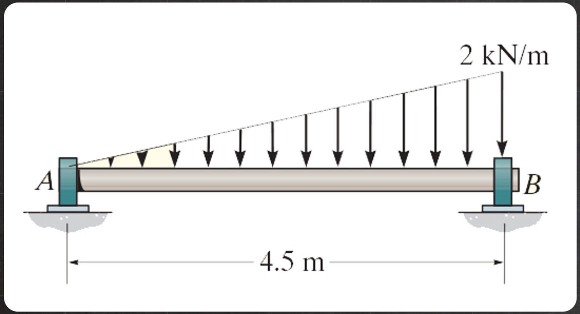


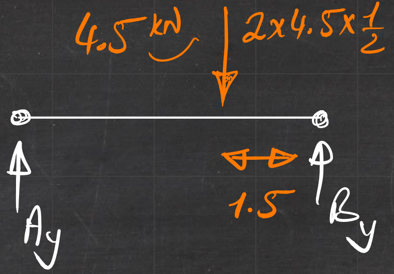
$$+\uparrow \sum F_y = 0$$

$$A_y + B_y - 4.5 = 0$$

$$\circlearrowleft^{+} \sum M_A = 0$$

$$B_y \times 4.5 - 3 \times 4.5 = 0$$





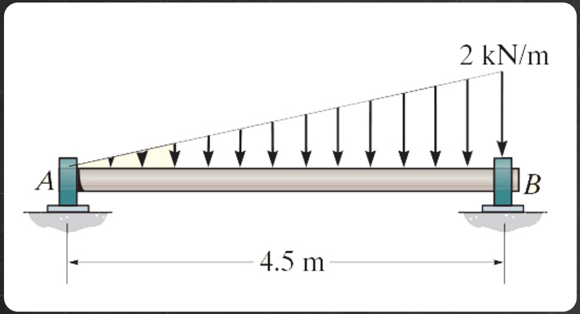
$$+\uparrow \sum F_y = 0$$

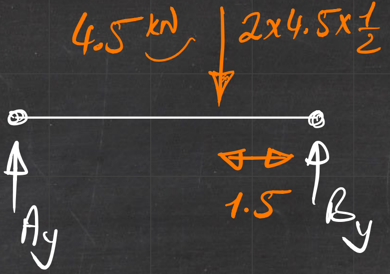
$$A_y + B_y - 4.5 = 0$$

$$\circlearrowleft + \sum M_A = 0$$

$$B_y \times 4.5 - 3 \times 4.5 = 0$$

$$\Rightarrow B_y = 3 \text{ kN} \Rightarrow A_y = 1.5 \text{ kN}$$





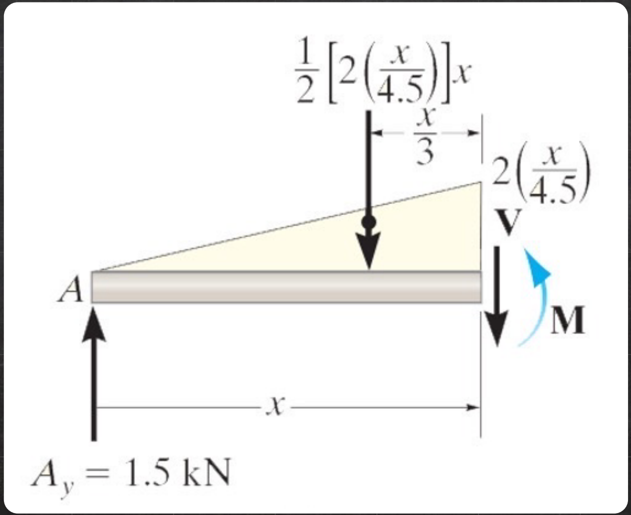
$+\uparrow \sum F_y = 0$

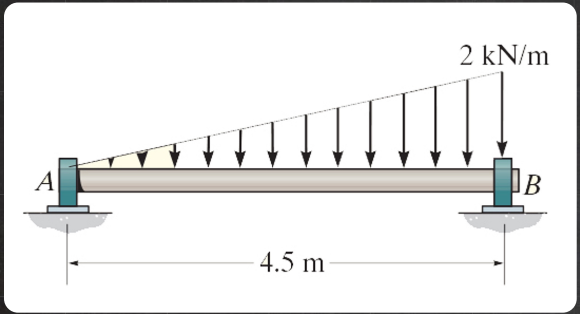
$A_y + B_y - 4.5 = 0$

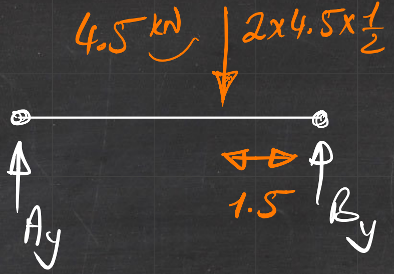
$\circlearrowleft + \sum M_A = 0$

$B_y \times 4.5 - 3 \times 4.5 = 0$

$\Rightarrow B_y = 3$ kN $\Rightarrow A_y = 1.5$ kN







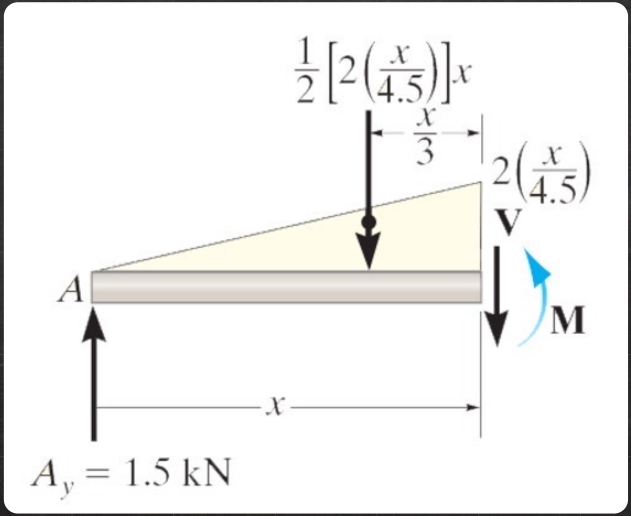
$+\uparrow \sum F_y = 0$

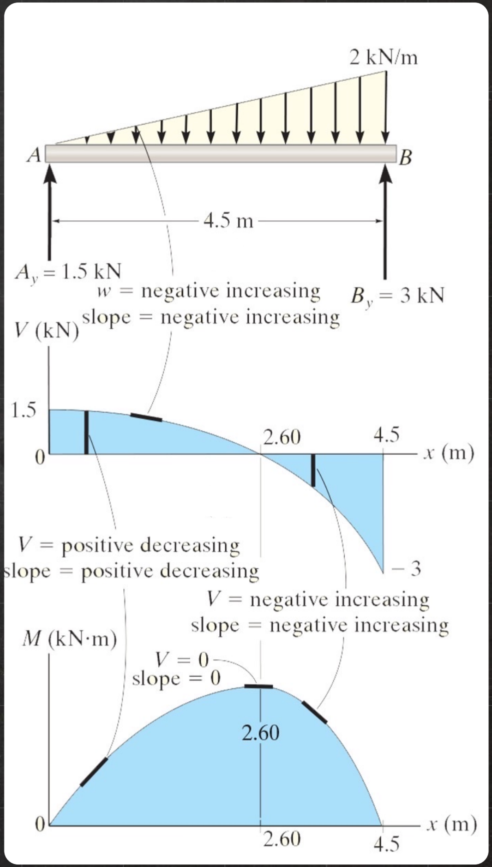
$A_y + B_y - 4.5 = 0$

$\circlearrowleft + \sum M_A = 0$

$B_y \times 4.5 - 3 \times 4.5 = 0$

$\Rightarrow B_y = 3$ kN $\Rightarrow A_y = 1.5$ kN





MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

Bending ii

Sections … 6.2

Chap. 6
[Hibbeler 9th edition]

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