MECHANICS AND MATERIALS I

23

Ali Javili

Bilkent University

MECHANICS AND MATERIALS I

Energy Methods

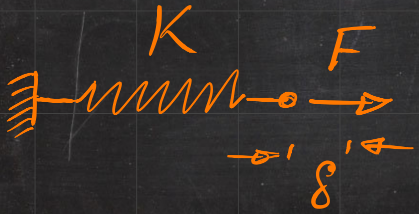
Sections … 14.1 – 14.3 … 14.5 … 14.6

Chap. 14

[Hibbeler 9th edition]

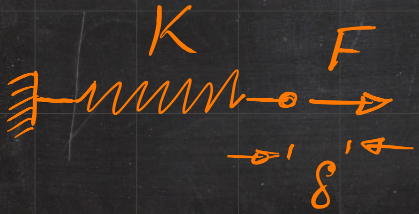
EXTERNAL vs. INTERNAL ENERGY

EXTERNAL vs. INTERNAL ENERGY



EXTERNAL VS. INTERNAL ENERGY

↳ Internal Energy : Stored Energy : Strain Energy ⟶ U_i



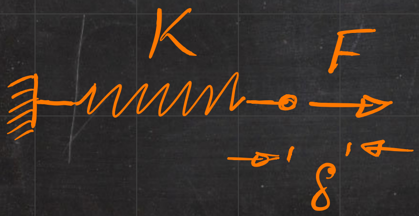
$$U_i = \frac{1}{2} K \delta^2$$

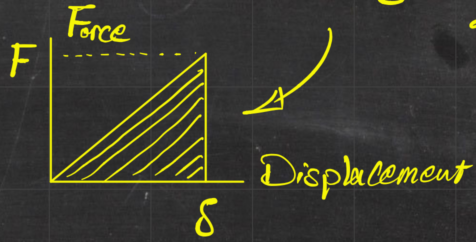
EXTERNAL vs. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy $\leftarrow U_i$

External Energy : Work $\leftarrow U_e$

$$U_e = \frac{1}{2} F \delta$$





$$U_i = \frac{1}{2} K \delta^2$$

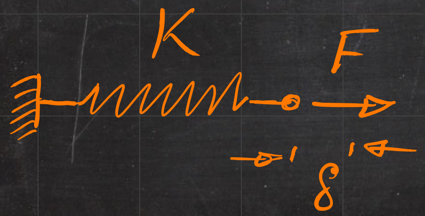
EXTERNAL VS. INTERNAL ENERGY

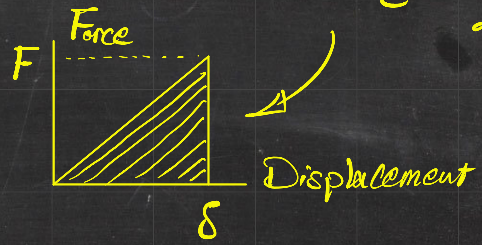
Internal Energy : Stored Energy : Strain Energy ⟵ $\mathcal{U}_i$

External Energy : Work ⟵ $\mathcal{U}_e$

$$\mathcal{U}_e = \frac{1}{2} F \delta$$

$$\mathcal{U}_i = \frac{1}{2} K \delta^2$$





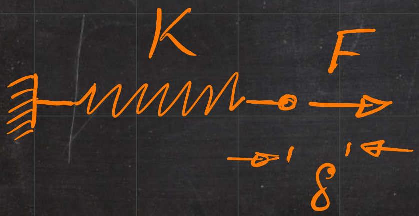
$$\mathcal{U}_e = \mathcal{U}_i \iff F = K\delta$$

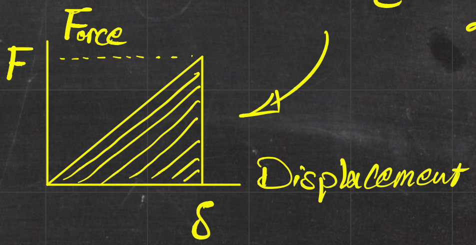
External vs. Internal Energy

Internal Energy : Stored Energy : Strain Energy ← U_i

External Energy : Work ← U_e

$$U_e = \frac{1}{2} F \delta$$





$$U_i = \frac{1}{2} K \delta^2$$

Conservation of Energy → $U_e = U_i \iff F = K\delta$ ← Equilibrium of Forces

EXTERNAL VS. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy ← U_i

External Energy : Work ← U_e

Conservation of Energy → $U_e = U_i \iff F = K\delta$ ← Equilibrium of Forces

EXTERNAL VS. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy ← $\mathcal{U}_i$

External Energy : Work ← $\mathcal{U}_e$

STRESS-STRAIN

CONTRACTION

Conservation of Energy → $\mathcal{U}_e = \mathcal{U}_i \iff F = K\delta$ ← Equilibrium of Forces

EXTERNAL vs. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy $\leftarrow \mathcal{U}_i$

External Energy : Work $\leftarrow \mathcal{U}_e$

STRESS-STRAIN CONTRACTION $\begin{cases} \sigma\text{-}\varepsilon \\ \tau\text{-}\gamma \end{cases}$

Conservation of Energy $\rightarrow$ $\mathcal{U}_e = \mathcal{U}_i \iff F = K\delta \leftarrow$ Equilibrium of Forces

EXTERNAL VS. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy ← U_i

External Energy : Work ← U_e

LOAD - DEFORMATION
CONTRACTION

STRESS - STRAIN
CONTRACTION

$\begin{cases} \sigma - \varepsilon \\ \tau - \gamma \end{cases}$

Equilibrium of Forces

Conservation of Energy → $U_e = U_i \iff F = K\delta$

EXTERNAL vs. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy ⟵ $\mathcal{U}_i$

External Energy : Work ⟵ $\mathcal{U}_e$

LOAD-DEFORMATION CONTRACTION
$\left\{\begin{array}{l} F - \delta \leftarrow \text{AXIAL LOADING} \\ T - \phi \leftarrow \text{TORSION} \\ M - \theta \leftarrow \text{BENDING} \end{array}\right.$

(neglect transverse shear)

STRESS-STRAIN CONTRACTION
$\left\{\begin{array}{l} \sigma - \varepsilon \\ \tau - \gamma \end{array}\right.$

Conservation of Energy ⟶ $\mathcal{U}_e = \mathcal{U}_i \iff F = K\delta$ ⟵ Equilibrium of Forces

EXTERNAL vs. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy ← U_i

External Energy : Work ← U_e

LOAD-DEFORMATION CONTRACTION
- $F - \delta$ ← AXIAL LOADING
- $T - \phi$ ← TORSION
- $M - \theta$ ← BENDING

(neglect transverse shear)

STRESS-STRAIN CONTRACTION
- $\sigma - \varepsilon$
- $\tau - \gamma$

First law of thermodynamics

Conservation of Energy → $U_e = U_i \iff F = K\delta$ ← Equilibrium of Forces

EXTERNAL VS. INTERNAL ENERGY

Internal Energy : Stored Energy : Strain Energy $\leftarrow \mathcal{U}_i$

External Energy : Work $\leftarrow \mathcal{U}_e$

LOAD-DEFORMATION CONTRACTION

- $F - \delta$ ← AXIAL LOADING
- $T - \phi$ ← TORSION
- $M - \theta$ ← BENDING

(neglect transverse shear)

STRESS-STRAIN CONTRACTION

- $\sigma - \varepsilon$
- $\tau - \gamma$

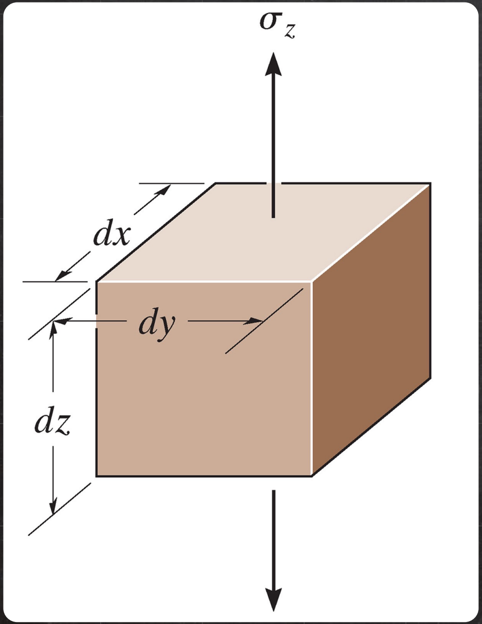
First law of thermodynamics

Remark: Elasticity → No Dissipation

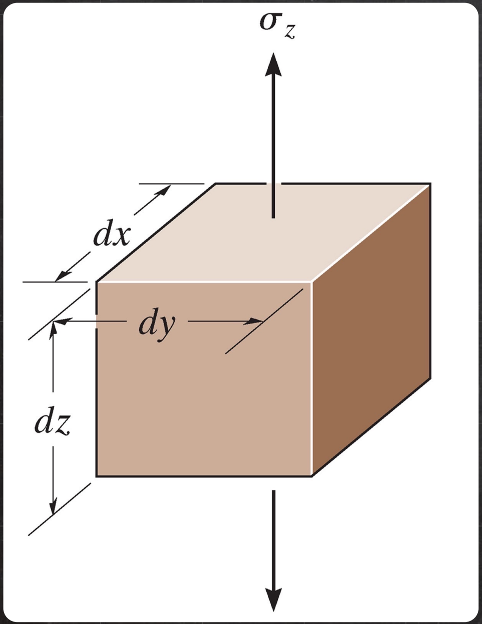
Conservation of Energy → $\mathcal{U}_e = \mathcal{U}_i \iff F = K\delta$ ← Equilibrium of Forces

STRAIN ENERGY

Strain Energy



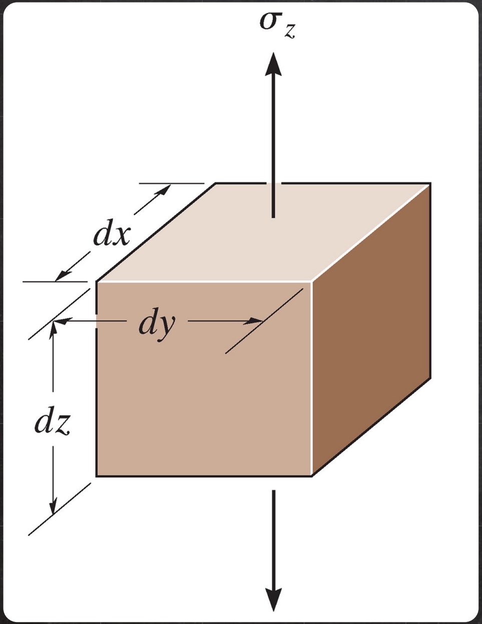
STRAIN ENERGY



Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N.m}{m^3}\right]$

$\frac{1}{2}\sigma\varepsilon$ INTERNAL ENERGY DENSITY

Strain Energy

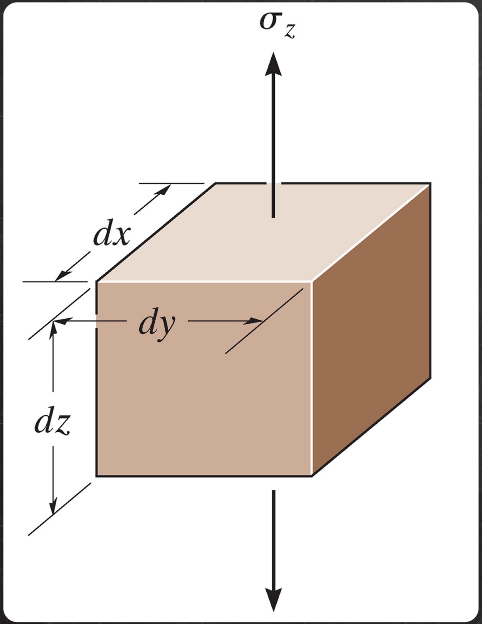


Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N.m}{m^3}\right]$

$\frac{1}{2}\sigma\varepsilon$ INTERNAL ENERGY DENSITY

$$u_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV$$

STRAIN ENERGY



Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N\cdot m}{m^3}\right]$

$\frac{1}{2}\sigma\varepsilon$ INTERNAL ENERGY DENSITY

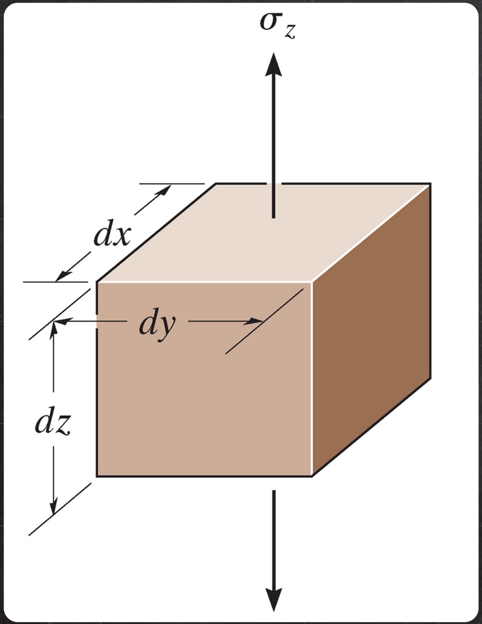
$$U_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV$$

$$= \int_V \frac{1}{2} E\varepsilon^2 \, dV$$

STRAIN ENERGY

Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N \cdot m}{m^3}\right]$

$\frac{1}{2}\sigma\varepsilon$ INTERNAL ENERGY DENSITY



$$U_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV$$

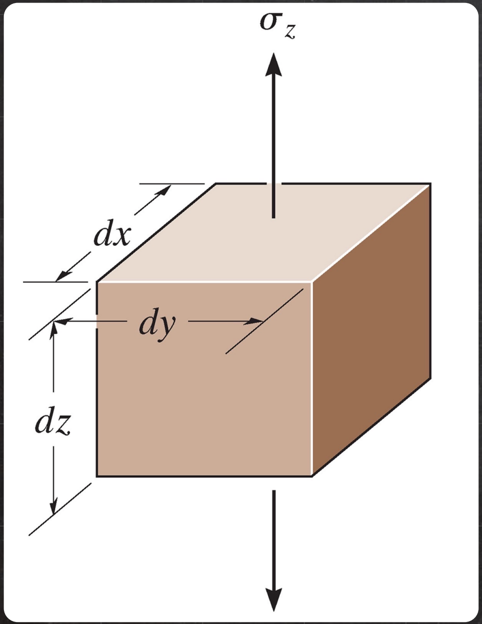
$$= \int_V \frac{1}{2} E\varepsilon^2 \, dV$$

$$= \int_V \frac{1}{2} \frac{\sigma^2}{E} \, dV$$

Strain Energy

Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N \cdot m}{m^3}\right]$

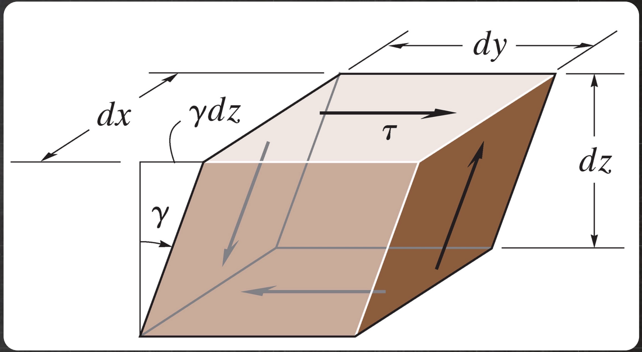
$\frac{1}{2}\sigma\varepsilon$ Internal energy density



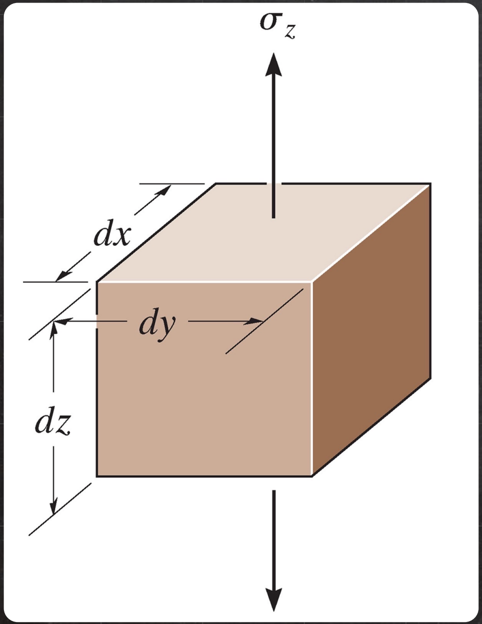
$$U_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV$$

$$= \int_V \frac{1}{2}E\varepsilon^2 \, dV$$

$$= \int_V \frac{1}{2}\frac{\sigma^2}{E} \, dV$$

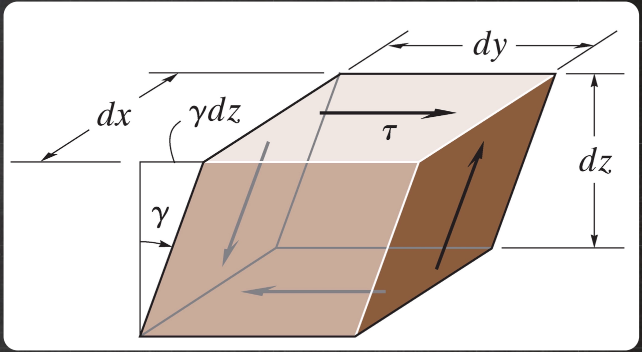


Strain Energy



Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N\cdot m}{m^3}\right]$

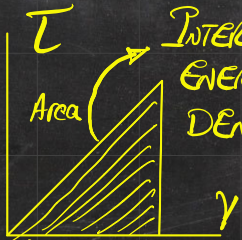
$\frac{1}{2}\sigma\varepsilon$ Internal Energy Density



$$U_i = \int_V \frac{1}{2}\sigma\varepsilon\, dV$$

$$= \int_V \frac{1}{2} E\varepsilon^2\, dV$$

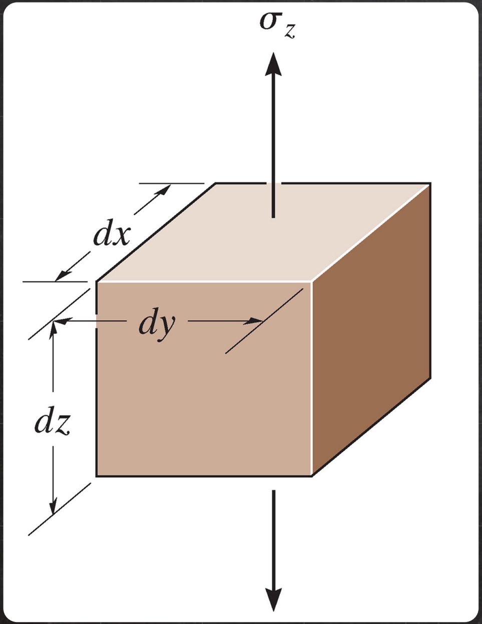
$$= \int_V \frac{1}{2} \frac{\sigma^2}{E}\, dV$$



Internal Energy Density

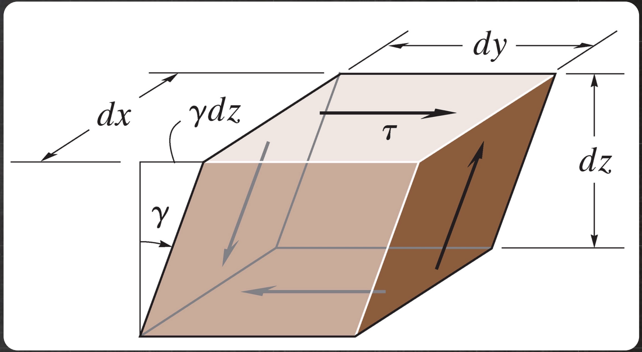
$\frac{1}{2}\tau\gamma$

STRAIN ENERGY



Area : $\frac{1}{2}\sigma\varepsilon \left[\frac{N}{m^2}\right] \equiv \left[\frac{N\cdot m}{m^3}\right]$

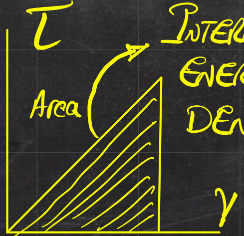
$\frac{1}{2}\sigma\varepsilon$ INTERNAL ENERGY DENSITY



$$U_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV$$

$$= \int_V \frac{1}{2}E\varepsilon^2 \, dV$$

$$= \int_V \frac{1}{2}\frac{\sigma^2}{E} \, dV$$



$\frac{1}{2}\tau\gamma$

INTERNAL ENERGY DENSITY

$$U_i = \int_V \frac{1}{2}\tau\gamma \, dV$$

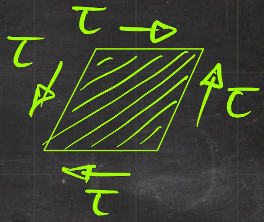
$$= \int_V \frac{1}{2}G\gamma^2 \, dV$$

$$= \int_V \frac{1}{2}\frac{\tau^2}{G} \, dV$$

Strain Energy

$$u_i = \int_V \frac{1}{2}\sigma\varepsilon\, dV = \int_V \frac{1}{2}E\varepsilon^2\, dV = \int_V \frac{1}{2}\frac{\sigma^2}{E}\, dV$$

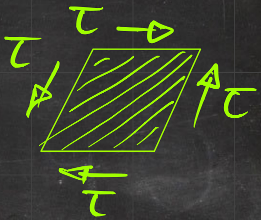
$$u_i = \int_V \frac{1}{2}\tau\gamma\, dV = \int_V \frac{1}{2}G\gamma^2\, dV = \int_V \frac{1}{2}\frac{\tau^2}{G}\, dV$$



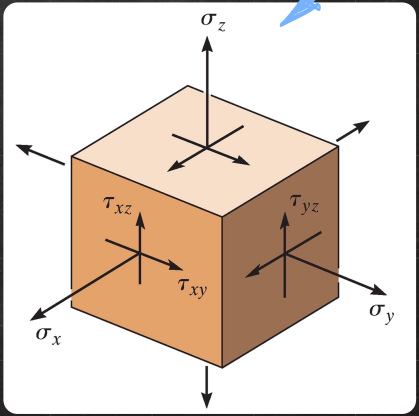
STRAIN ENERGY

$$u_i = \int_V \frac{1}{2}\sigma\varepsilon\, dV = \int_V \frac{1}{2}E\varepsilon^2\, dV = \int_V \frac{1}{2}\frac{\sigma^2}{E}\, dV$$

$$u_i = \int_V \frac{1}{2}\tau\gamma\, dV = \int_V \frac{1}{2}G\gamma^2\, dV = \int_V \frac{1}{2}\frac{\tau^2}{G}\, dV$$



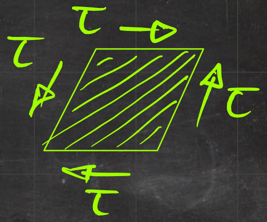
MULTIAXIAL STRESS STATE



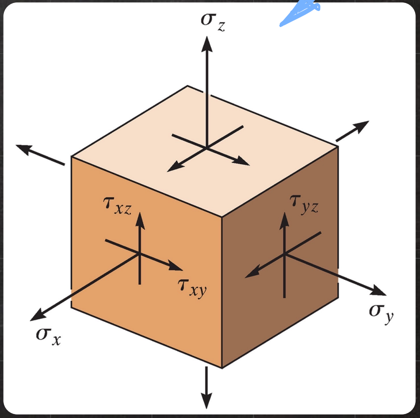
Strain Energy

$$\mathcal{U}_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV = \int_V \frac{1}{2} E \varepsilon^2 \, dV = \int_V \frac{1}{2}\frac{\sigma^2}{E} \, dV$$

$$\mathcal{U}_i = \int_V \frac{1}{2}\tau\gamma \, dV = \int_V \frac{1}{2} G \gamma^2 \, dV = \int_V \frac{1}{2}\frac{\tau^2}{G} \, dV$$



MULTIAXIAL STRESS STATE



$$\mathcal{U}_i = \int_V \Big[\frac{1}{2}\sigma_x\varepsilon_x + \frac{1}{2}\sigma_y\varepsilon_y + \frac{1}{2}\sigma_z\varepsilon_z + \frac{1}{2}\tau_{xy}\gamma_{xy} + \frac{1}{2}\tau_{xz}\gamma_{xz} + \frac{1}{2}\tau_{yz}\gamma_{yz} \Big] \, dV$$

STRAIN ENERGY

$$\mathcal{U}_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV = \int_V \frac{1}{2}E\varepsilon^2 \, dV = \int_V \frac{1}{2}\frac{\sigma^2}{E} \, dV$$

$$\mathcal{U}_i = \int_V \frac{1}{2}\tau\gamma \, dV = \int_V \frac{1}{2}G\gamma^2 \, dV = \int_V \frac{1}{2}\frac{\tau^2}{G} \, dV$$

EXTERNAL WORK

$\hookrightarrow \mathcal{U}_e \rightsquigarrow \begin{cases} \text{AXIAL LOADING} \\ \text{TORSION} \\ \text{BENDING} \end{cases}$

STRAIN ENERGY

$$\mathcal{U}_i = \int_V \frac{1}{2}\sigma\varepsilon \, dV = \int_V \frac{1}{2}E\varepsilon^2 \, dV = \int_V \frac{1}{2}\frac{\sigma^2}{E} \, dV$$

$$\mathcal{U}_i = \int_V \frac{1}{2}\tau\gamma \, dV = \int_V \frac{1}{2}G\gamma^2 \, dV = \int_V \frac{1}{2}\frac{\tau^2}{G} \, dV$$

$\mathcal{U}_i$

What comes out from setting

$\mathcal{U}_i = \mathcal{U}_e$

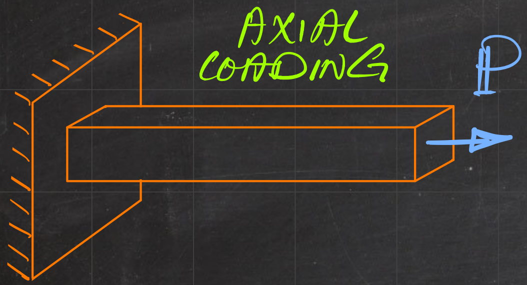
?

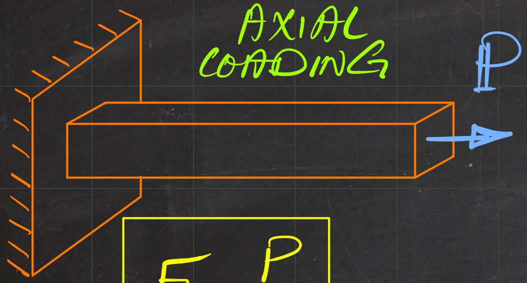
EXTERNAL WORK

$\rightarrow \mathcal{U}_e \rightsquigarrow$

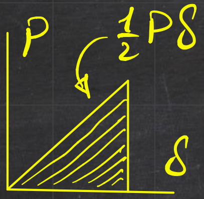
- AXIAL LOADING
- TORSION
- BENDING

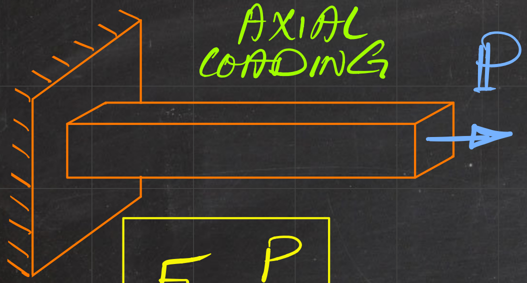
$\mathcal{U}_e$


AXIAL
LOADING
P

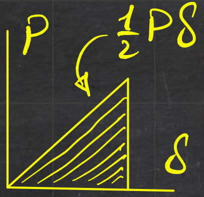


$$\boxed{\sigma = \frac{P}{A}}$$

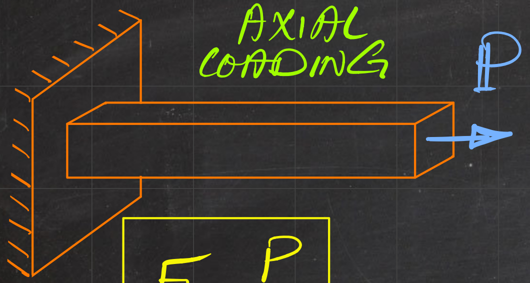




$$\boxed{\sigma = \frac{P}{A}}$$



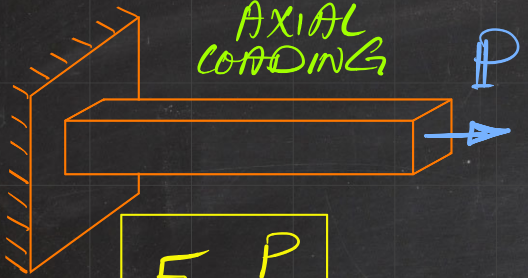
$$\frac{1}{2} P \delta = \int_V \frac{1}{2} \frac{\sigma^2}{E} \, dV$$



$$\boxed{\sigma = \frac{P}{A}}$$

$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

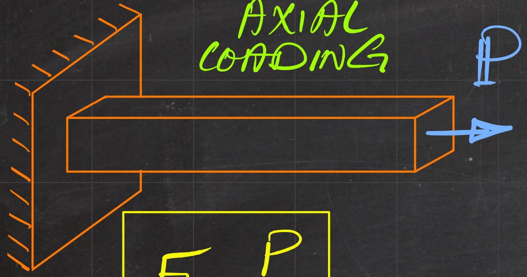


$$\boxed{\sigma = \frac{P}{A}}$$

$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

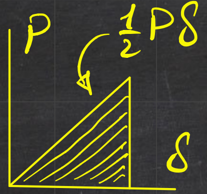
$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}\,A\,dx$$

$$= \frac{1}{2}P^2L/EA$$



$$\boxed{\sigma = \frac{P}{A}}$$

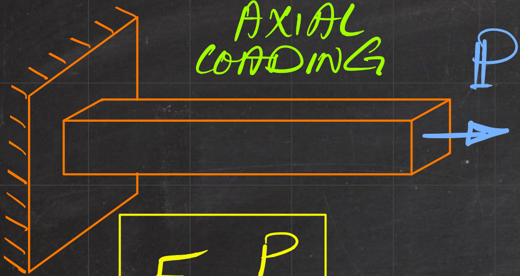
$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

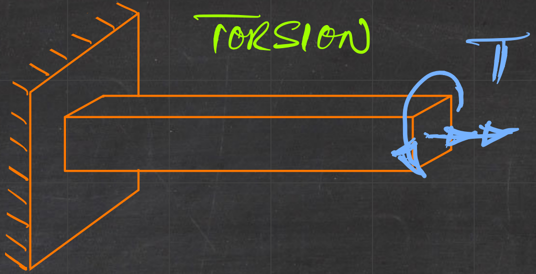


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

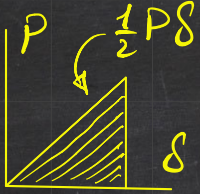
$$= \frac{1}{2}P^2L/EA$$





$$\boxed{\sigma = \frac{P}{A}}$$

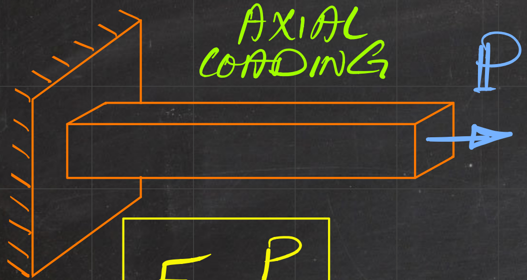
$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

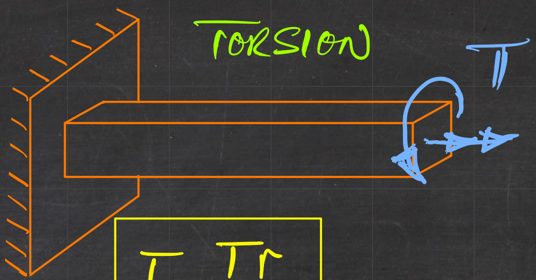


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

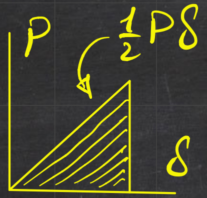
$$= \frac{1}{2}P^2L/EA$$



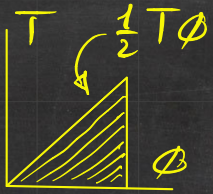


$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$



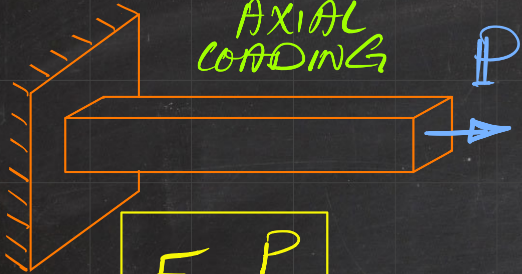
$$\boxed{\tau = \frac{Tr}{J}}$$

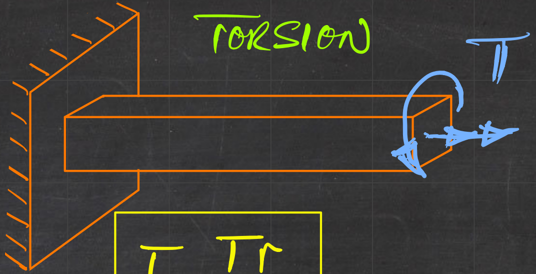


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

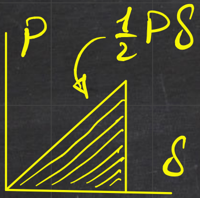
$$= \frac{1}{2}P^2L/EA$$



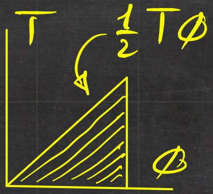


$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$



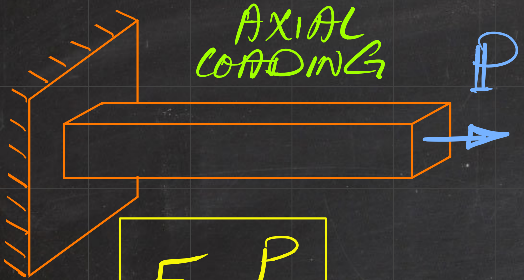
$$\boxed{\tau = \frac{Tr}{J}}$$

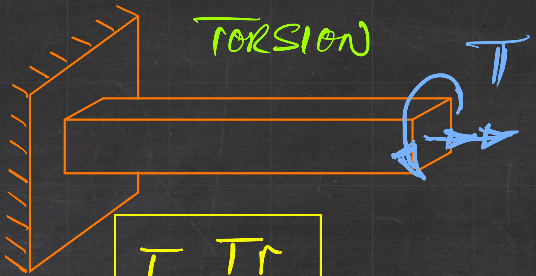


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV \qquad \frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

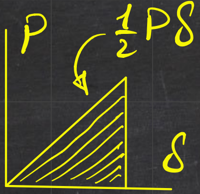
$$= \frac{1}{2}P^2L/EA$$



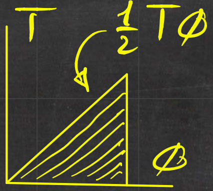


$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$



$$\boxed{\tau = \frac{Tr}{J}}$$



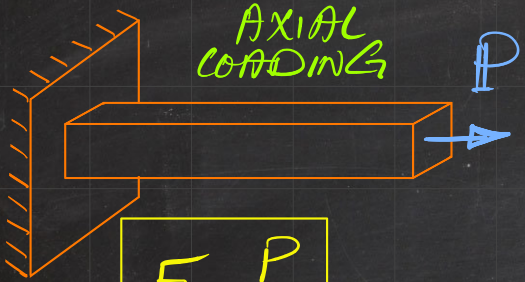
$$\frac{1}{2} P \delta = \int_V \frac{1}{2} \frac{\sigma^2}{E} \, dV$$

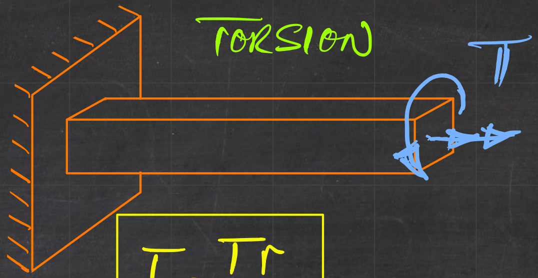
$$= \int_0^L \frac{1}{2} \frac{P^2}{A^2} \frac{1}{E} A \, dx$$

$$= \frac{1}{2} P^2 L / EA$$

$$\frac{1}{2} T \phi = \int_V \frac{1}{2} \frac{\tau^2}{G} \, dV \qquad J$$

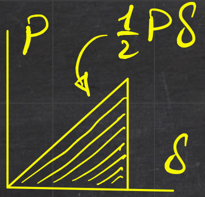
$$= \int_0^L \int_A \frac{1}{2} \frac{T^2 r^2}{J^2} \frac{1}{G} \, dA \, dx$$



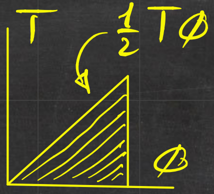


$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$



$$\boxed{\tau = \frac{Tr}{J}}$$



$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

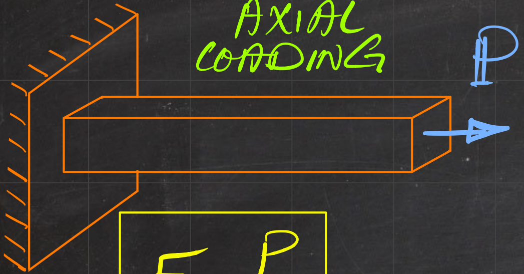
$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

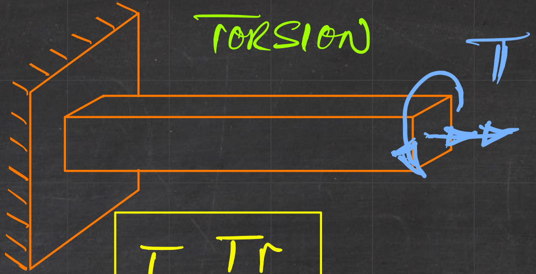
$$= \frac{1}{2}P^2L/EA$$

$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV \qquad J$$

$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

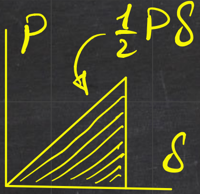
$$= \frac{1}{2}T^2L/GJ$$





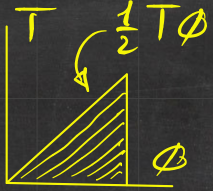
$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$



$$\boxed{\tau = \frac{Tr}{J}}$$

$$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$



$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

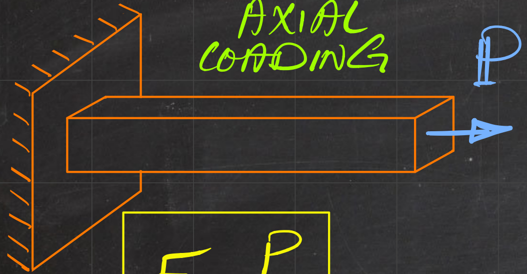
$$= \frac{1}{2}P^2L/EA$$

$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV \qquad J$$

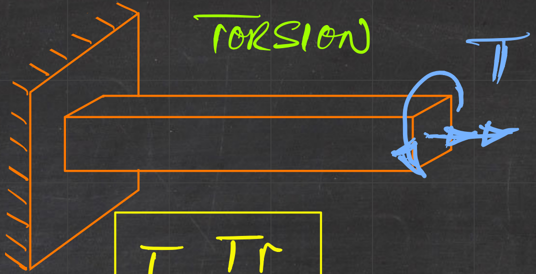
$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

$$= \frac{1}{2}T^2L/GJ$$

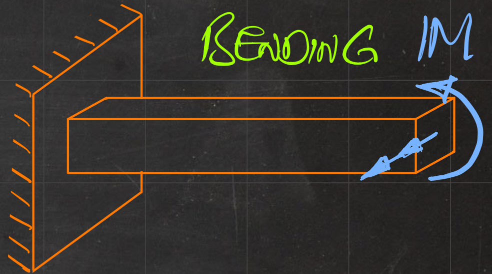
AXIAL LOADING



TORSION

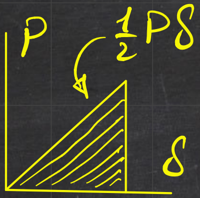


BENDING



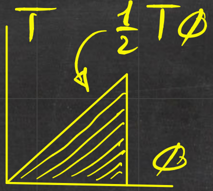
$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$



$$\boxed{\tau = \frac{Tr}{J}}$$

$$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$



$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

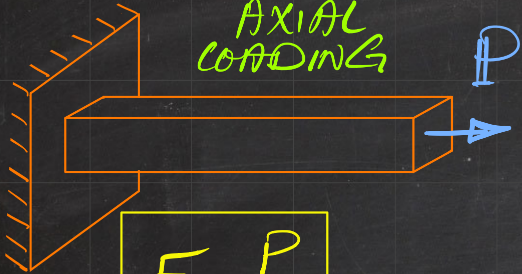
$$= \frac{1}{2}P^2L/EA$$

$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV \qquad J$$

$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

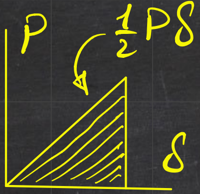
$$= \frac{1}{2}T^2L/GJ$$

AXIAL LOADING

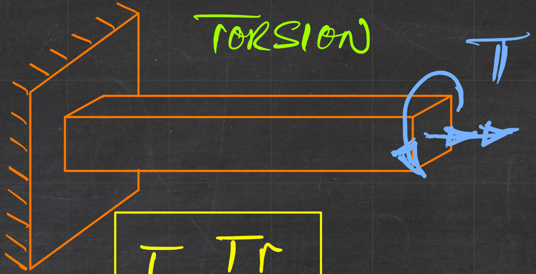


$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

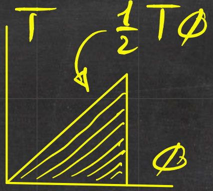


TORSION

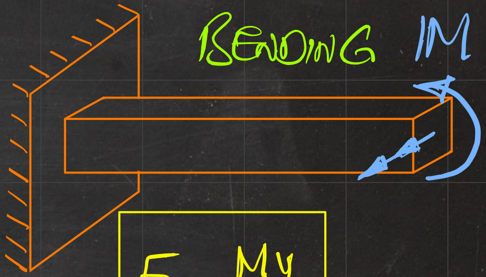


$$\boxed{\tau = \frac{Tr}{J}}$$

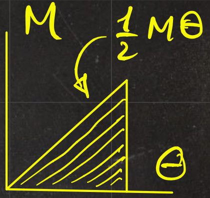
$$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$



BENDING



$$\boxed{\sigma = -\frac{My}{I}}$$



$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

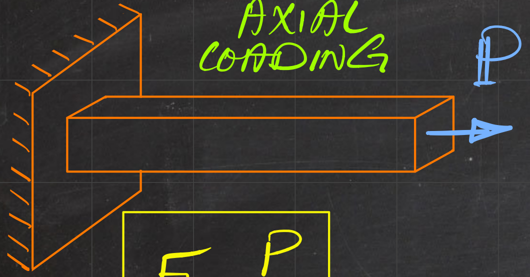
$$= \frac{1}{2}P^2L/EA$$

$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV \qquad J$$

$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

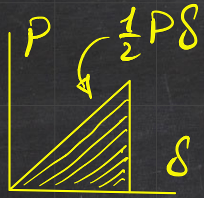
$$= \frac{1}{2}T^2L/GJ$$

AXIAL LOADING



$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

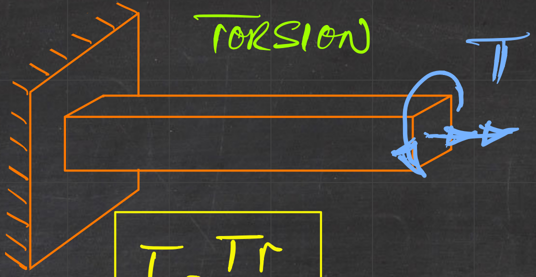


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

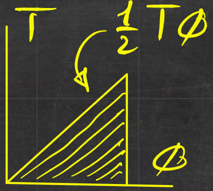
$$= \frac{1}{2}P^2L/EA$$

TORSION



$$\boxed{\tau = \frac{Tr}{J}}$$

$$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$

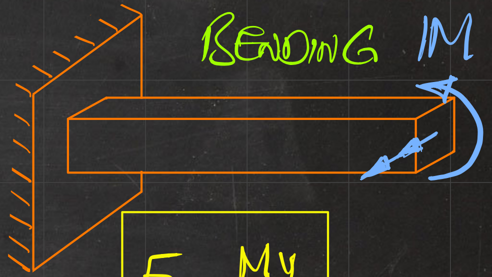


$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV \qquad J$$

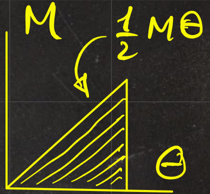
$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

$$= \frac{1}{2}T^2L/GJ$$

BENDING

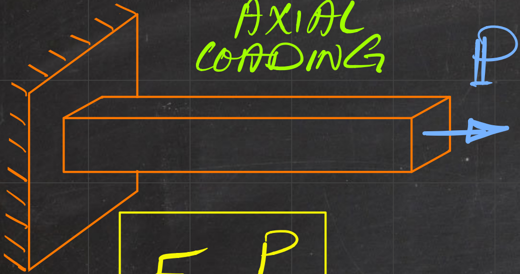


$$\boxed{\sigma = -\frac{My}{I}}$$



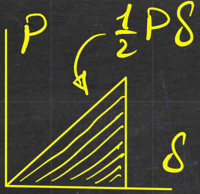
$$\frac{1}{2}M\theta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

AXIAL LOADING



$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

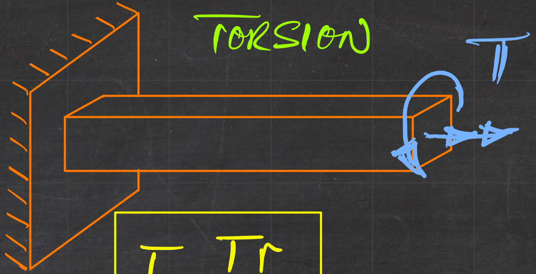


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

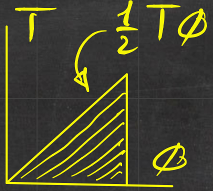
$$= \frac{1}{2}P^2L/EA$$

TORSION



$$\boxed{\tau = \frac{Tr}{J}}$$

$$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$

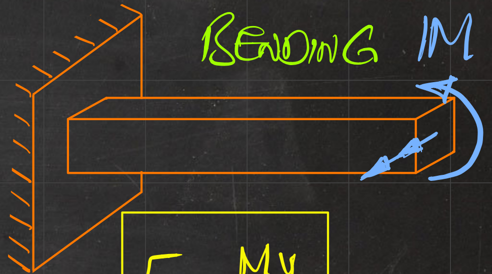


$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV \qquad J$$

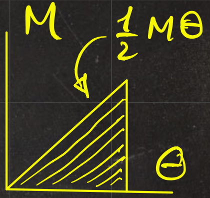
$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

$$= \frac{1}{2}T^2L/GJ$$

BENDING



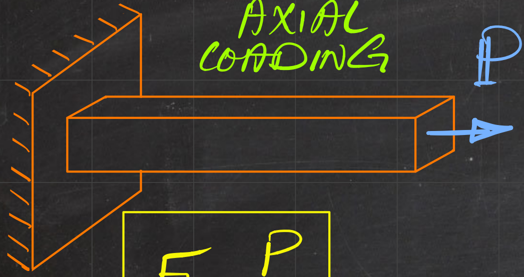
$$\boxed{\sigma = -\frac{My}{I}}$$



$$\frac{1}{2}M\theta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV \qquad I$$

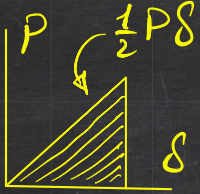
$$= \int_0^L\int_A \frac{1}{2}\frac{M^2y^2}{I^2}\frac{1}{E}\,dA\,dx$$

AXIAL LOADING



$$\boxed{\sigma = \frac{P}{A}}$$

$$\Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

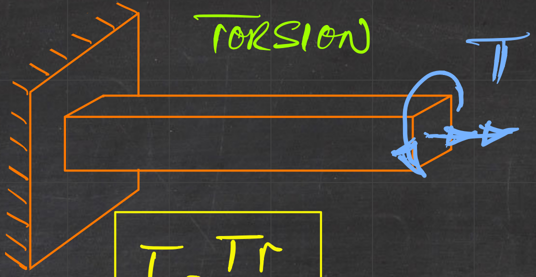


$$\frac{1}{2}P\delta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L \frac{1}{2}\frac{P^2}{A^2}\frac{1}{E}A\,dx$$

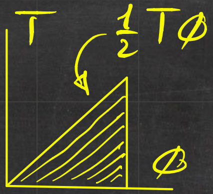
$$= \frac{1}{2}P^2L/EA$$

TORSION



$$\boxed{\tau = \frac{Tr}{J}}$$

$$\Rightarrow \boxed{\phi = \frac{TL}{GJ}}$$

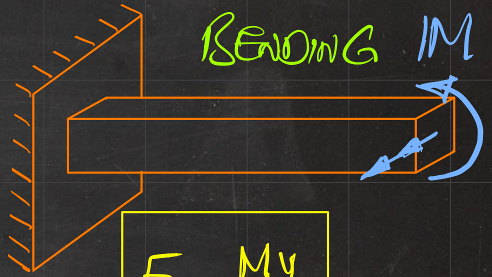


$$\frac{1}{2}T\phi = \int_V \frac{1}{2}\frac{\tau^2}{G}\,dV$$

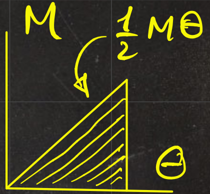
$$= \int_0^L\int_A \frac{1}{2}\frac{T^2r^2}{J^2}\frac{1}{G}\,dA\,dx$$

$$= \frac{1}{2}T^2L/GJ$$

BENDING



$$\boxed{\sigma = -\frac{My}{I}}$$

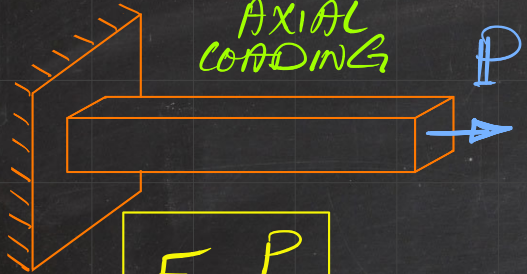


$$\frac{1}{2}M\theta = \int_V \frac{1}{2}\frac{\sigma^2}{E}\,dV$$

$$= \int_0^L\int_A \frac{1}{2}\frac{M^2y^2}{I^2}\frac{1}{E}\,dA\,dx$$

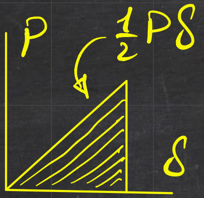
$$= \frac{1}{2}M^2L/EI$$

AXIAL LOADING



$$\sigma = \frac{P}{A}$$

$$\Rightarrow \quad \delta = \frac{PL}{EA}$$

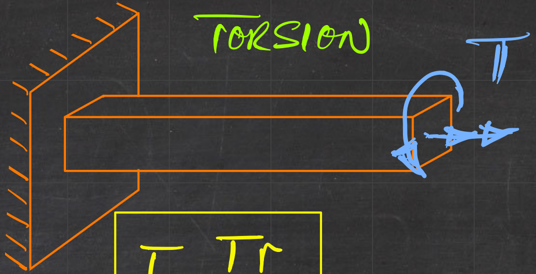


$$\frac{1}{2} P \delta = \int_V \frac{1}{2} \frac{\sigma^2}{E} \, dV$$

$$= \int_0^L \frac{1}{2} \frac{P^2}{A^2} \frac{1}{E} A \, dx$$

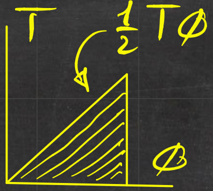
$$= \frac{1}{2} P^2 L / EA$$

TORSION



$$\tau = \frac{Tr}{J}$$

$$\Rightarrow \quad \phi = \frac{TL}{GJ}$$

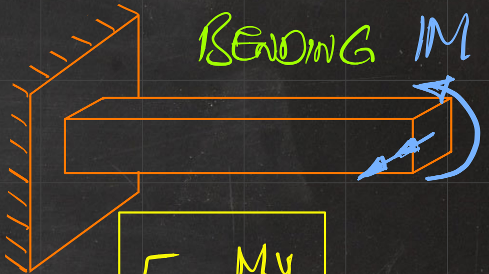


$$\frac{1}{2} T \phi = \int_V \frac{1}{2} \frac{\tau^2}{G} \, dV$$

$$= \int_0^L \int_A \frac{1}{2} \frac{T^2 r^2}{J^2} \frac{1}{G} \, dA \, dx \qquad J$$

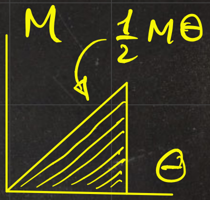
$$= \frac{1}{2} T^2 L / GJ$$

BENDING



$$\sigma = -\frac{My}{I}$$

$$\Rightarrow \quad \theta = \frac{ML}{EI}$$



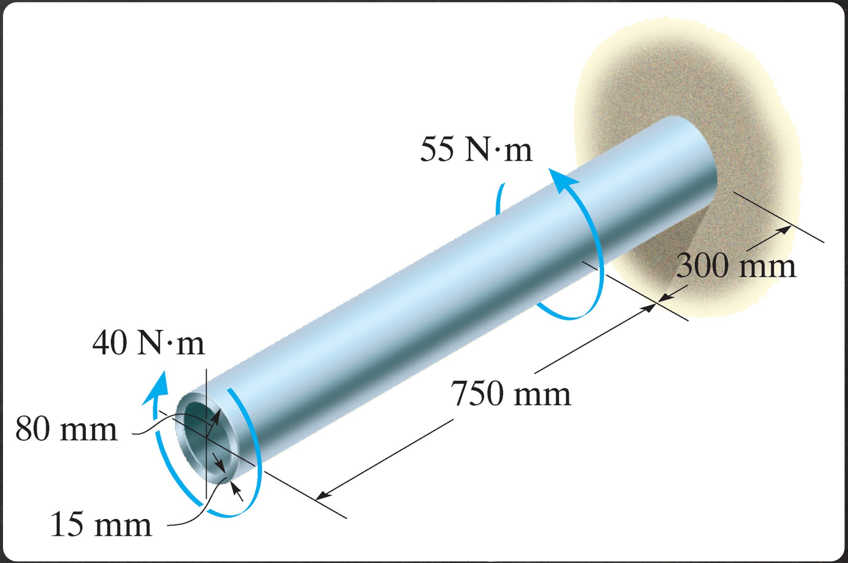
$$\frac{1}{2} M \theta = \int_V \frac{1}{2} \frac{\sigma^2}{E} \, dV$$

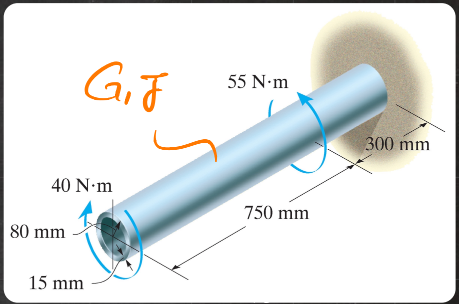
$$= \int_0^L \int_A \frac{1}{2} \frac{M^2 y^2}{I^2} \frac{1}{E} \, dA \, dx \qquad I$$

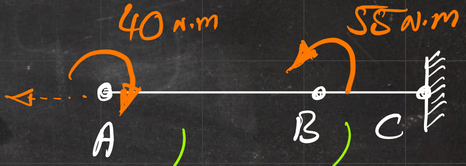
$$= \frac{1}{2} M^2 L / EI$$

Exercise 1 . [similar to … P. 732 … 14.5]

FOR THE SHAFT SHOWN IN THE FIGURE, COMPUTE THE STRAIN ENERGY AND EXTERNAL WORK. ASSUME G AND J AS GIVEN CONSTANTS.

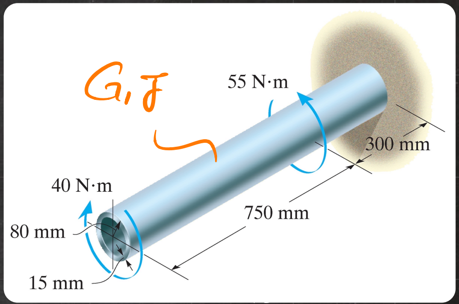


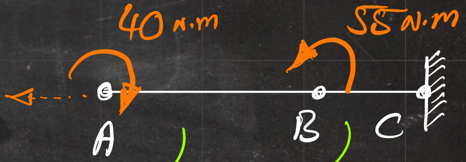




$T_{AB} = -40 \qquad T_{BC} = +15$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\, dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

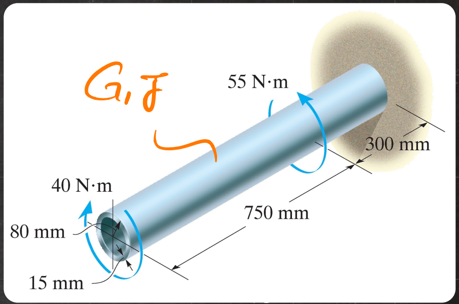




$T_{AB} = -40$ $T_{BC} = +15$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\,dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

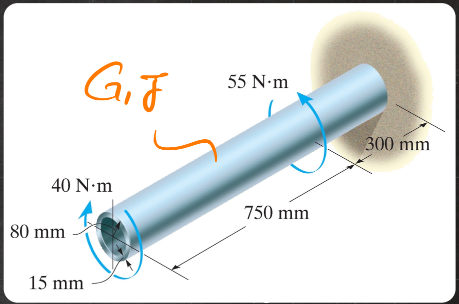
$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ}$$

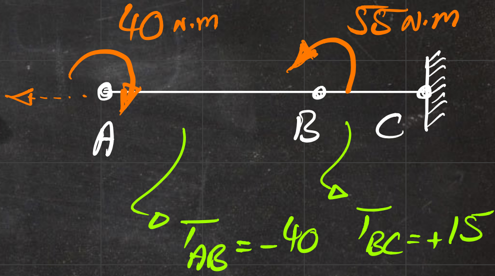


$T_{AB} = -40$ $T_{BC} = +15$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\,dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

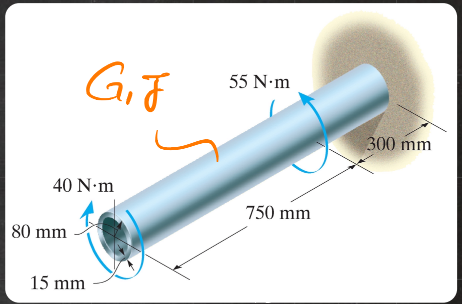
$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$

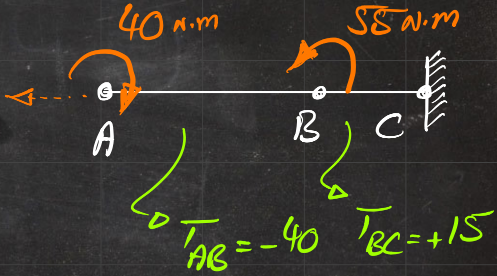




$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\, dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$

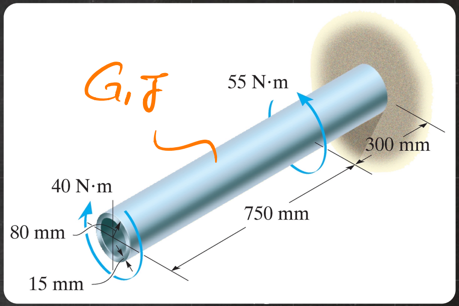


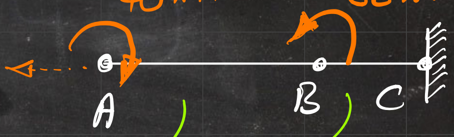


$$U_e = \frac{1}{2} T_A \phi_A + \frac{1}{2} T_B \phi_B$$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\,dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$





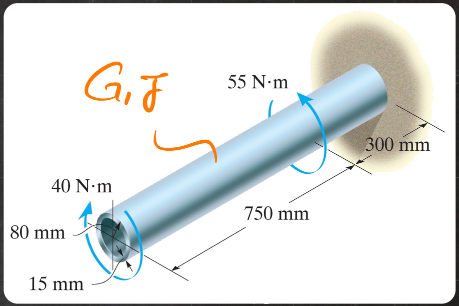
$T_{AB} = -40$ $\quad T_{BC} = +15$

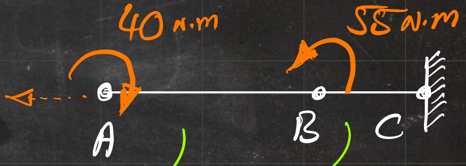
$$U_e = \frac{1}{2}T_A\phi_A + \frac{1}{2}T_B\phi_B \quad \phi_B = \phi_{B/C}$$

$$\phi_A = \phi_{A/B} + \phi_{B/C}$$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\,dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$





$T_{AB} = -40 \qquad T_{BC} = +15$

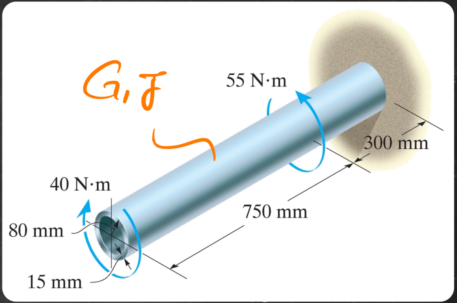
$$U_e = \frac{1}{2} T_A \phi_A + \frac{1}{2} T_B \phi_B \quad 2o \quad \phi_B = \phi_{B/C} = \frac{\overset{15}{T_{BC}}\,\overset{0.3}{L_{BC}}}{GJ} = \frac{4.5}{GJ}$$

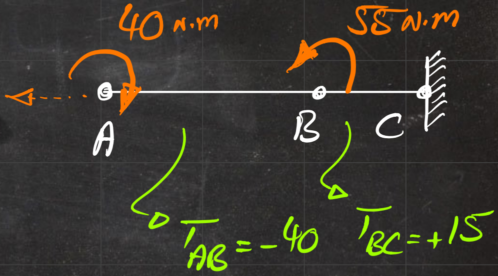
$$\phi_A = \phi_{A/B} + \phi_{B/C}$$

$$= \frac{\overset{-40}{T_{AB}}\,\overset{0.75}{L_{AB}}}{GJ} + \frac{\overset{15}{T_{BC}}\,\overset{0.3}{L_{BC}}}{GJ} = -\frac{25.5}{GJ}$$

$$\mathcal{U}_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\,dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$



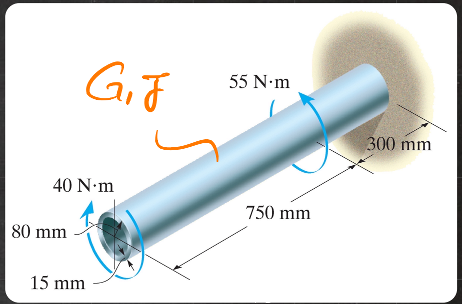


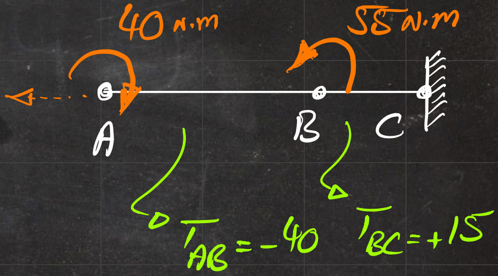
$$\mathcal{U}_e = \frac{1}{2} T_A \phi_A + \frac{1}{2} T_B \phi_B \quad 2_o \quad \frac{4.5}{GJ}$$

$$-\frac{25.5}{GJ}$$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\, dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$





$$U_e = \frac{1}{2} T_A \phi_A + \frac{1}{2} T_B \phi_B$$

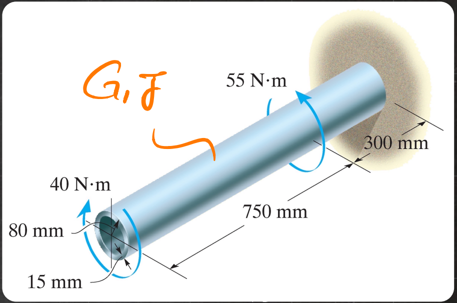
$\phi_B \to \frac{4.5}{GJ}$

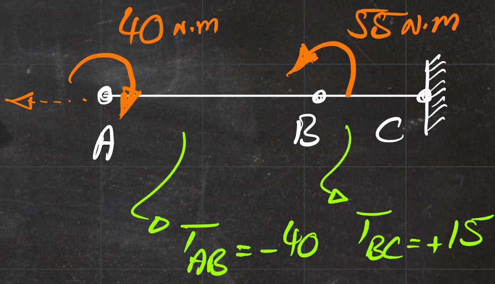
$\phi_A \to -\frac{25.5}{GJ}$

$$\Rightarrow U_e = \frac{1}{2} \times (-40) \times \left(-\frac{25.5}{GJ}\right) + \frac{1}{2} \times 55 \times \frac{4.5}{GJ}$$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\,dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$





$$U_e = \frac{1}{2} T_A \phi_A + \frac{1}{2} T_B \phi_B$$

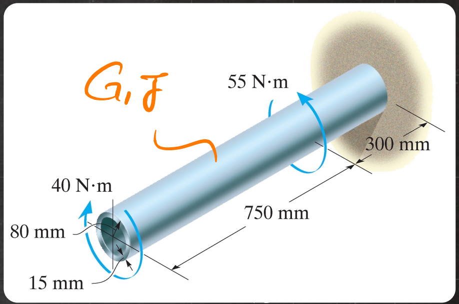
$$\phi_B = \frac{4.5}{GJ}$$

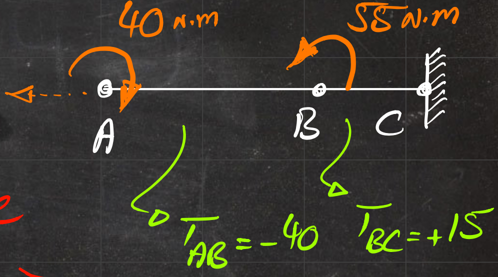
$$\phi_A = -\frac{25.5}{GJ}$$

$$\Rightarrow U_e = \frac{1}{2} \times (-40) \times \left(-\frac{25.5}{GJ}\right) + \frac{1}{2} \times 55 \times \frac{4.5}{GJ} \Rightarrow U_e = \frac{633.75}{GJ}$$

$$U_i = \frac{1}{2}\int_V \frac{\tau^2}{G}\, dV = \frac{1}{2}\sum \frac{T^2 L}{GJ}$$

$$= \frac{1}{2}\,\frac{(-40)^2 \times 0.75}{GJ} + \frac{1}{2}\,\frac{15 \times 0.3}{GJ} = \frac{633.75}{GJ}$$





$$U_e = \frac{1}{2} T_A \phi_A + \frac{1}{2} T_B \phi_B$$

$$\phi_B \to \frac{4.5}{GJ} \qquad \phi_A \to -\frac{25.5}{GJ}$$

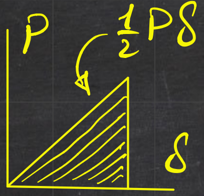
$$U_i = U_e \quad \checkmark$$

$$\Rightarrow U_e = \frac{1}{2} \times (-40) \times \left(-\frac{25.5}{GJ}\right) + \frac{1}{2} \times 55 \times \frac{4.5}{GJ} \Rightarrow U_e = \frac{633.75}{GJ}$$

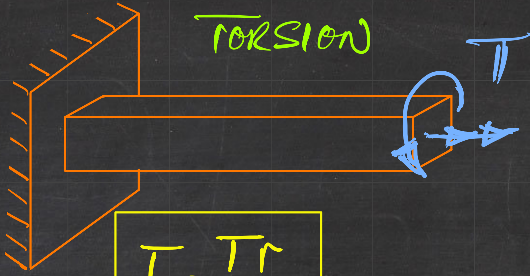
AXIAL LOADING

$$\sigma = \frac{P}{A}$$

$$\delta = \frac{PL}{EA}$$

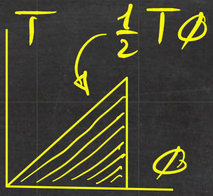


TORSION

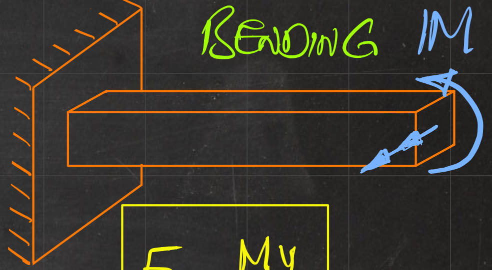


$$\tau = \frac{Tr}{J}$$

$$\phi = \frac{TL}{GJ}$$

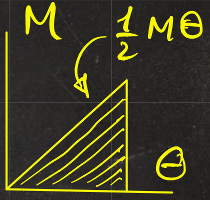


BENDING

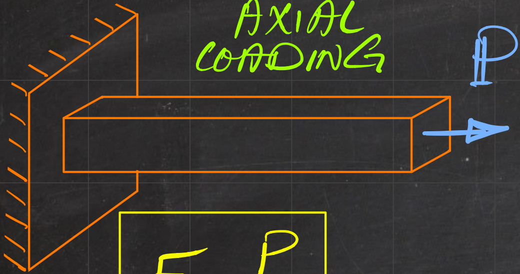


$$\sigma = -\frac{My}{I}$$

$$\theta = \frac{ML}{EI}$$



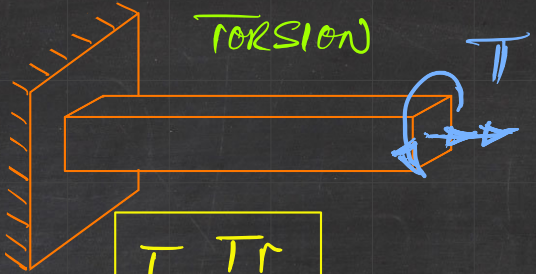
AXIAL LOADING



$$\sigma = \frac{P}{A}$$

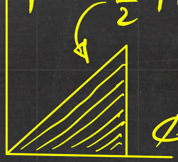
$$\delta = \frac{PL}{EA}$$

TORSION

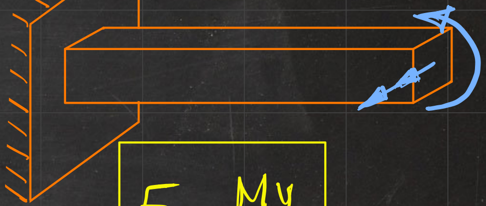


$$\tau = \frac{Tr}{J}$$

$$\phi = \frac{TL}{GJ}$$

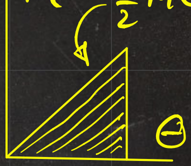


BENDING



$$\sigma = -\frac{My}{I}$$

$$\theta = \frac{ML}{EI}$$



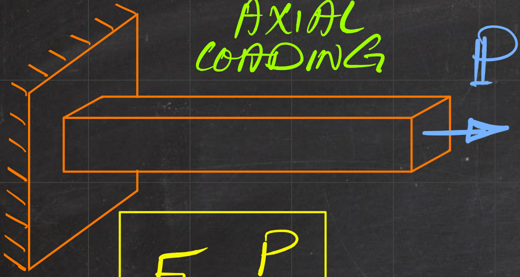
FOR ANY EXAMPLE,

$$U_i = U_e$$

WE CAN SHOW THAT

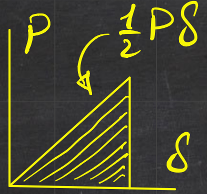
ENERGY CONSERVATION HOLDS!

AXIAL LOADING

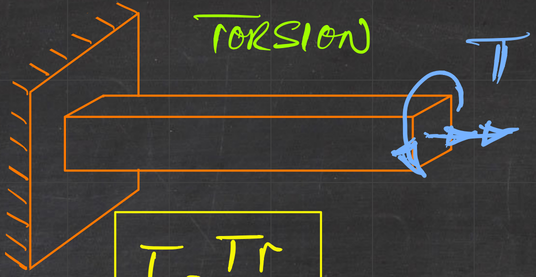


$$\sigma = \frac{P}{A}$$

$$\delta = \frac{PL}{EA}$$

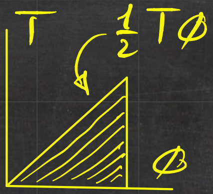


TORSION

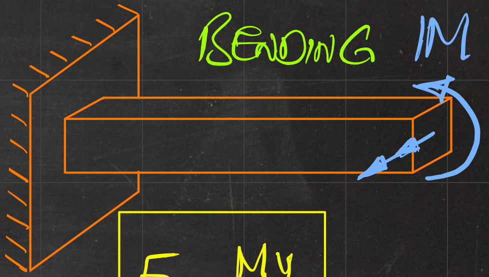


$$\tau = \frac{Tr}{J}$$

$$\phi = \frac{TL}{GJ}$$

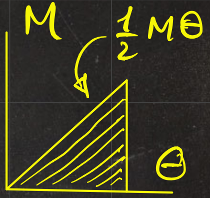


BENDING



$$\sigma = -\frac{My}{I}$$

$$\theta = \frac{ML}{EI}$$



FOR ANY EXAMPLE, WE CAN SHOW THAT

$$U_i = U_e$$

ENERGY CONSERVATION HOLDS!

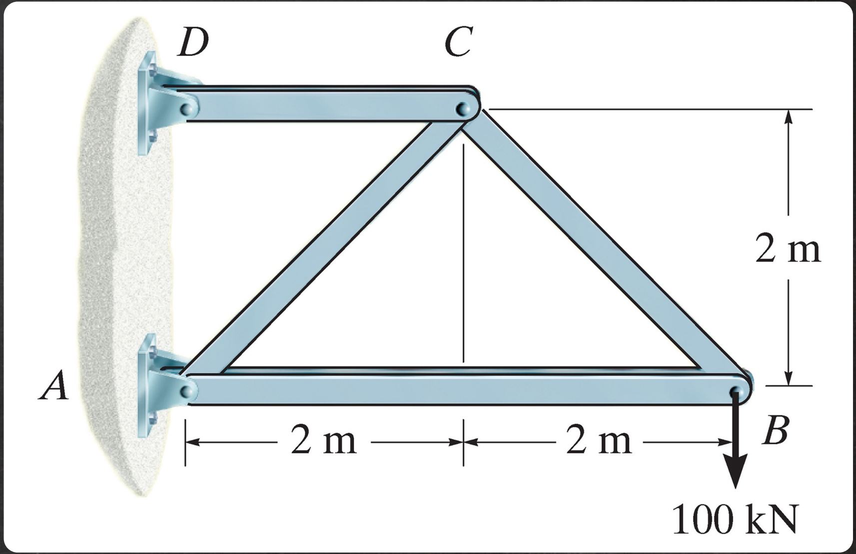
$\Rightarrow$ NEXT, WE EMPLOY THIS RELATION AS STARTING POINT TO COMPUTE DEFORMATION

(e.g. truss structures)

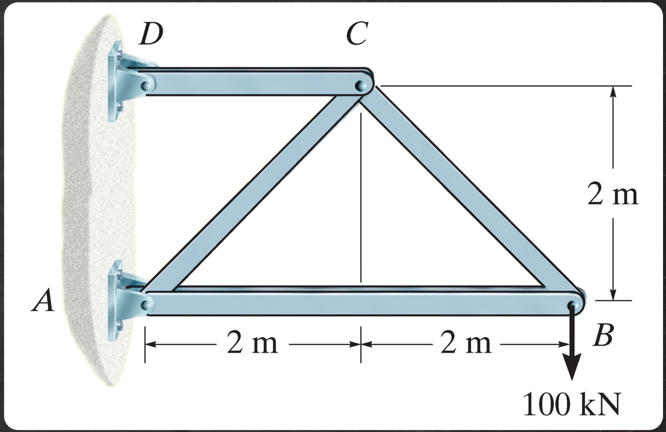
Exercise 2 . [similar to ... P. 762 ... 14.11]

DETERMINE THE VERTICAL DISPLACEMENT OF JOINT B OF THE TRUSS STRUCTURE SHOWN IN THE FIGURE.

THE CROSS-SECTIONAL AREA OF EACH MEMBER IS $A = 400\ mm^2$ AND $E = 200\ GPa$.



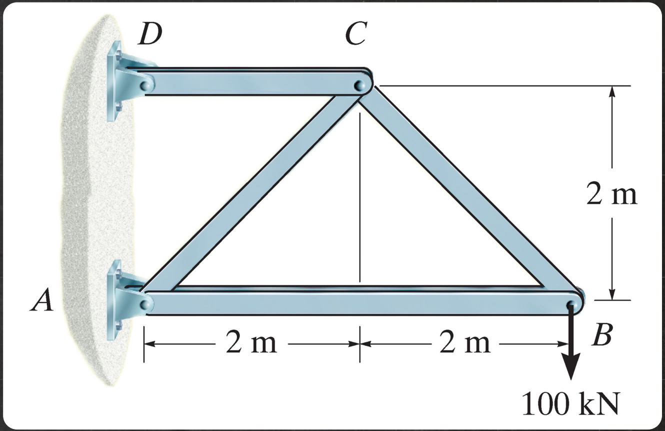
$\mathcal{U}_e = \mathcal{U}_i$



$$U_e = U_i$$

$$\rightsquigarrow \frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

displacement at B in the same direction as F_B

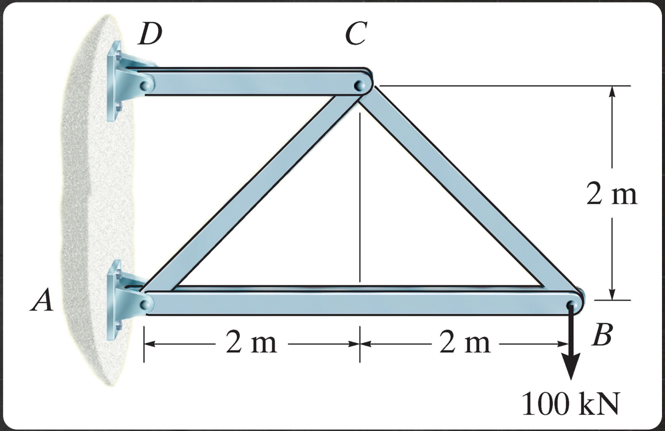


$$U_e = U_i$$

$$\leadsto \frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

displacement at B in the same direction as F_B

$$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$$

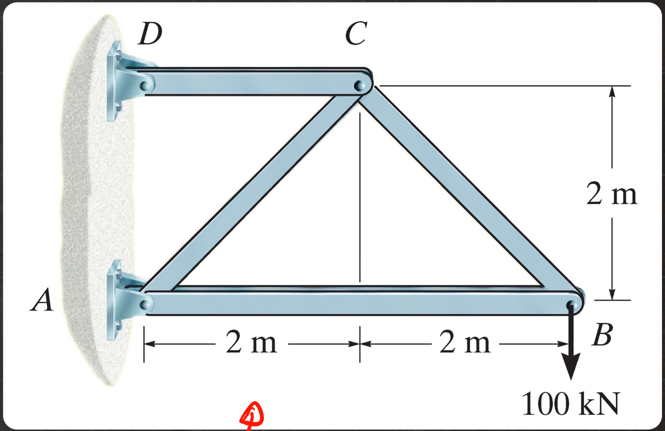


$$\mathcal{U}_e = \mathcal{U}_i$$

displacement at B in the same direction as F_B

$$\frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

$$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$$



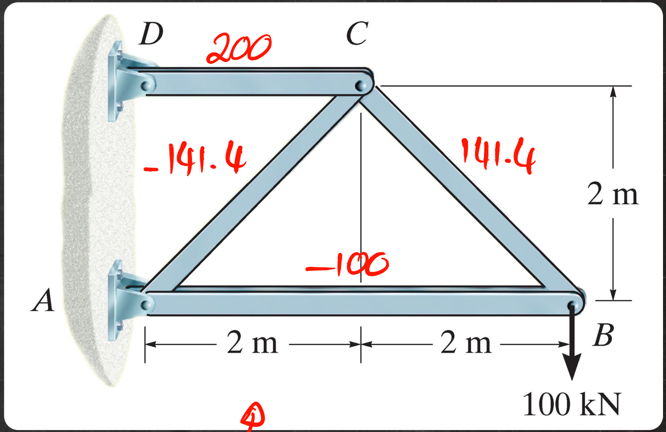
COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$$\mathcal{U}_e = \mathcal{U}_i$$

$$\hookrightarrow \frac{1}{2} F_B \, v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

displacement at B in the same direction as F_B

$$\Rightarrow F_B \, v_B = \sum \frac{F^2 L}{EA}$$



COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100$ kN , $F_{BC} = 141.4$ kN

$F_{AC} = -141.4$ kN , $F_{CD} = 200$ kN

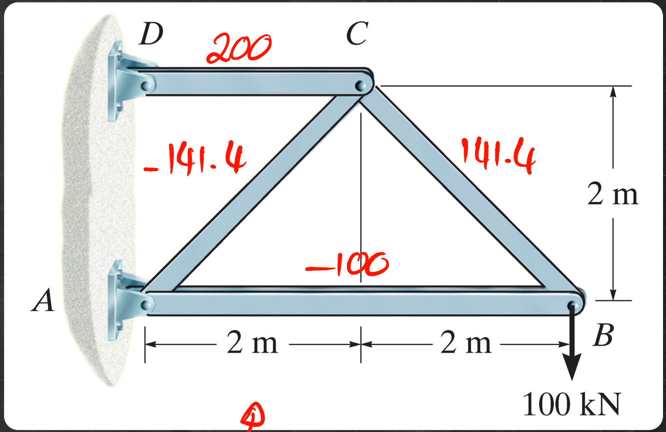
$$\mathcal{U}_e = \mathcal{U}_i$$

$$\frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

displacement at B in the same direction as F_B

$$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$$

MEMBER	F (kN)	L (m)	A (mm^2)	E (GPa)	F^2L/EA
AB					
BC					
AC					
CD					



COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100$ kN, $F_{BC} = 141.4$ kN

$F_{AC} = -141.4$ kN, $F_{CD} = 200$ kN

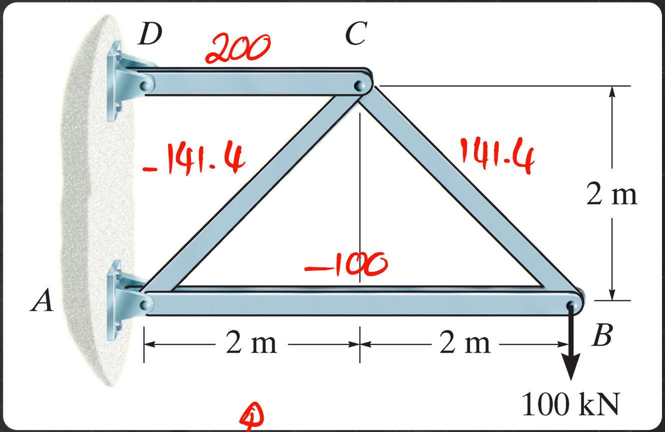
$$U_e = U_i$$

displacement at B in the same direction as F_B

$$\frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

$$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$$

MEMBER	F (kN)	L (m)	A (mm^2)	E (GPa)	F^2L/EA
AB	-100	4	400	200	500
BC	141.4	2.828	400	200	707
AC	-141.4	2.828	400	200	707
CD	200	2	400	200	1000



COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100$ kN , $F_{BC} = 141.4$ kN

$F_{AC} = -141.4$ kN , $F_{CD} = 200$ kN

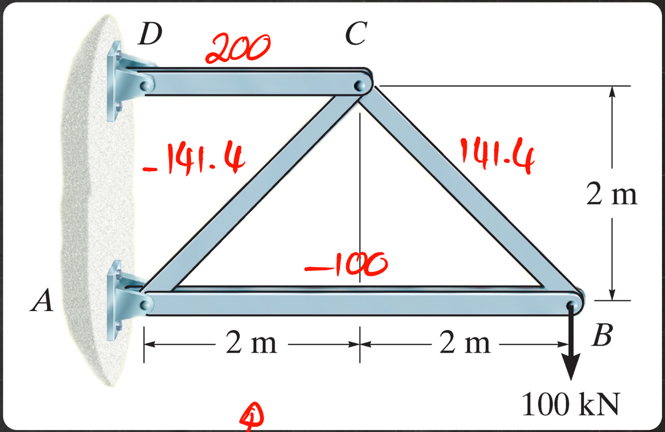
$U_e = U_i$

$\rightarrow \frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$

displacement at B in the same direction as F_B

$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$

MEMBER	F (kN)	L (m)	A (mm^2)	E (GPa)	F^2L/EA
AB	-100	4	400	200	500
BC	141.4	2.828	400	200	707
AC	-141.4	2.828	400	200	707
CD	200	2	400	200	1000
					Σ 2915



COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100\ kN$, $F_{BC} = 141.4\ kN$

$F_{AC} = -141.4\ kN$, $F_{CD} = 200\ kN$

$$\mathcal{U}_e = \mathcal{U}_i$$

$$\hookrightarrow \frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

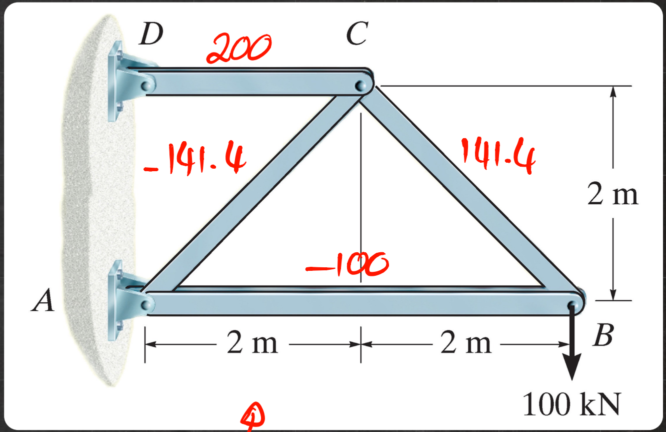
displacement at B in the same direction as F_B

$$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$$

MEMBER	F (kN)	L (m)	A (mm^2)	E (GPa)	F^2L/EA
AB	−100	4	400	200	500
BC	141.4	2.828	400	200	707
AC	−141.4	2.828	400	200	707
CD	200	2	400	200	1000

$$F_B v_B = 2915 \Leftarrow \sum 2915$$

$\hookrightarrow$ 100 kN



COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100$ kN, $F_{BC} = 141.4$ kN

$F_{AC} = -141.4$ kN, $F_{CD} = 200$ kN

$$U_e = U_i$$

$$\hookrightarrow \frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$$

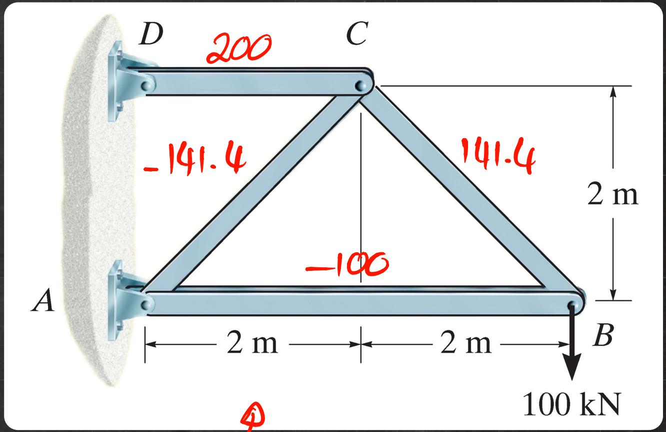
displacement at B in the same direction as F_B

$$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$$

MEMBER	F (kN)	L (m)	A (mm^2)	E (GPa)	F^2L/EA
AB	−100	4	400	200	500
BC	141.4	2.828	400	200	707
AC	−141.4	2.828	400	200	707
CD	200	2	400	200	1000

$$v_B = 29.15 \text{ mm} \Leftarrow F_B v_B = 2915 \Leftarrow \sum 2915$$

$\hookrightarrow 100$ kN



COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100$ kN, $F_{BC} = 141.4$ kN

$F_{AC} = -141.4$ kN, $F_{CD} = 200$ kN

$U_e = U_i$

$\Rightarrow \frac{1}{2} F_B v_B = \sum \frac{1}{2} F \frac{FL}{EA}$

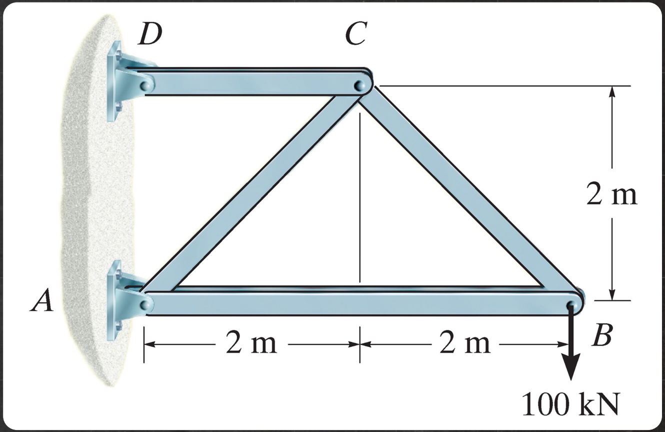
displacement at B in the same direction as F_B

$\Rightarrow F_B v_B = \sum \frac{F^2 L}{EA}$

MEMBER	F; kN	L; m	A; mm²	E; GPa	F^2L/EA
AB	-100	4	400	200	500
BC	141.4	2.828	400	200	707
AC	-141.4	2.828	400	200	707
CD	200	2	400	200	1000

$v_B = 29.15$ mm $\Leftarrow F_B v_B = 2915 \Leftarrow \sum 2915$

F_B → 100 kN



* HOW TO COMPUTE v_B IN A DIFFERENT DIRECTION?
* HOW TO COMPUTE v FOR ANY OTHER POINT? e.g. v_C?

Method of Virtual Forces

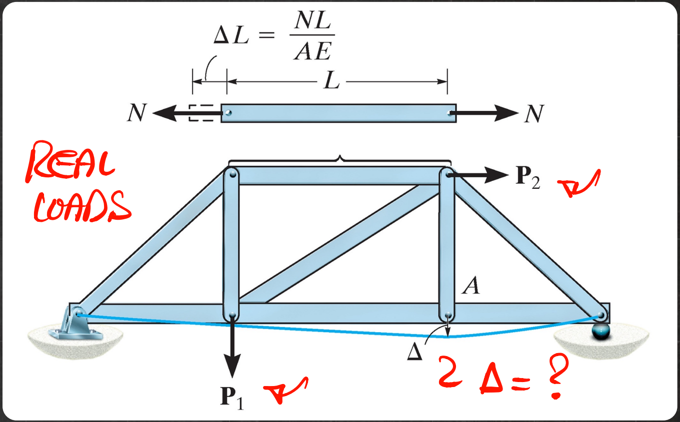
$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

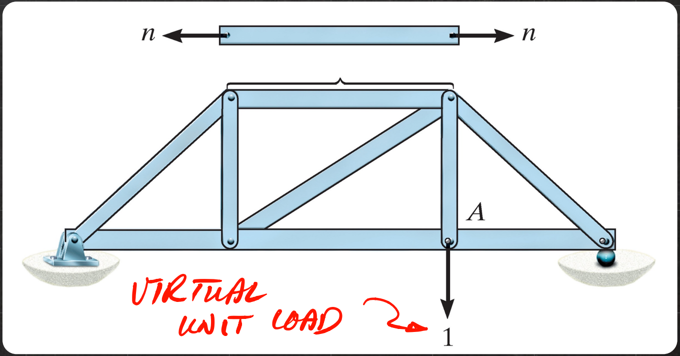
1: External Virtual Load acting on the joint of interest in the same direction as Δ

Δ: Joint Displacement due to Real Loads

n: Internal Virtual Force due to Virtual Unit Load "1"

N: Internal Force due to Real Loads

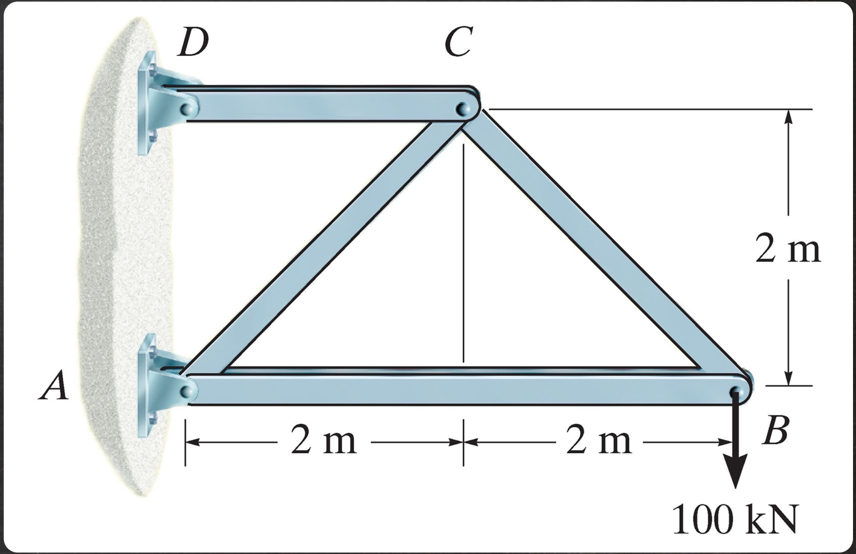


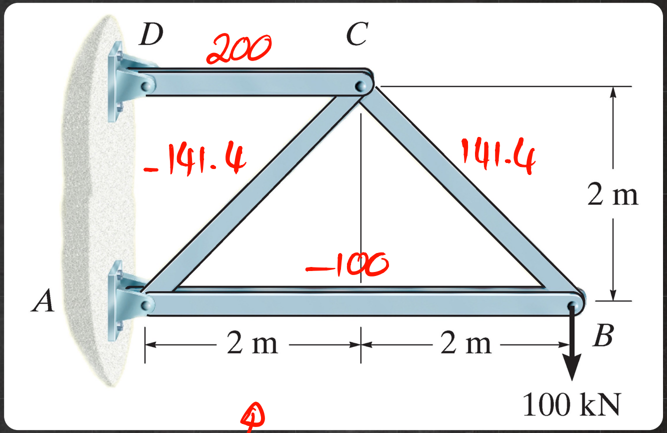


Exercise 3 . [similar to … P. 762 … 14.11]

DETERMINE THE VERTICAL DISPLACEMENT OF JOINT C OF THE TRUSS STRUCTURE SHOWN IN THE FIGURE.

THE CROSS-SECTIONAL AREA OF EACH MEMBER IS $A = 400\ mm^2$ AND $E = 200\ GPa$.





COMPUTE INTERNAL FORCES USING METHOD OF JOINTS

$F_{AB} = -100$ kN , $F_{BC} = 141.4$ kN

$F_{AC} = -141.4$ kN , $F_{CD} = 200$ kN

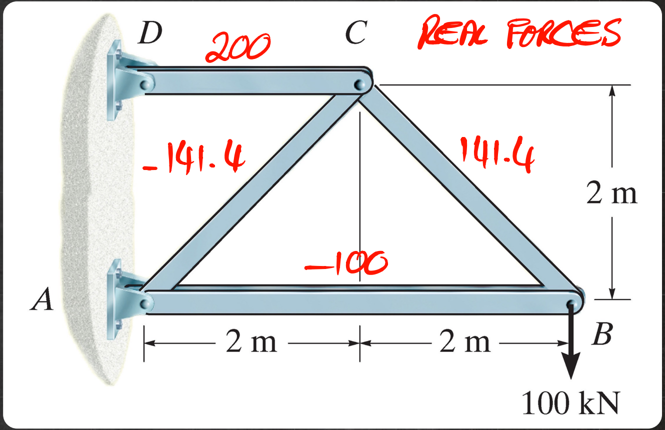
$$1.\ \Delta = \sum n \frac{NL}{EA}$$

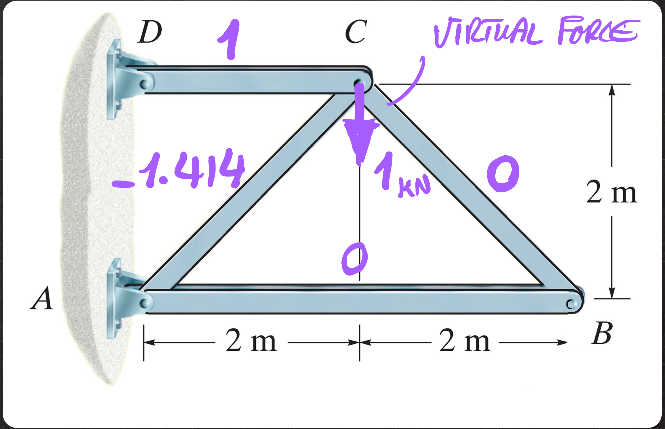
1: External Virtual Load acting on the joint of interest in the same direction as Δ

Δ: Joint Displacement due to Real Loads

n: Internal Virtual Force due to Virtual Unit Load "1"

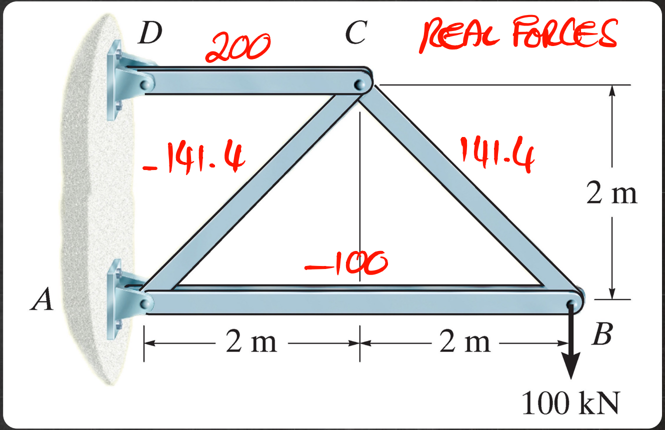
N: Internal Force due to Real Loads

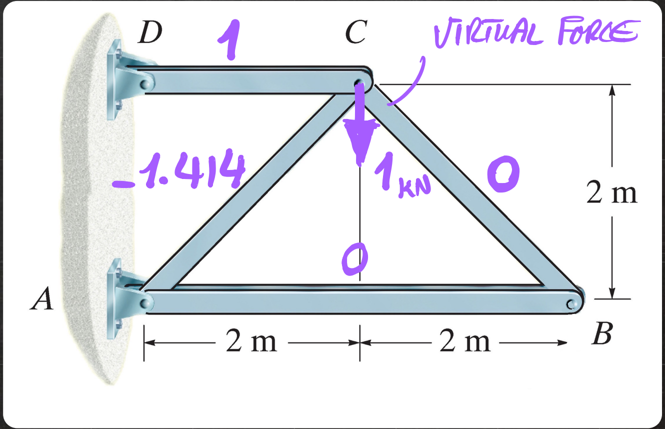




$$1 \cdot \Delta = \sum n \frac{NL}{EA}$$

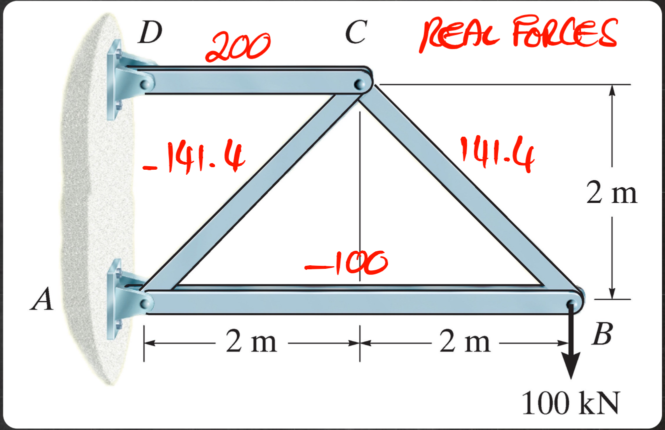
MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB						
BC						
AC						
CD						

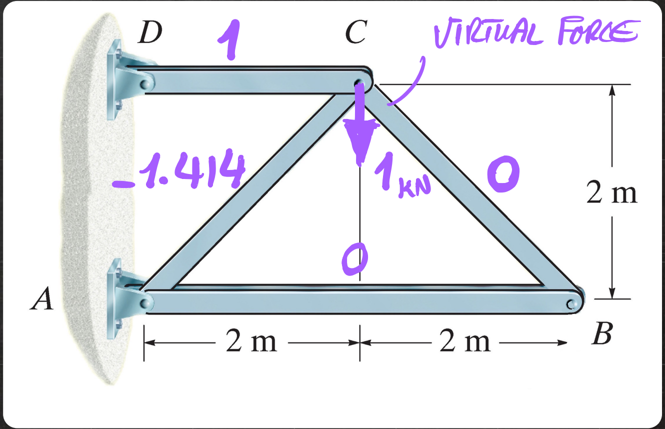




$$1.\ \boxed{\Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	0	-100	4	400	200	0
BC	0	141.4	2.828	400	200	0
AC	-1.414	-141.4	2.828	400	200	7.07
CD	1	200	2	400	200	5

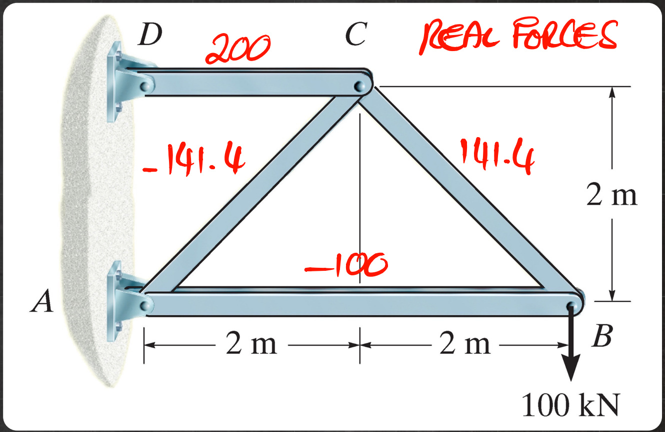


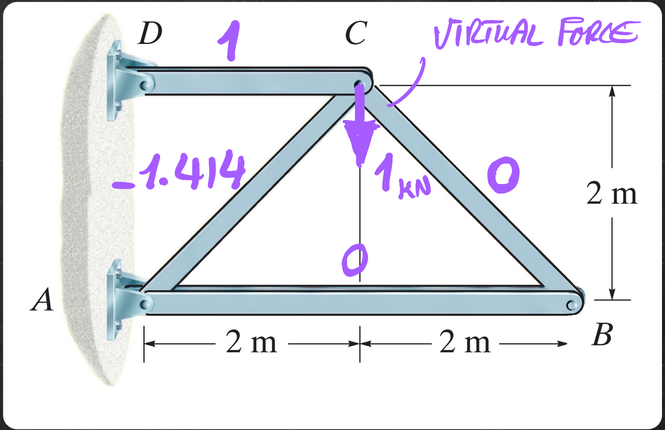


$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	0	-100	4	400	200	0
BC	0	141.4	2.828	400	200	0
AC	-1.414	-141.4	2.828	400	200	7.07
CD	1	200	2	400	200	5

$\sum 12.07$ N.m



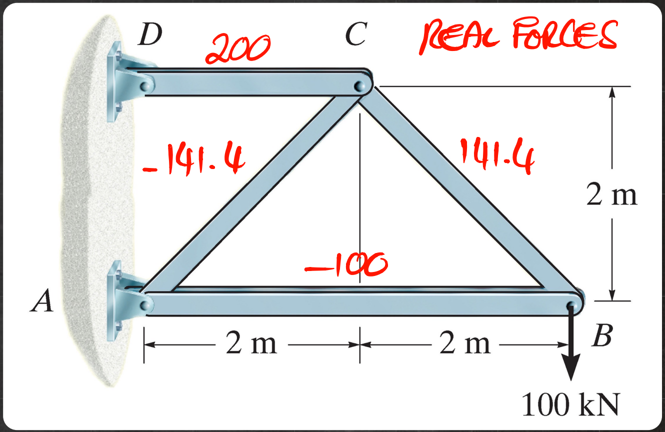


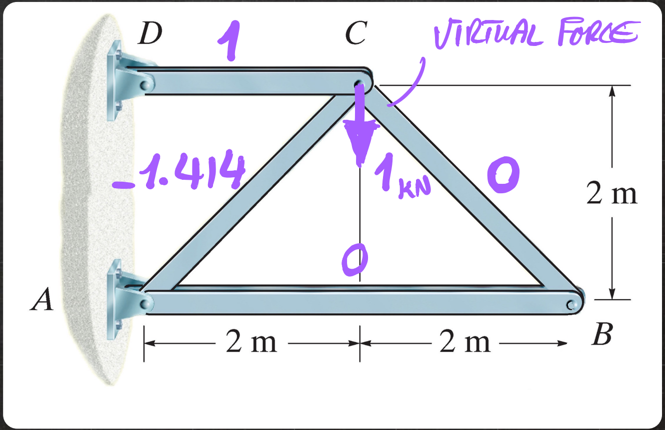
$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	0	-100	4	400	200	0
BC	0	141.4	2.828	400	200	0
AC	-1.414	-141.4	2.828	400	200	7.07
CD	1	200	2	400	200	5

$$1 \cdot \Delta = 12.07 \Leftarrow \sum 12.07$$

kN, mm; N.m



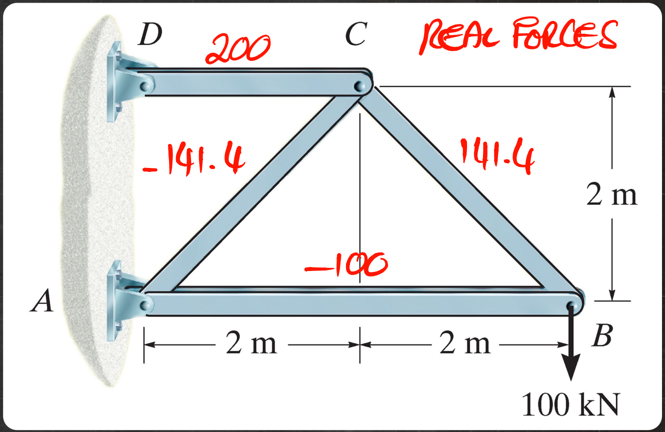


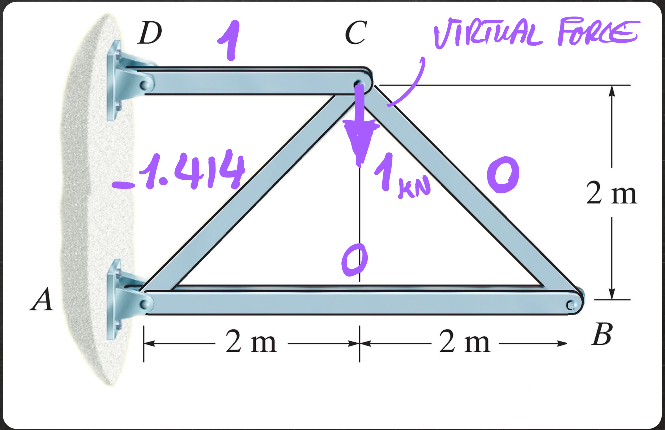
$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	0	-100	4	400	200	0
BC	0	141.4	2.828	400	200	0
AC	-1.414	-141.4	2.828	400	200	7.07
CD	1	200	2	400	200	5
						$\sum 12.07$ N.m

$\downarrow \Delta = 12.07$ mm $\Leftarrow$ $1 \cdot \Delta = 12.07$ $\Leftarrow$ $\sum 12.07$

DOWNWARDS $\hookleftarrow$ kN $\hookleftarrow$ mm   N.m

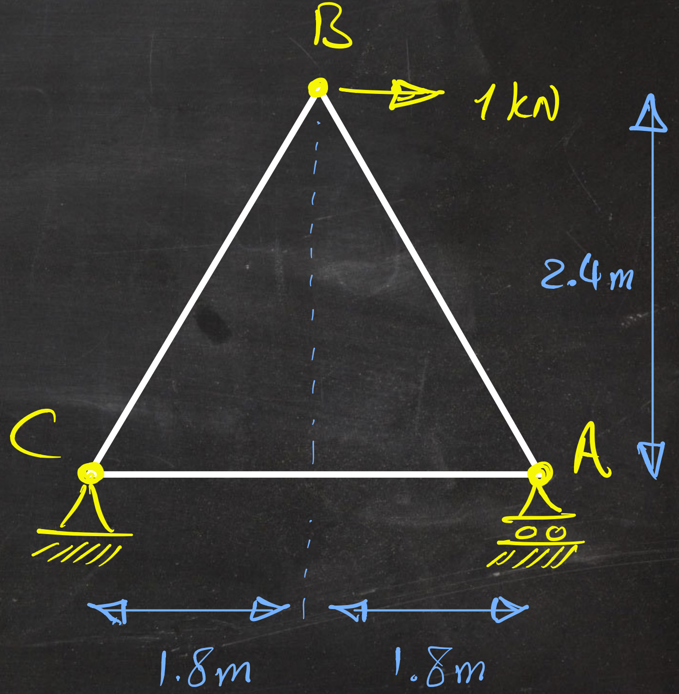




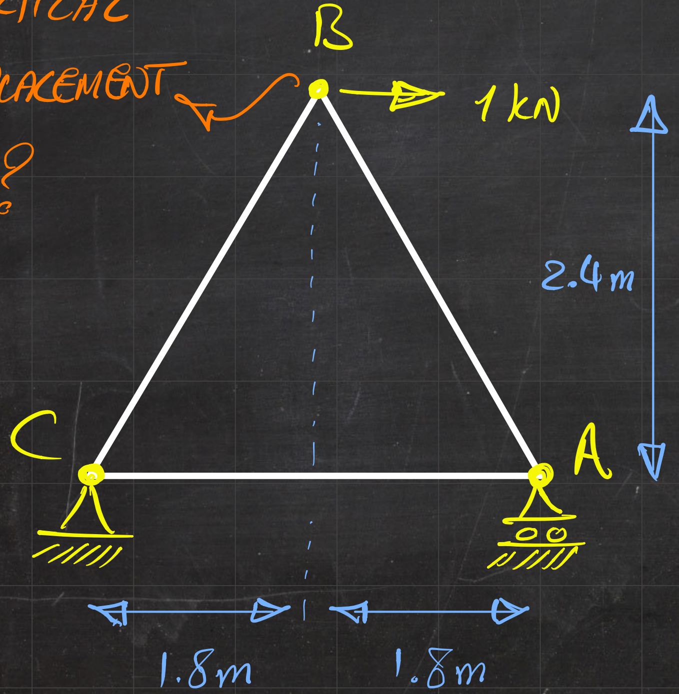
Exercise 4

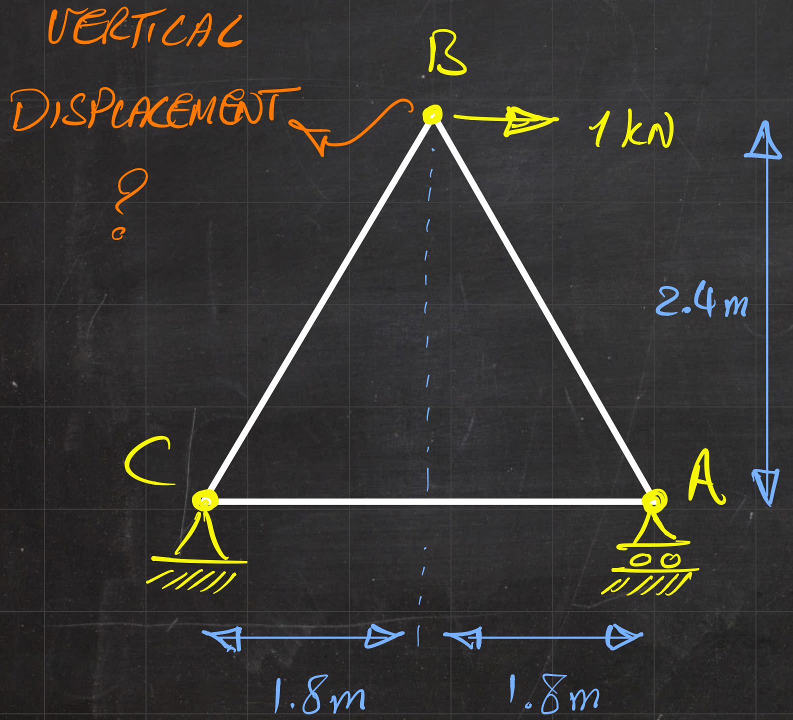
DETERMINE THE VERTICAL DISPLACEMENT OF JOINT B OF THE TRUSS STRUCTURE SHOWN IN THE FIGURE.

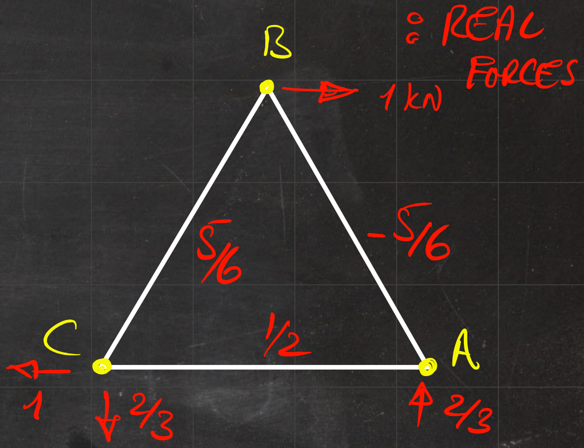
THE CROSS-SECTIONAL AREA OF EACH MEMBER IS $A = 1250\,mm^2$ AND $E = 200\,GPa$.

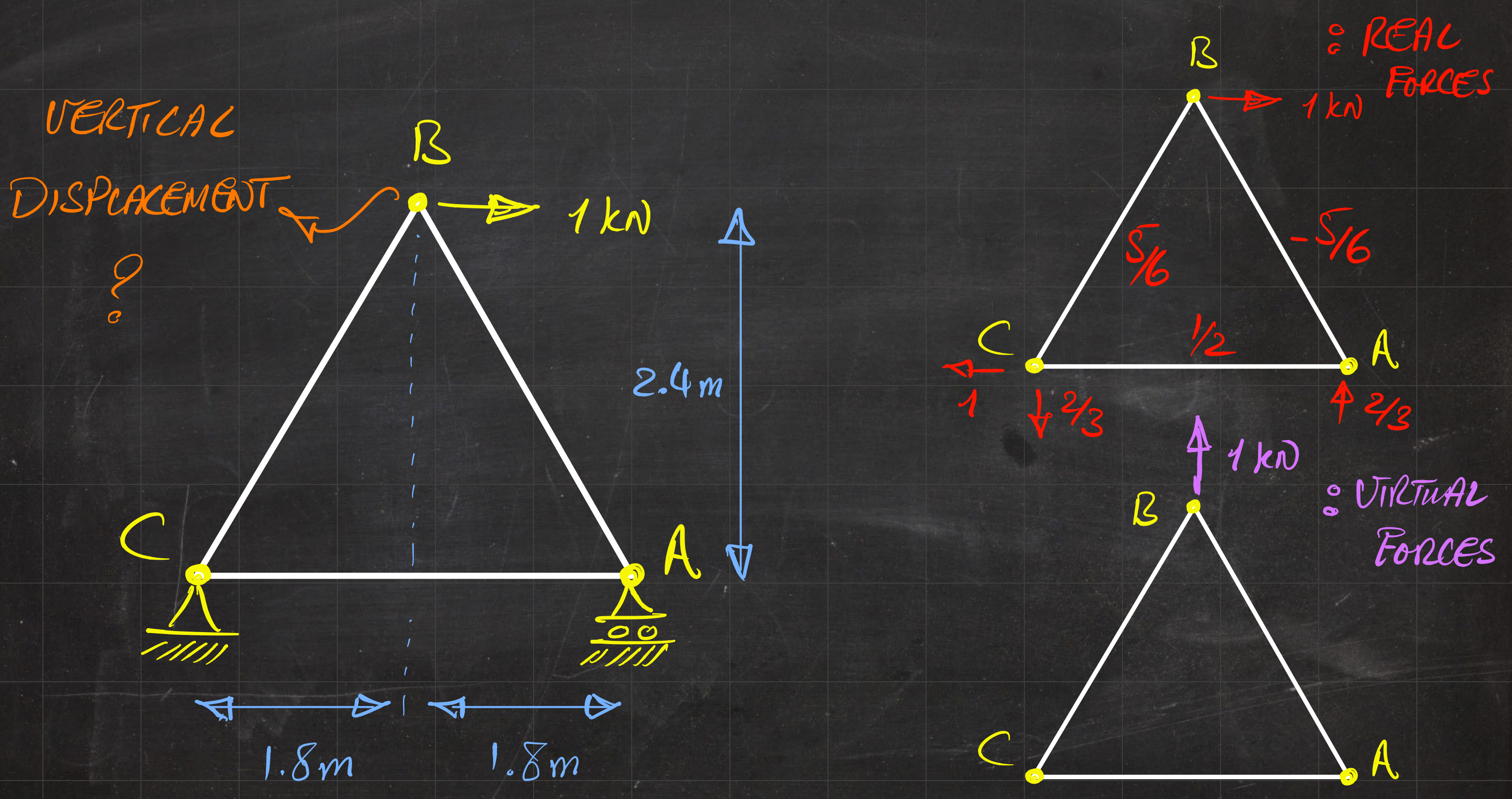


VERTICAL DISPLACEMENT ?

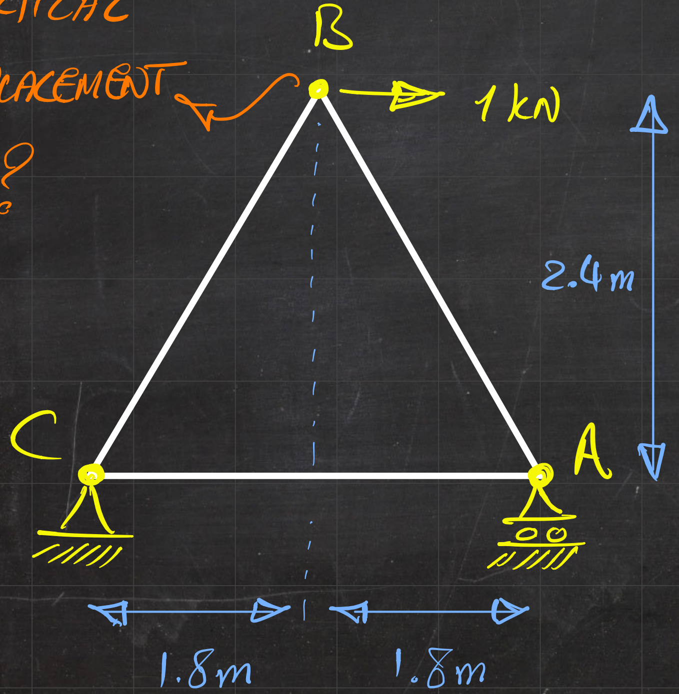


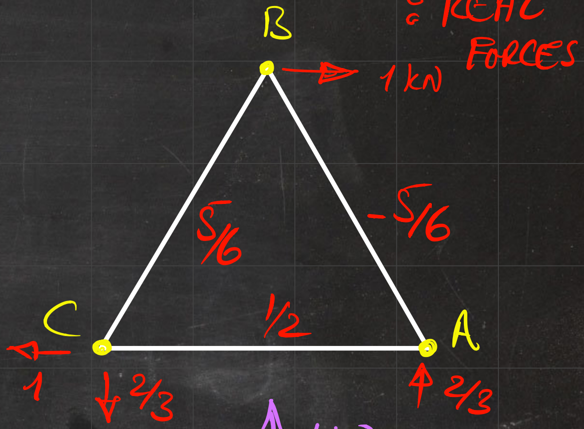

VERTICAL
DISPLACEMENT
?
B
1 kN
2.4 m
C
A
1.8m
1.8m

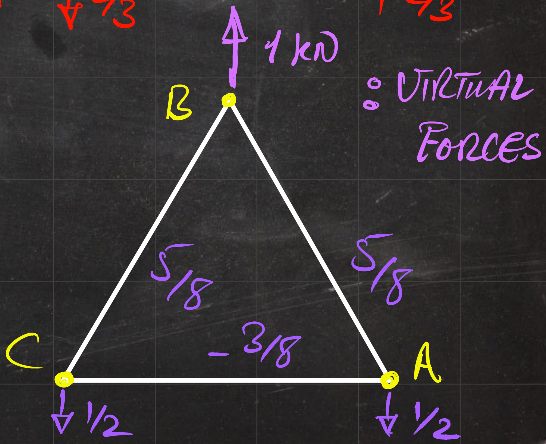

: REAL
FORCES
B
1 kN
5/6
-5/6
C
1/2
A
1
2/3
2/3


VERTICAL DISPLACEMENT ?
B
1 kN
2.4 m
C
A
1.8 m
1.8 m
: REAL FORCES
B
1 kN
5/6
-5/6
C
1/2
A
1
2/3
2/3
1 kN
: VIRTUAL FORCES
B
C
A

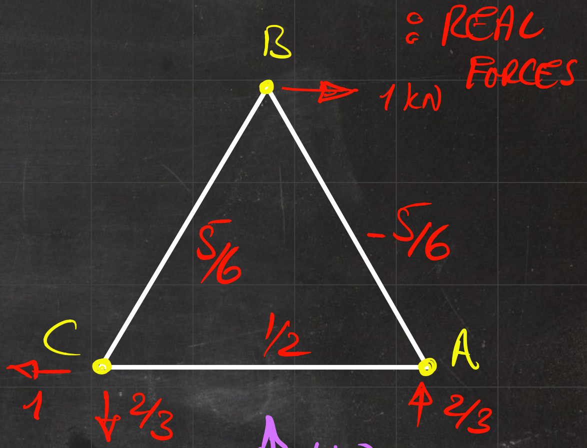
VERTICAL DISPLACEMENT ?

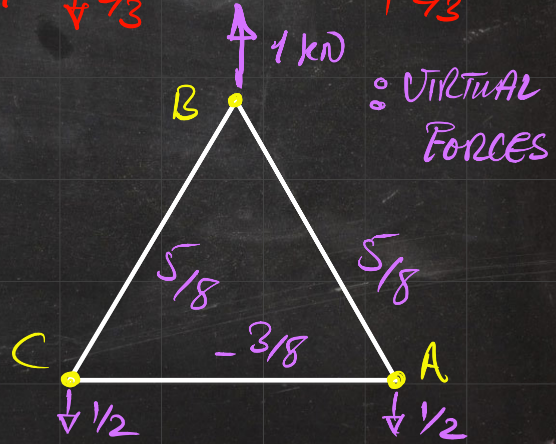






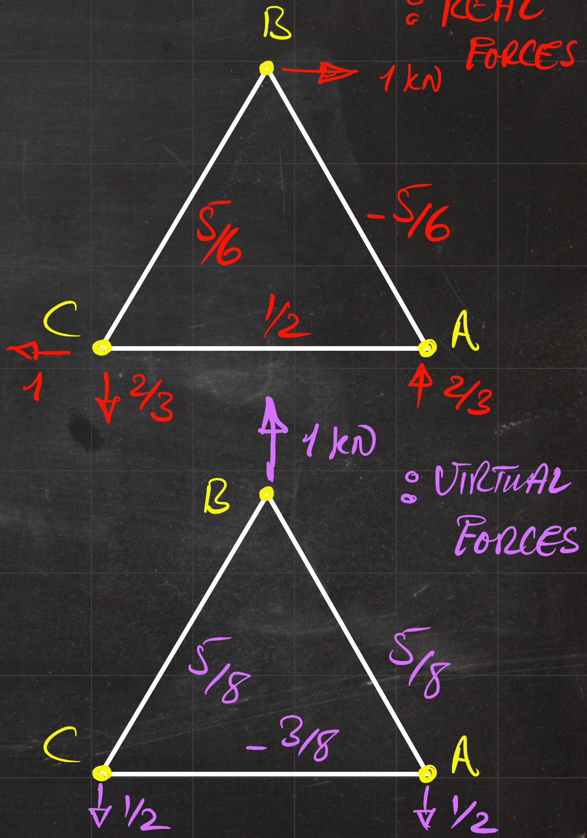
$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$





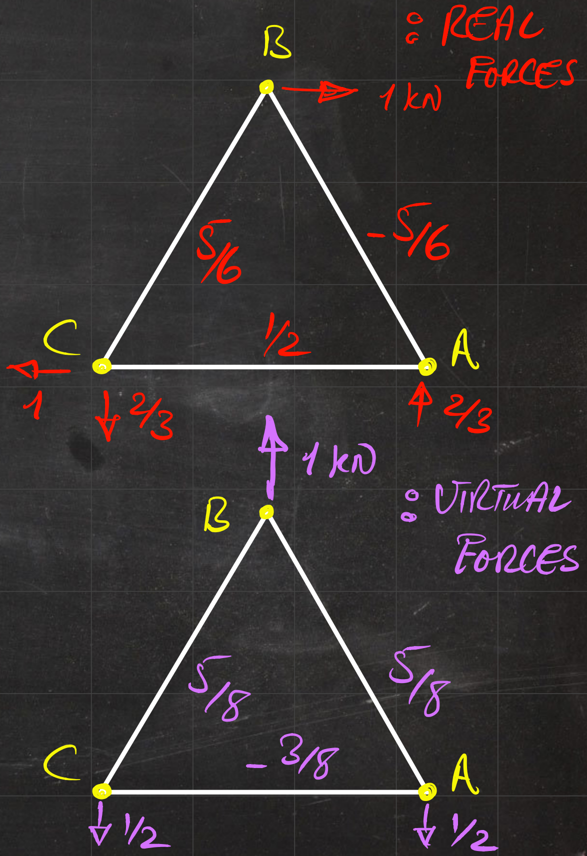
$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB						
BC						
AC						



$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

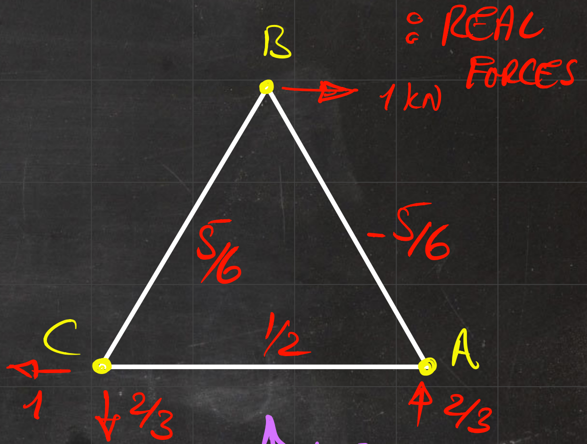
MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	5/8	-5/6	3	1250	200	-0.00625
BC	5/8	5/6	3	1250	200	+0.00625
AC	-3/8	1/2	3.6	1250	200	-0.0027

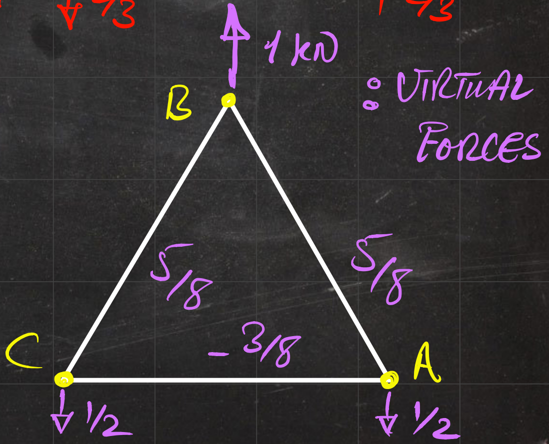


$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	$5/8$	$-5/6$	3	1250	200	-0.00625
BC	$5/8$	$5/6$	3	1250	200	$+0.00625$
AC	$-3/8$	$1/2$	3.6	1250	200	-0.0027

$\sum -0.0027$



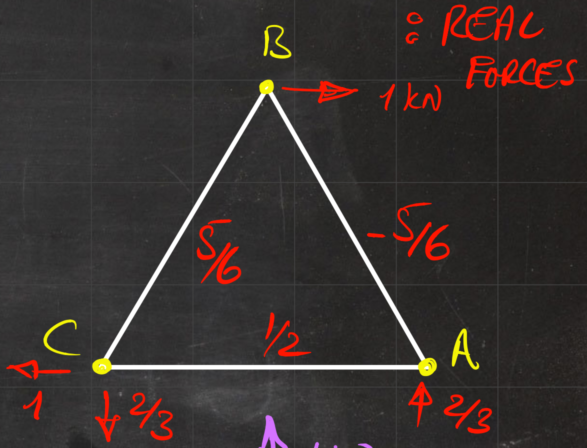


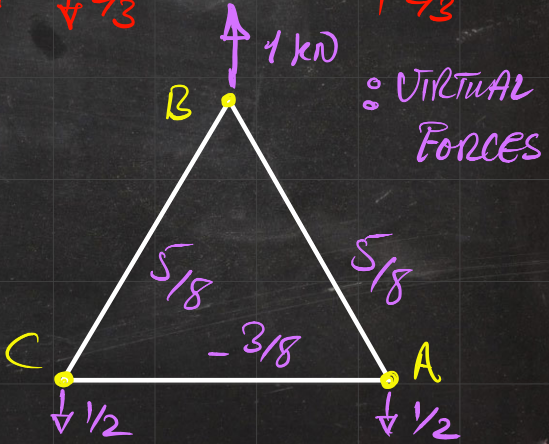
$$\boxed{1 \cdot \Delta = \sum n \frac{NL}{EA}}$$

MEMBER	n (kN)	N (kN)	L (m)	A (mm^2)	E (GPa)	$n\frac{NL}{EA}$ (N.m)
AB	$5/8$	$-5/6$	3	1250	200	-0.00625
BC	$5/8$	$5/6$	3	1250	200	$+0.00625$
AC	$-3/8$	$1/2$	3.6	1250	200	-0.0027

$$\downarrow v_B = -0.0027 \text{ mm} \Leftarrow \sum -0.0027$$

DOWNWARDS





MECHANICS AND MATERIALS I

Energy Methods

Sections … 14.1 – 14.3 … 14.5 … 14.6

Chap. 14

[Hibbeler 9th edition]

MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

23