MECHANICS AND MATERIALS I

7

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Bilkent University

MECHANICS AND MATERIALS I

Stress i

Sections ... 1.1 – 1.5

Chap. 1

[Hibbeler 9th edition]

Ali Javili

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Stress

STRESS

So far, we talked about Equilibrium of
- Points
- Rigid Bodies
- Structures made of Rigid Bodies

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STRESS

So far, we talked about Equilibrium of
- Points
- Rigid Bodies
- Structures made of Rigid Bodies

Now, we want to talk about Equilibrium of Deformable Bodies

$\sum F = 0, \sum M = 0$ <-- THEY APPEAR IN BALANCE EQUATIONS

We must distinguish between External Loading & Internal Loading

STRESS

after discussing resultant internal loading

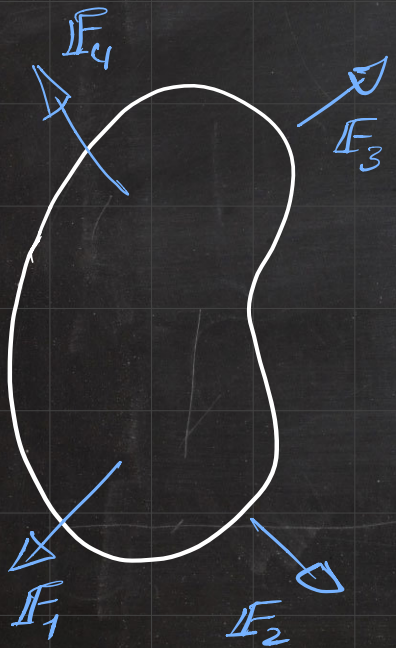
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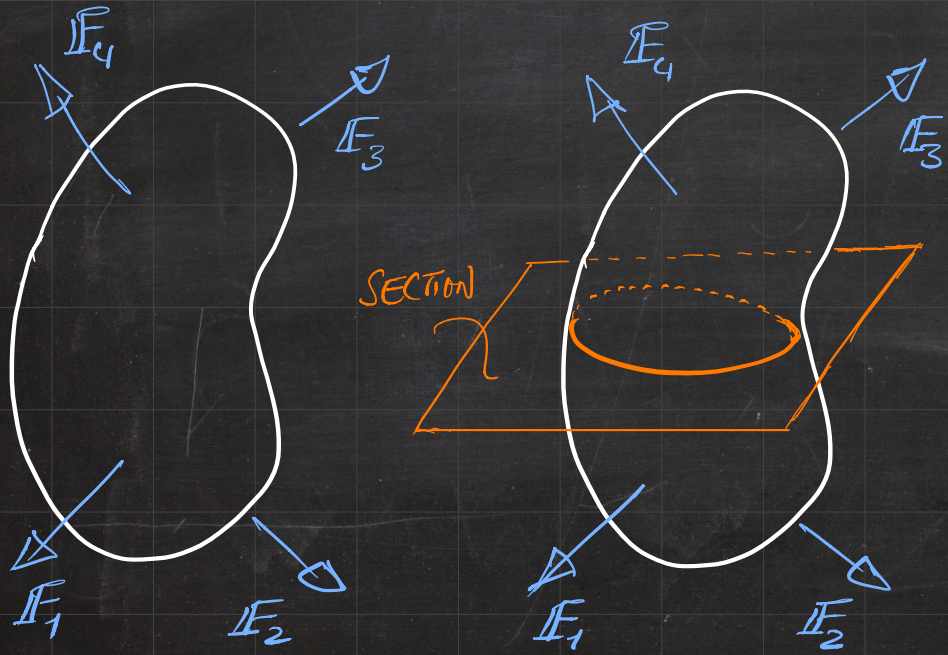
THEY APPEAR IN RESULTANT INTERNAL LOADING → STRESS

We must distinguish between External Loading & Internal Loading

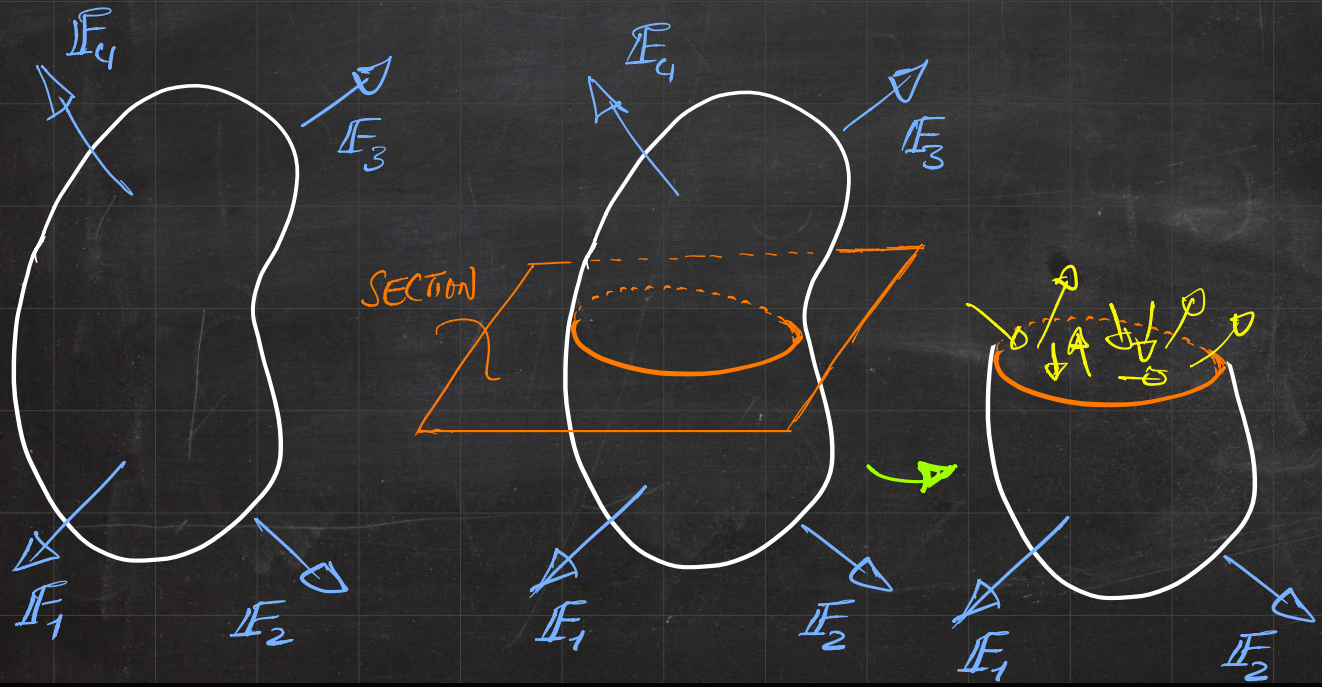
Internal Resultant Loadings



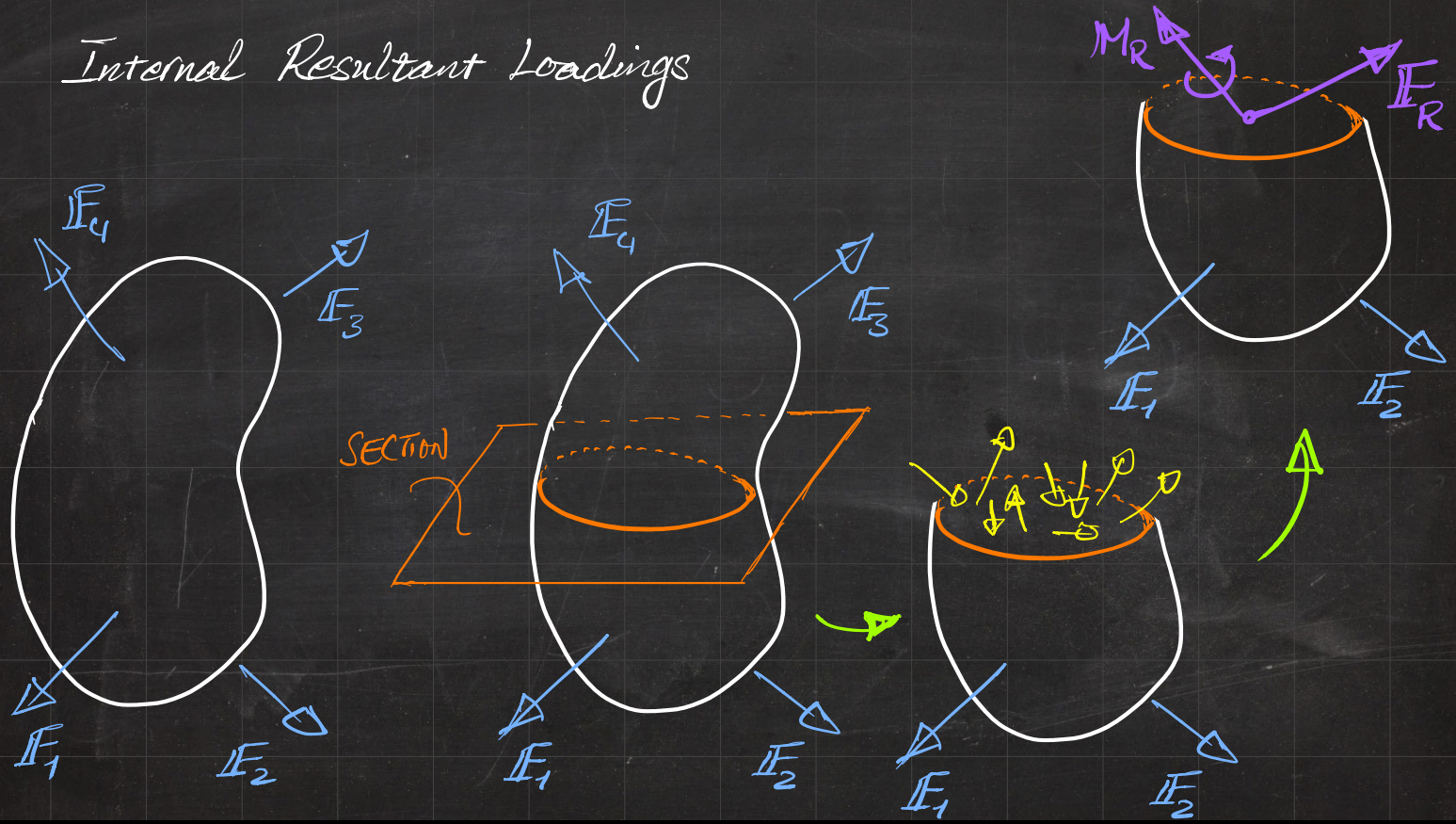
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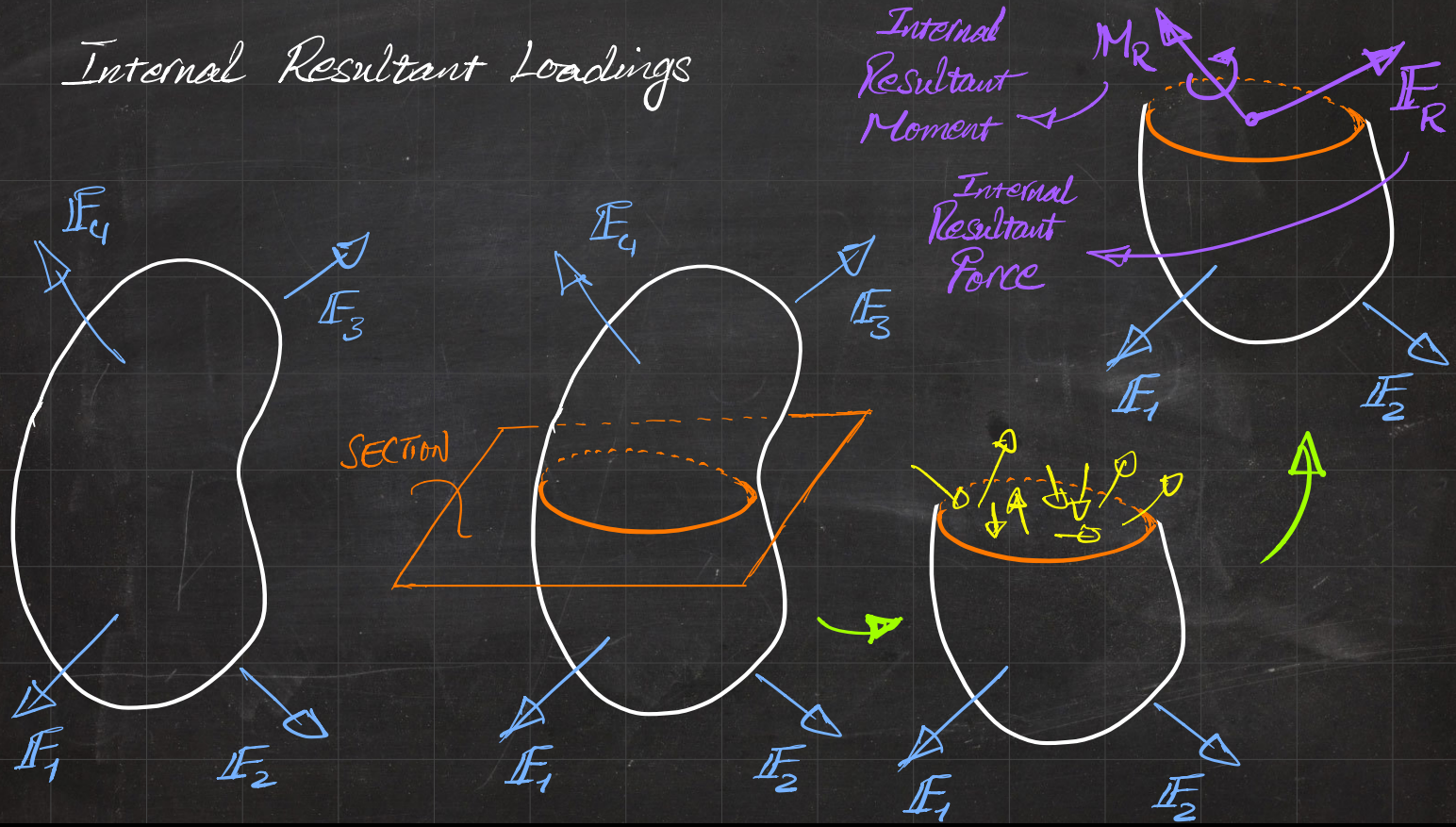
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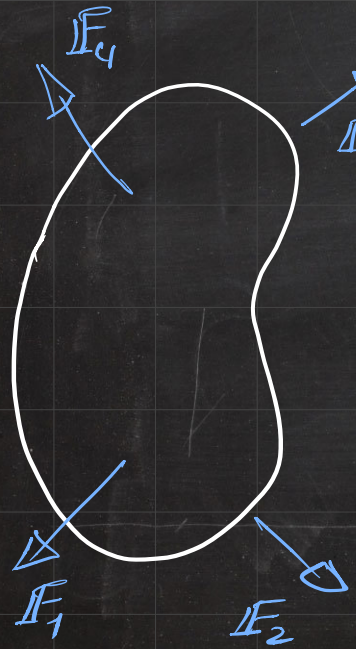
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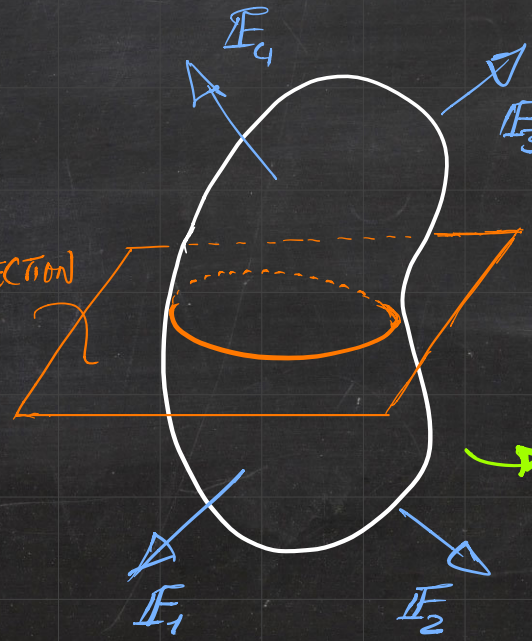


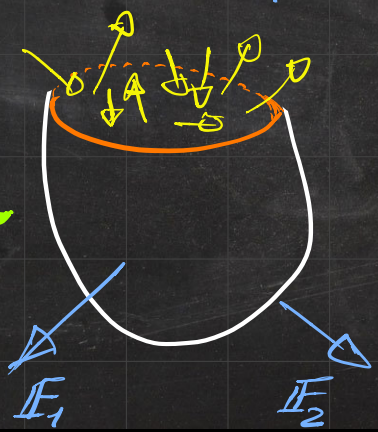
Internal Resultant Loadings



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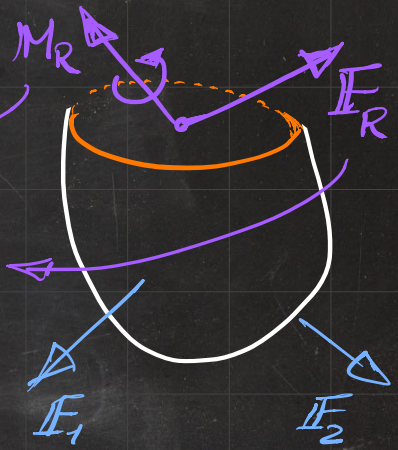






Internal Resultant Moment

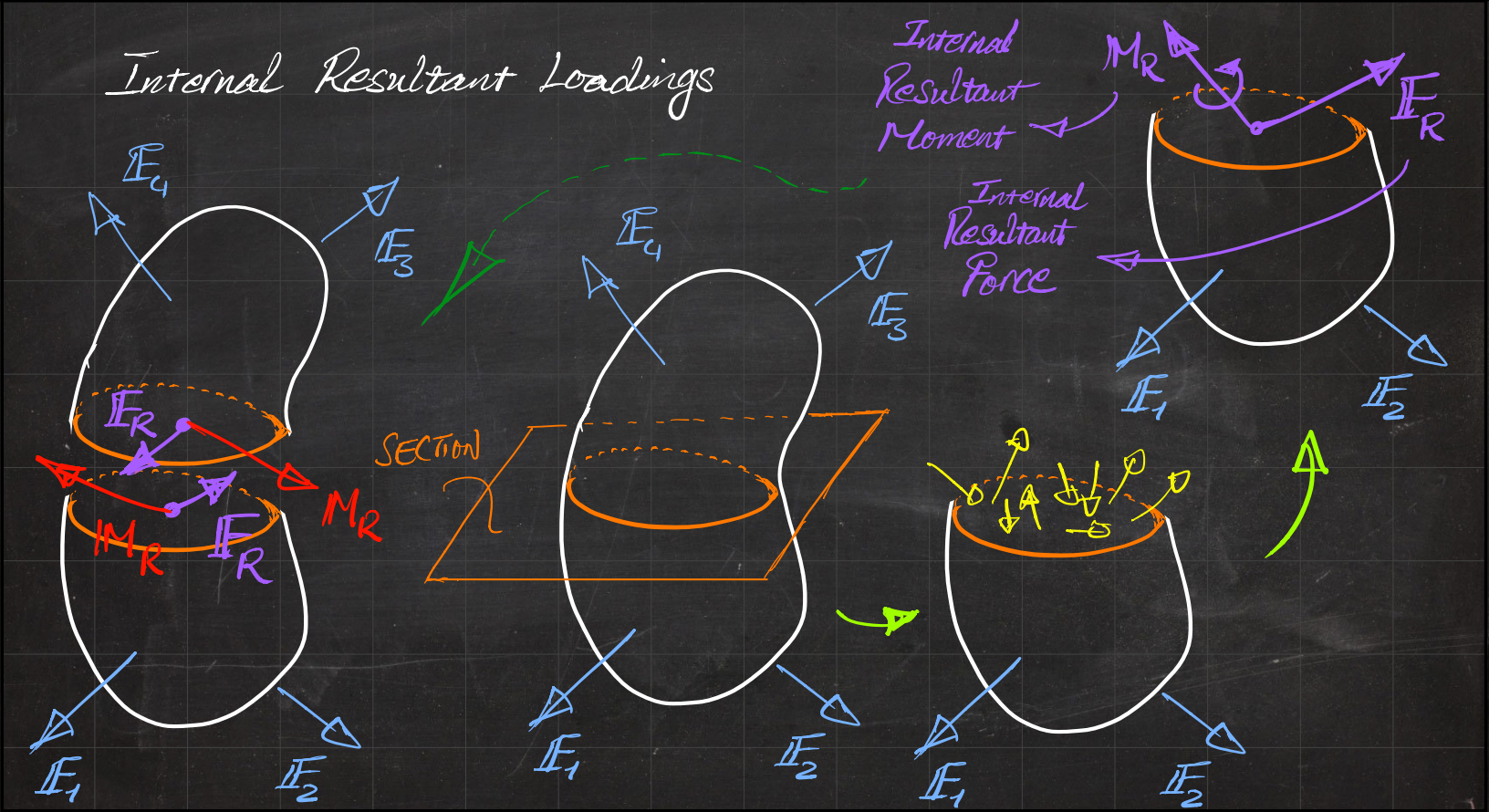
Internal Resultant Force

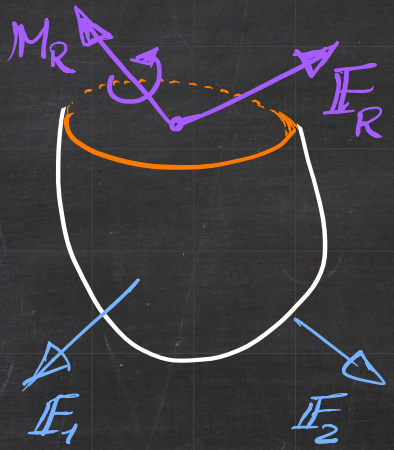


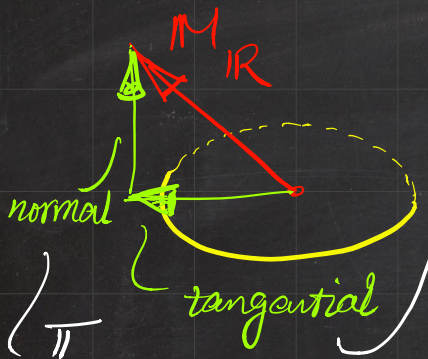
Resultant Forces and Moments are useful for calculations!

distributed in reality!

Internal Resultant Loadings

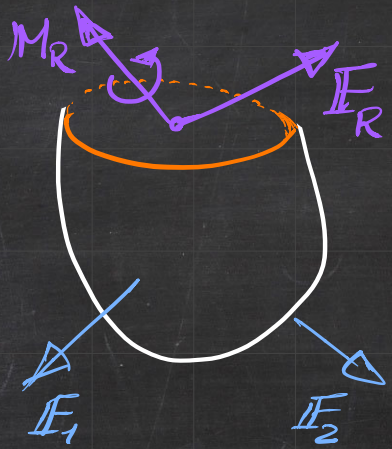



M_R
F_R
F_1
F_2



T
TORSIONAL MOMENT

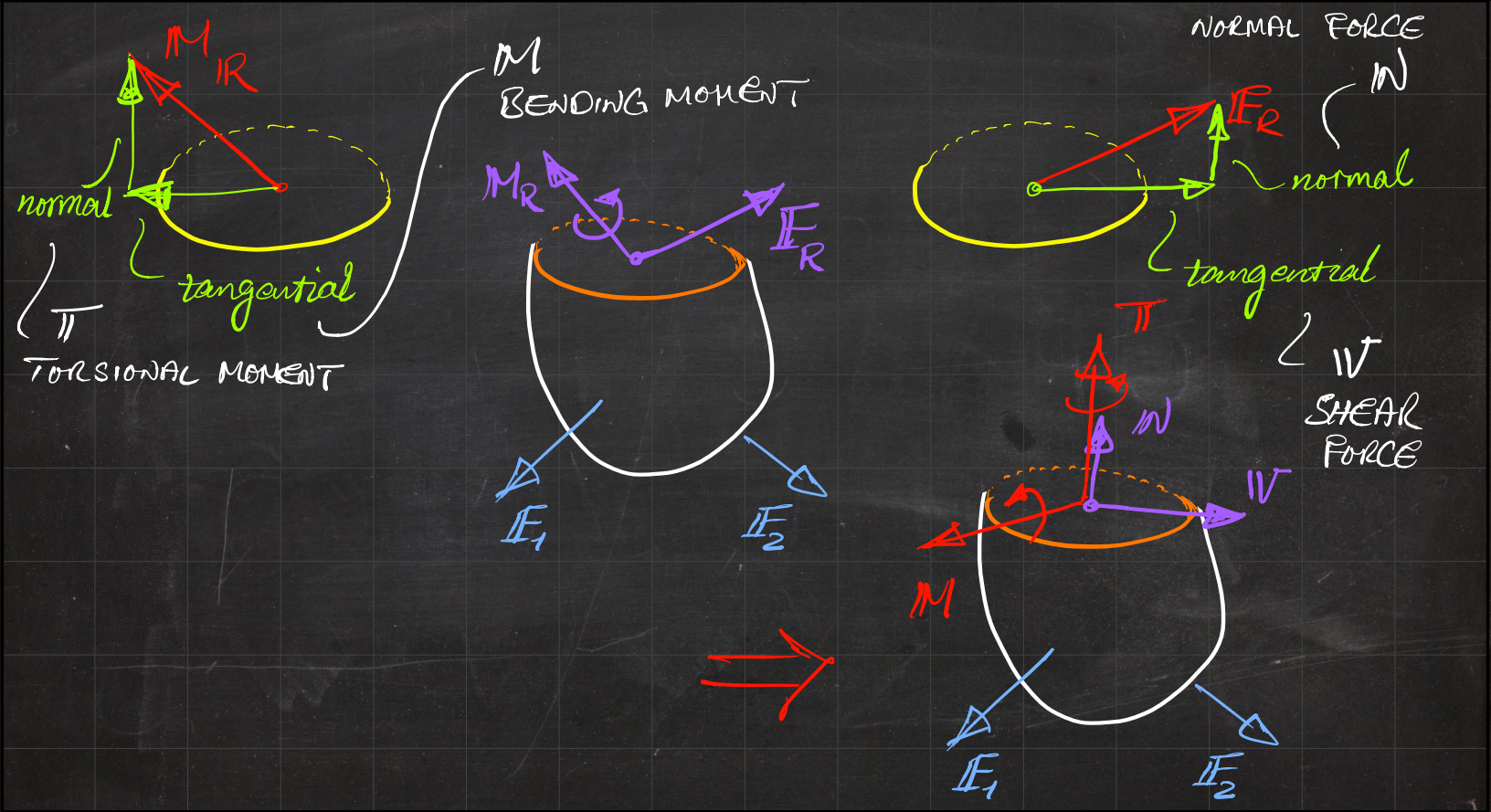
M
BENDING MOMENT



NORMAL FORCE
N



IV
SHEAR FORCE

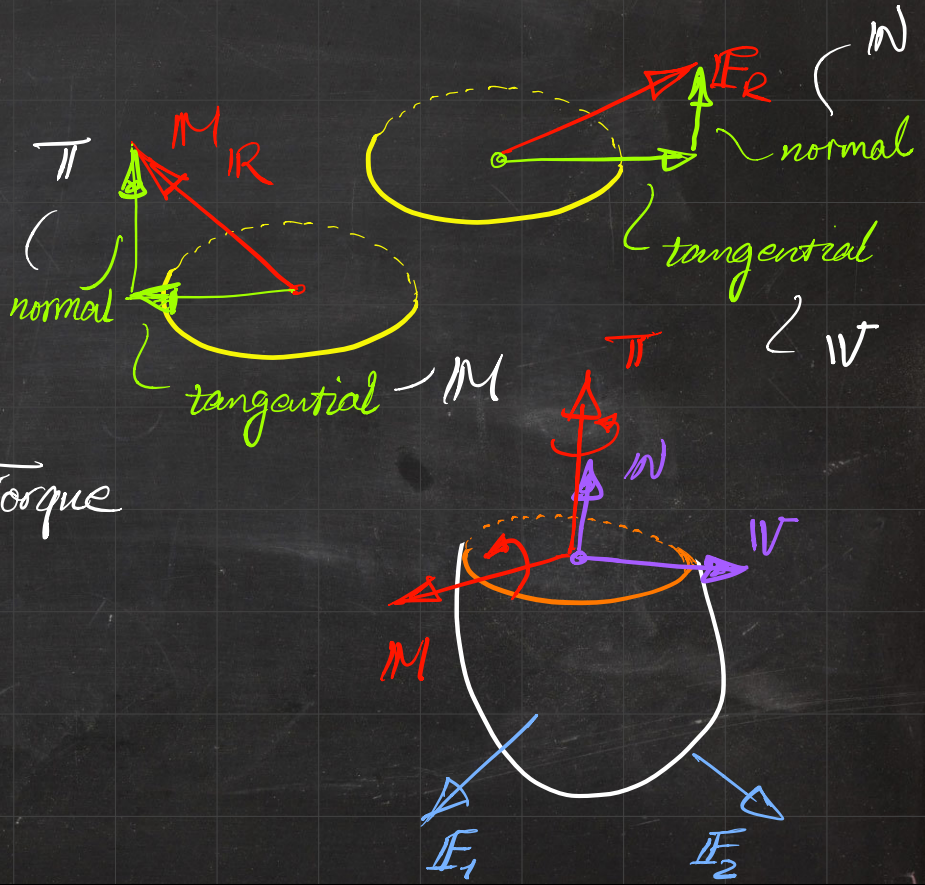

M
R
normal
tangential
T
TORSIONAL MOMENT
M
BENDING MOMENT
M_R
F_R
F_1
F_2
NORMAL FORCE
N
F_R
normal
tangential
T
N
V
M
V
SHEAR FORCE
F_1
F_2

N : Normal Force

V : Shear Force

T : Torsional Moment or Torque

M : Bending Moment



N : Normal Force

- Perpendicular to area
- Developed to resist against pull or push due to external forces

V : Shear Force

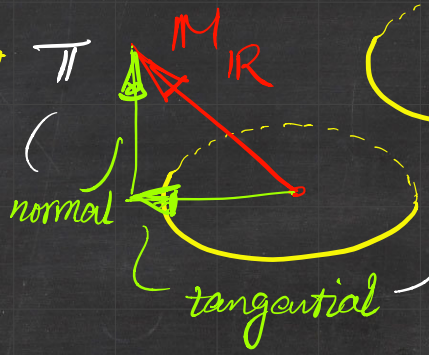
- Lies on area
- Developed when external loads tend to cause two segments to slide over one another

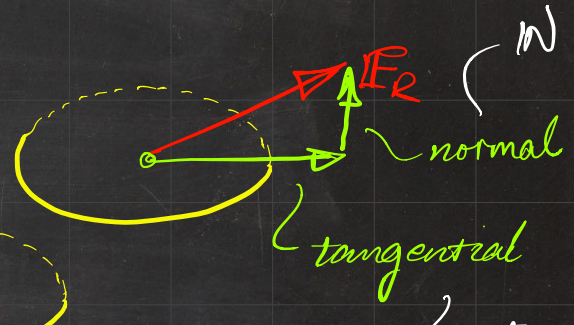
T : Torsional Moment or Torque

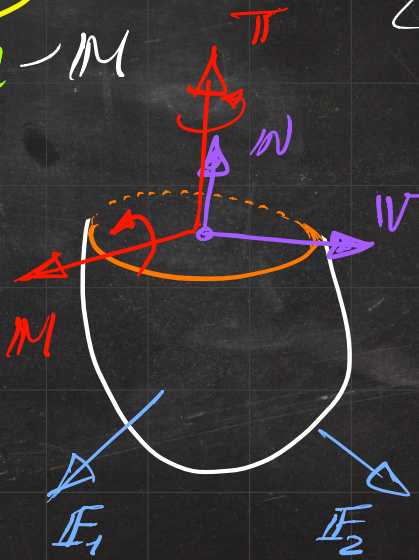
- Perpendicular Component
- Developed to resist against twist due to external forces

M : Bending Moment

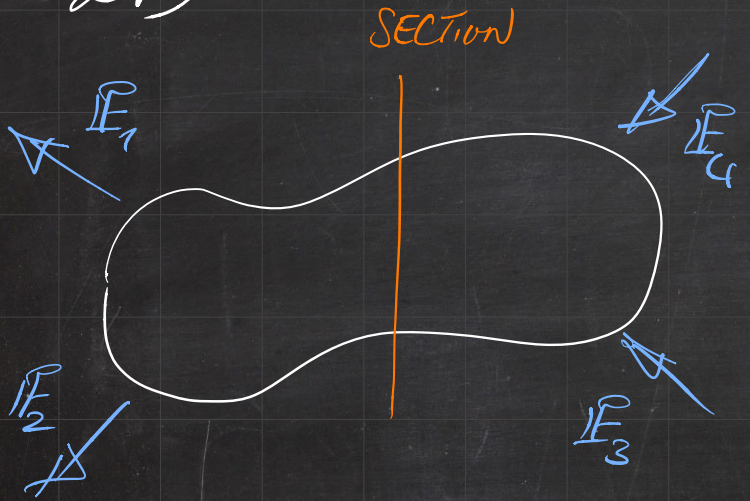
- Tangential Component
- Developed when external forces tend to bend the body about an axis lying within the plane of area



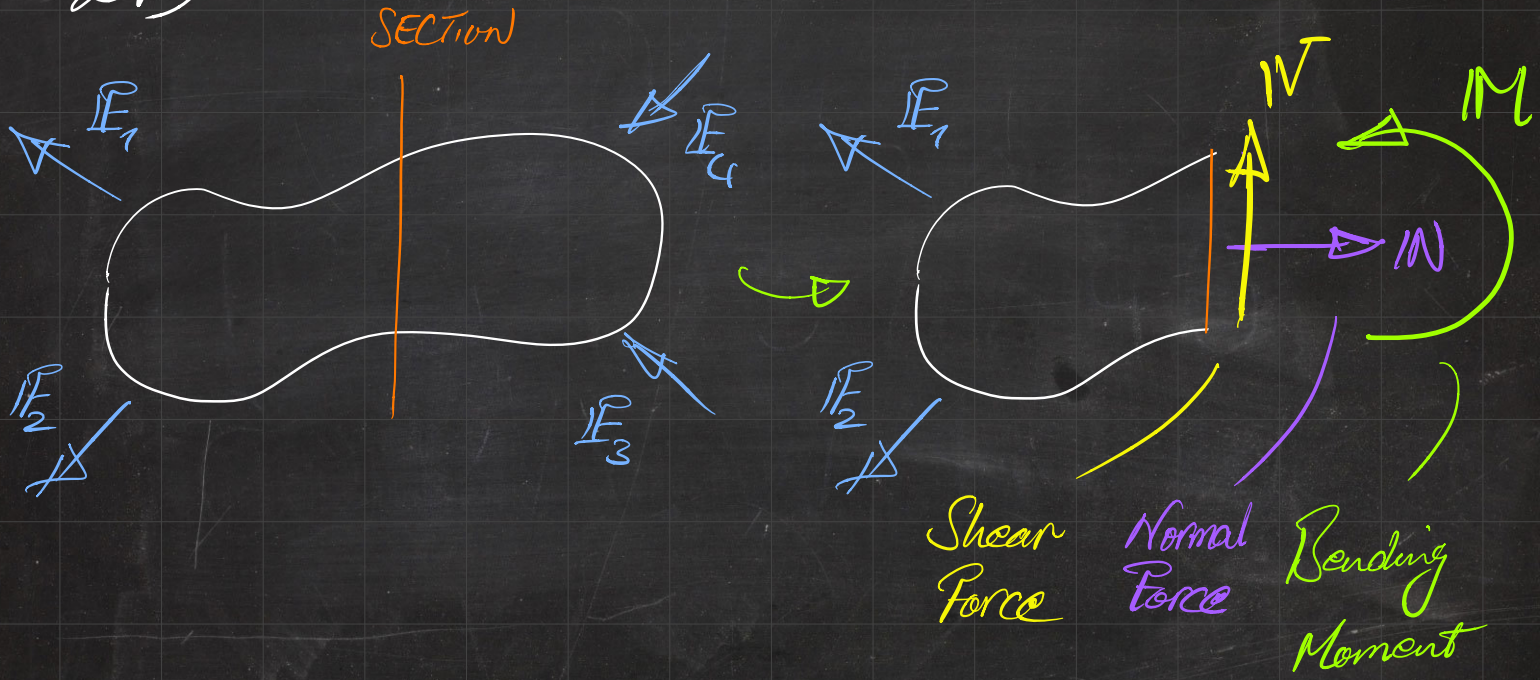




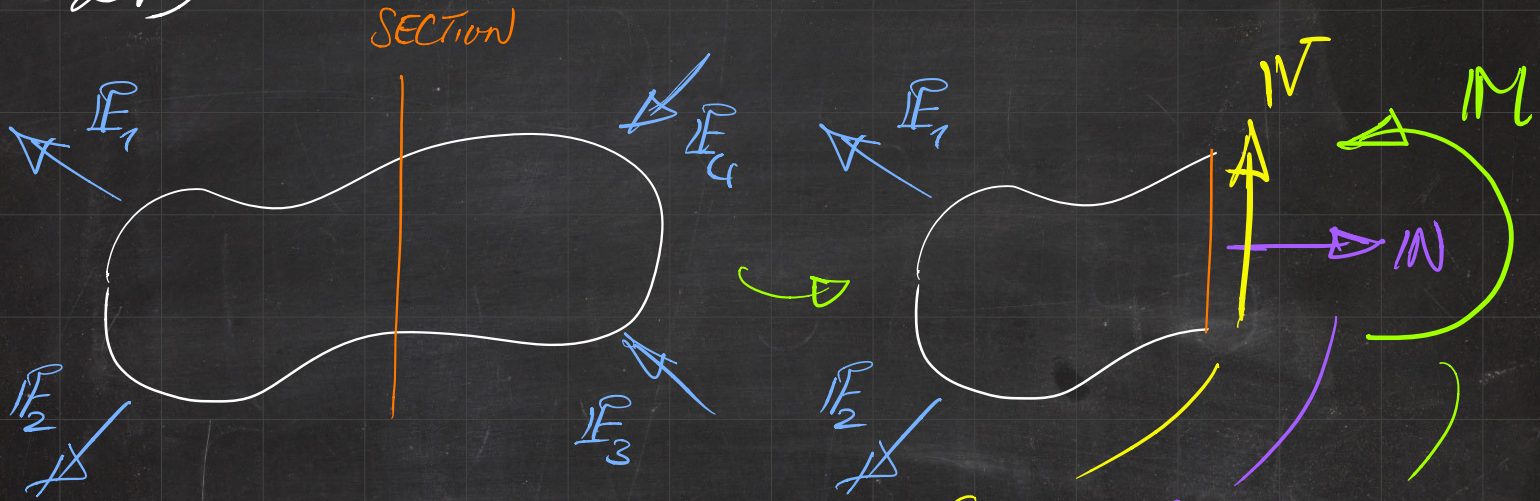
2D



2D

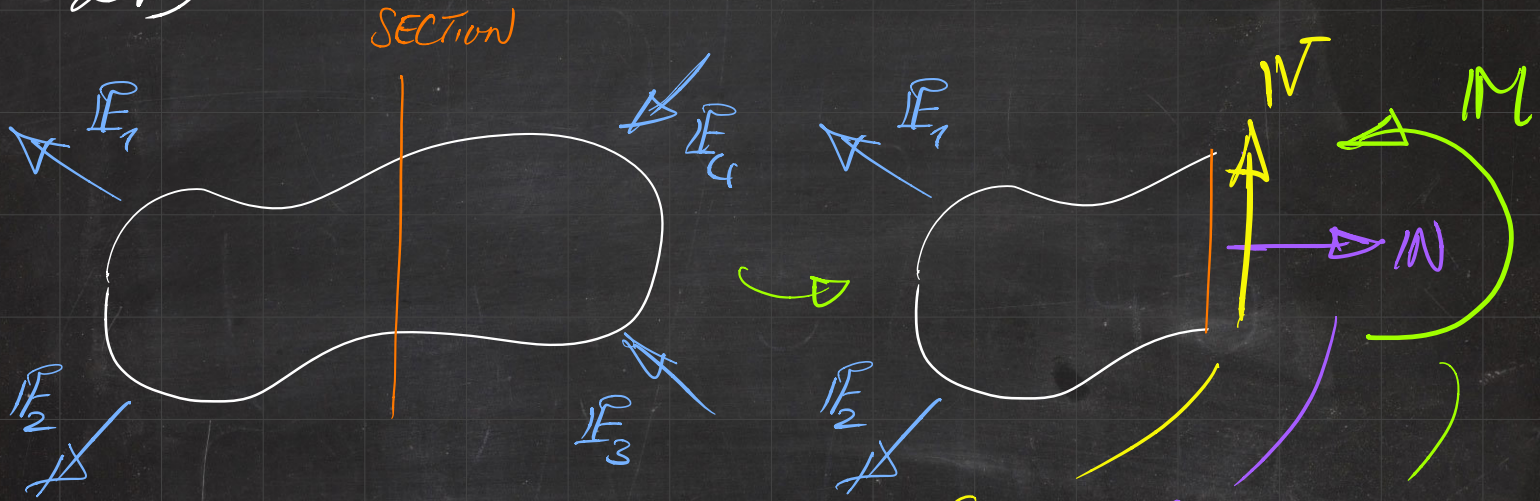

SECTION
F1
F4
F2
F3
F1
V
M
N
F2
Shear Force
Normal Force
Bending Moment

2D



$$\sum F_x = 0$$
$$\sum F_y = 0$$
$$\sum M_o = 0$$

2D



$\sum F_x = 0 \rightarrow N$

$\sum F_y = 0 \rightarrow V$

$\sum M_o = 0 \rightarrow M$

Shear Force

Normal Force

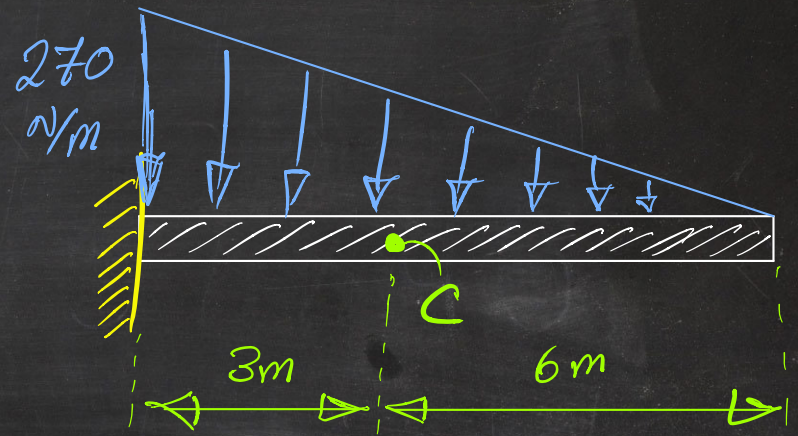
Bending Moment

Procedure for Analysis

- We are essentially using "METHOD OF SECTIONS".
- Divide the body/structure based on the segment of interest
- Draw FBD → including supports reactions & internal loads
- Impose equilibrium → $\sum \mathbf{F} = \mathbf{0}$, $\sum \mathbf{M} = \mathbf{0}$

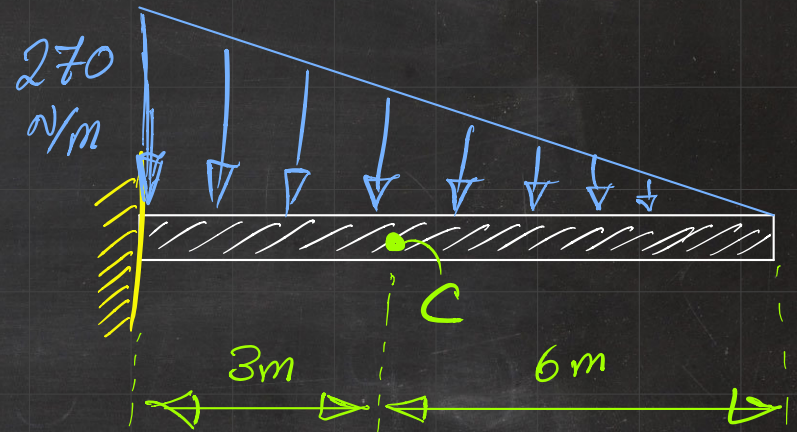
Exercise 1 . [similar to ... P. 11 ... 1.1]

DETERMINE THE RESULTANT INTERNAL LOADINGS ACTING ON THE CROSS SECTION AT C OF THE CANTILEVERED BEAM SHOWN IN THE FIGURE.



IMPORTANT

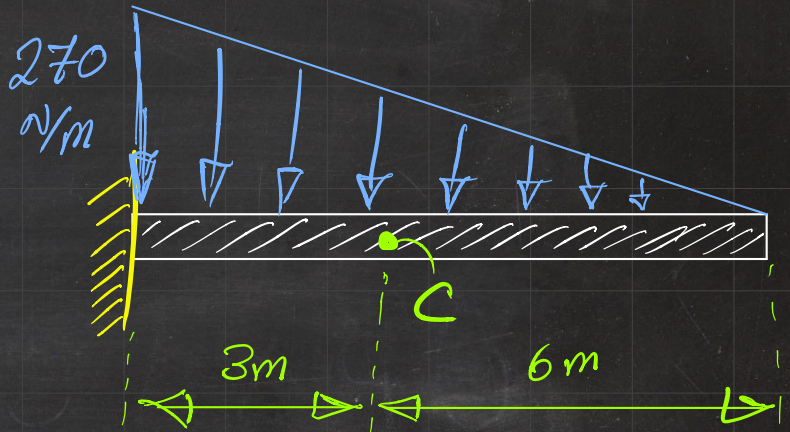
⊕ Convention for internal loadings



IMPORTANT

Convention for internal loadings

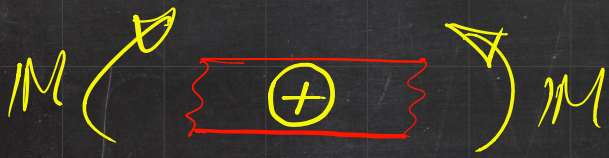


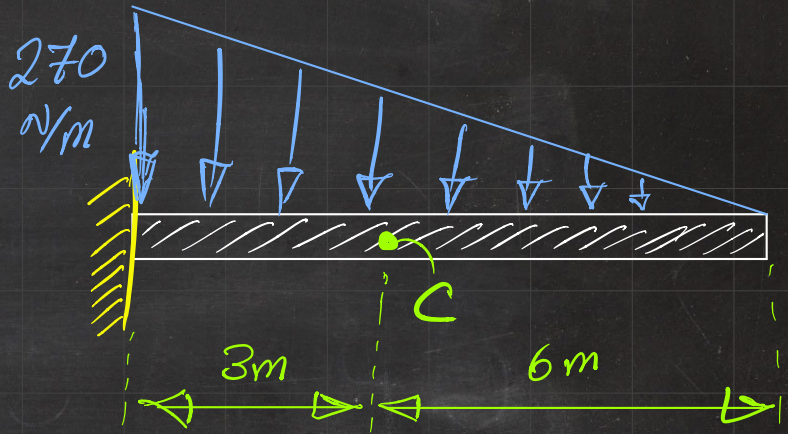


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Convention for internal loadings

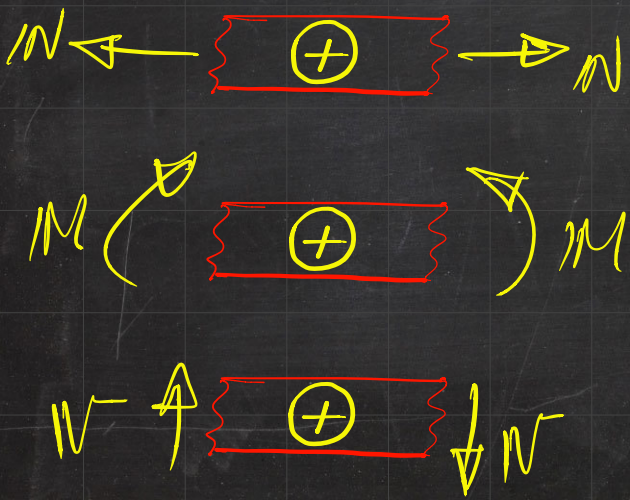


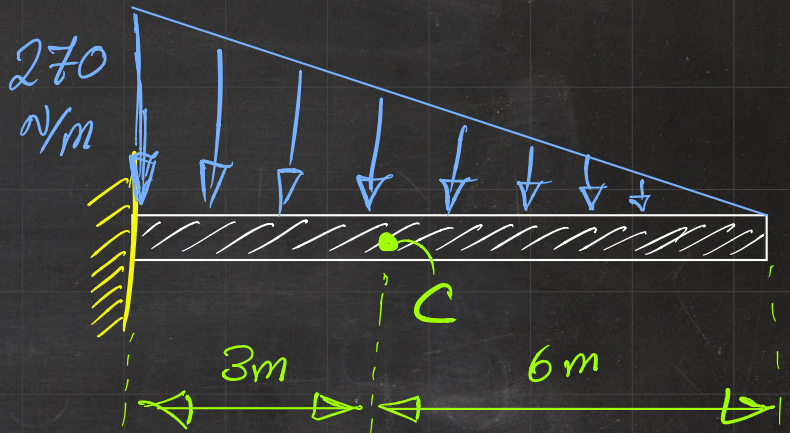




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Convention for internal loadings

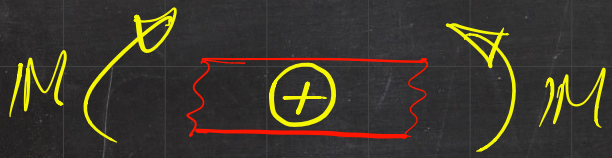


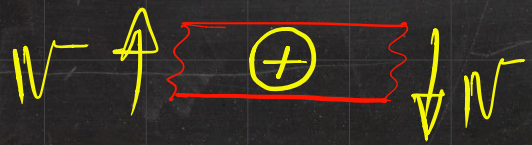


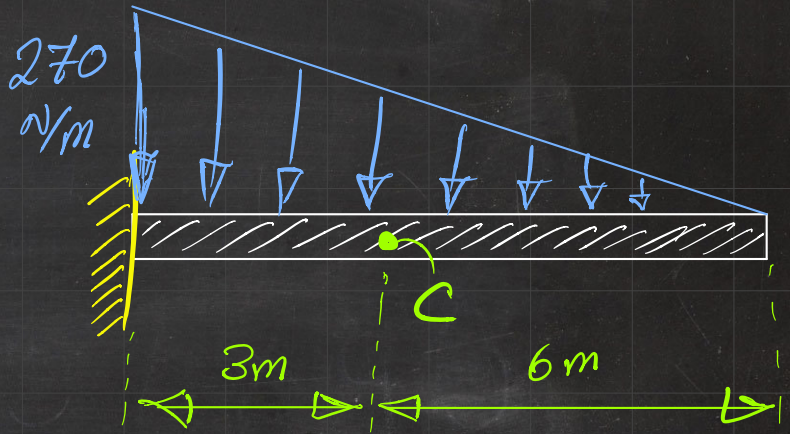
IMPORTANT

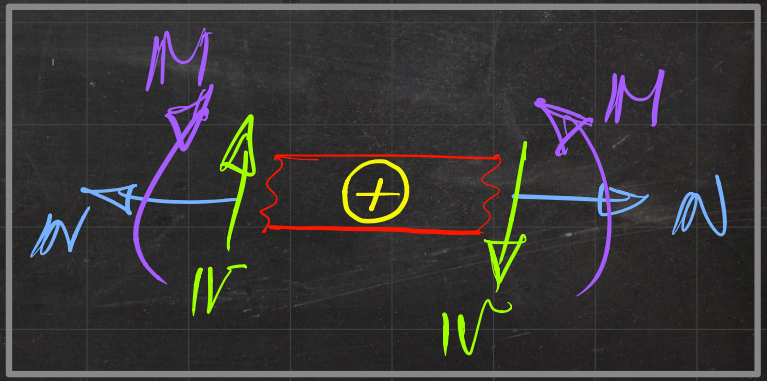
⊕ Convention for internal loadings





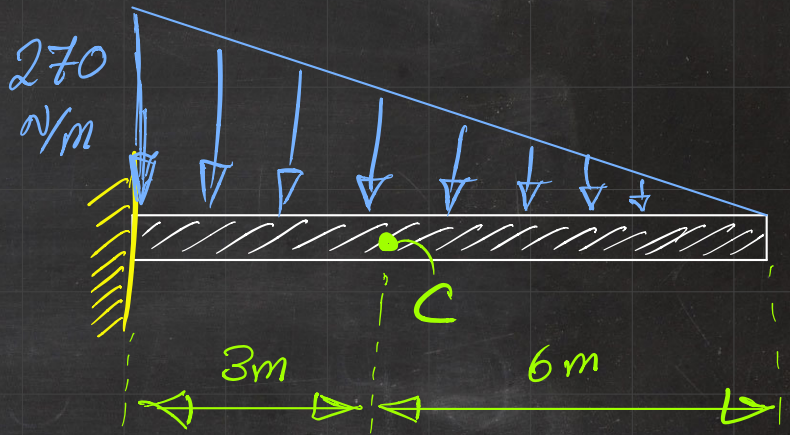


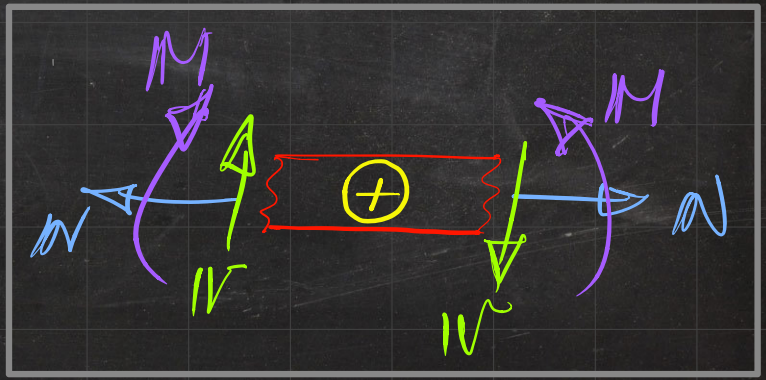




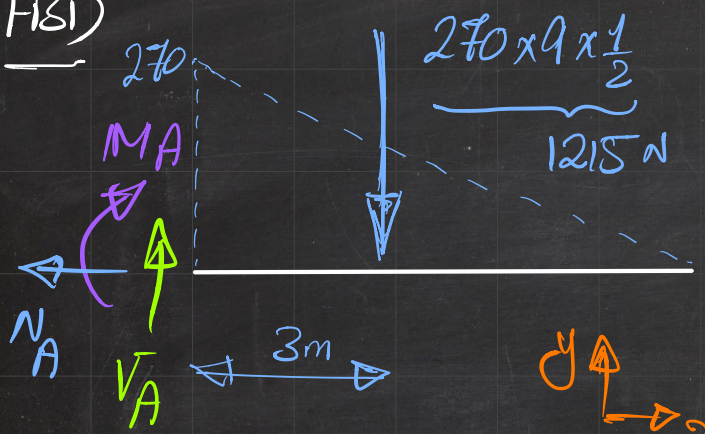
FBD

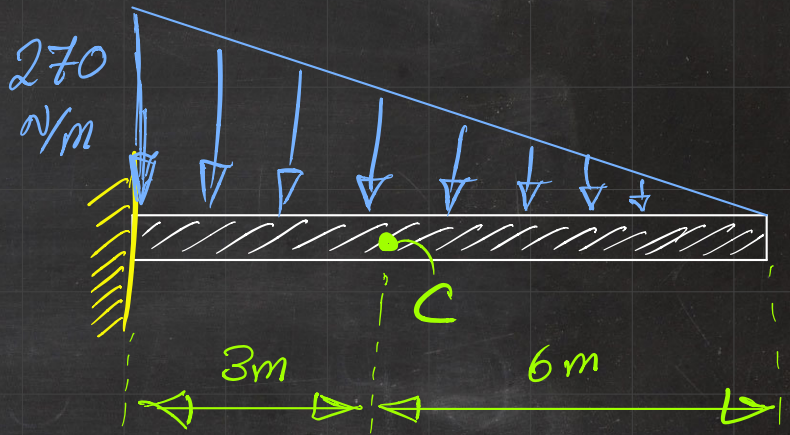






FBD

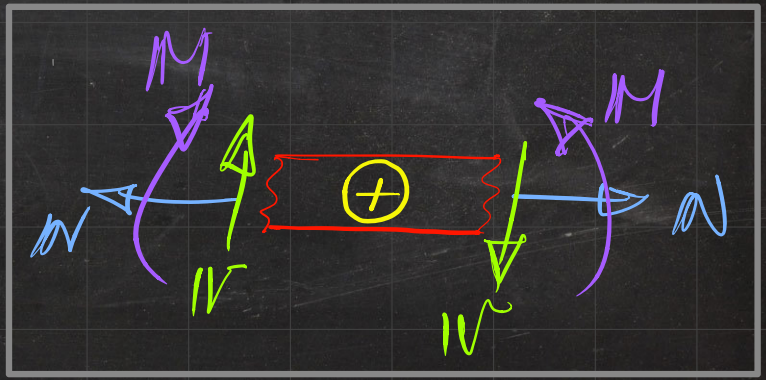




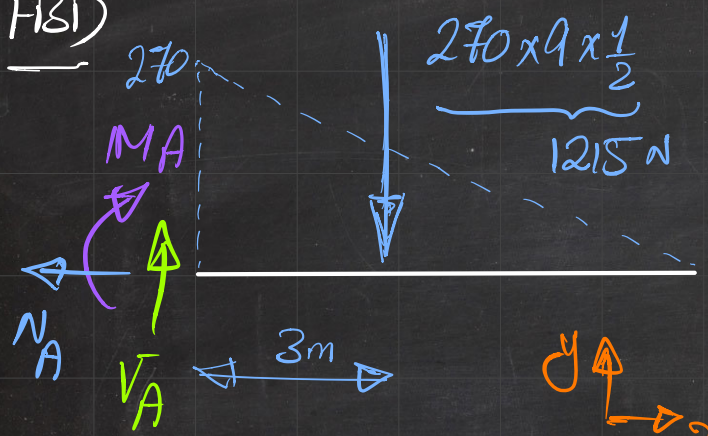
$$\overset{+}{\rightarrow} \sum F_x = 0$$

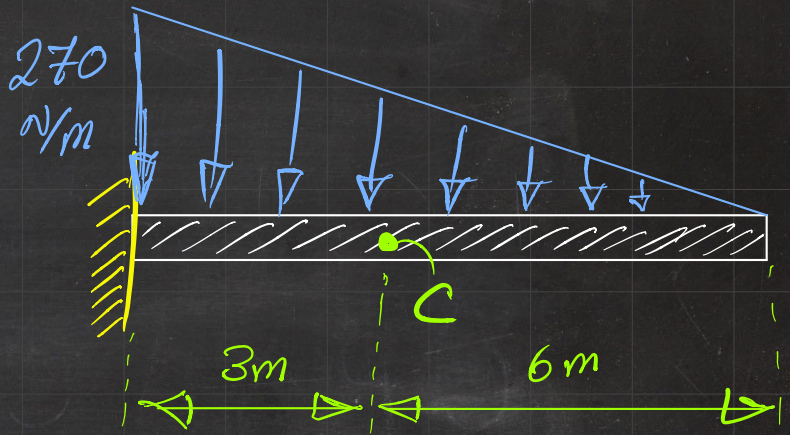
$$+\uparrow \sum F_y = 0$$

$$(+) \sum M_A = 0$$



FBD

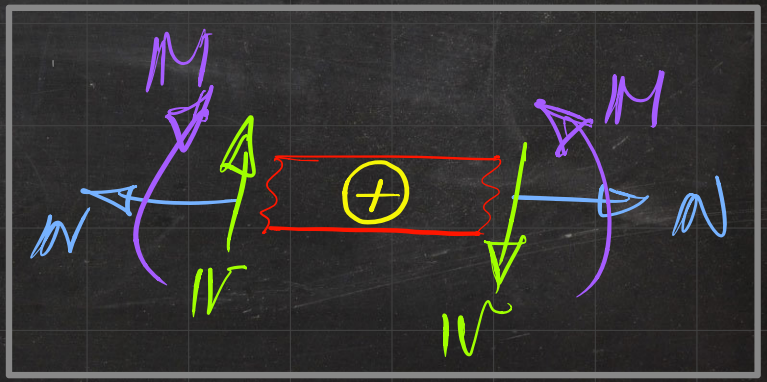




$$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -N_A = 0$$

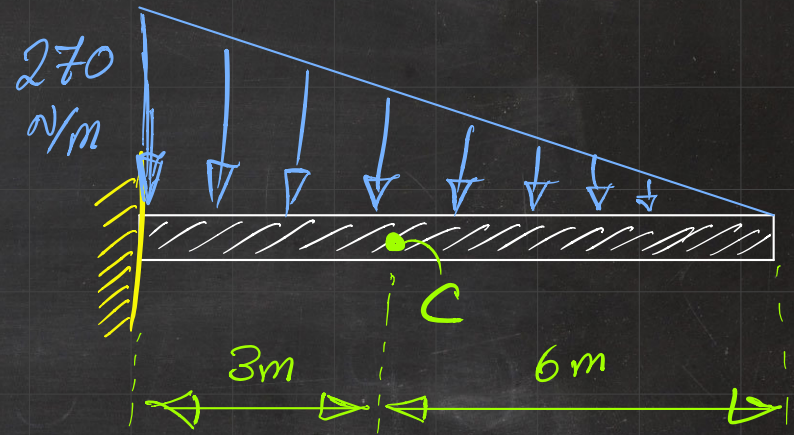
$$+\uparrow \sum F_y = 0 \Rightarrow V_A - 1215 = 0$$

$$(+) \sum M_A = 0 \Rightarrow -M_A - 1215 \times 3 = 0$$



FBD





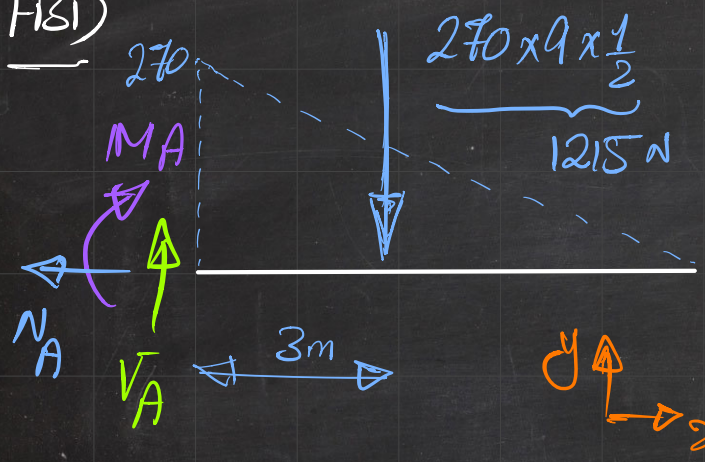
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow -N_A = 0 \Rightarrow N_A = 0$

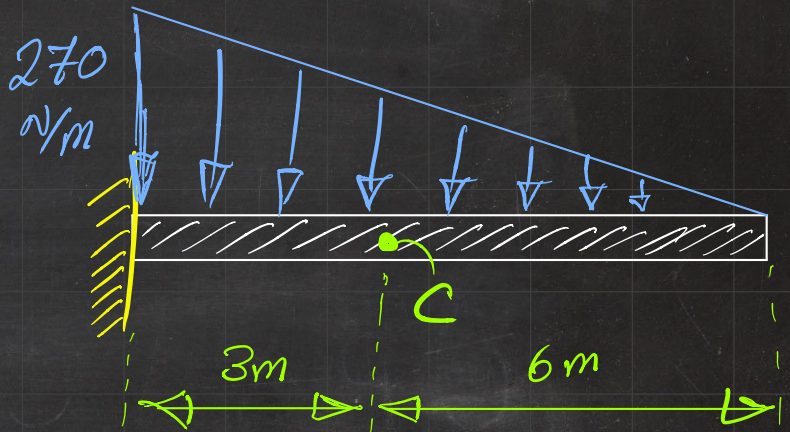
$+\uparrow \sum F_y = 0 \Rightarrow V_A - 1215 = 0 \Rightarrow V_A = 1215\ N$

$(+) \sum M_A = 0 \Rightarrow -M_A - 1215 \times 3 = 0 \Rightarrow M_A = -3645\ N.m \Rightarrow 3645\ (CCW)$

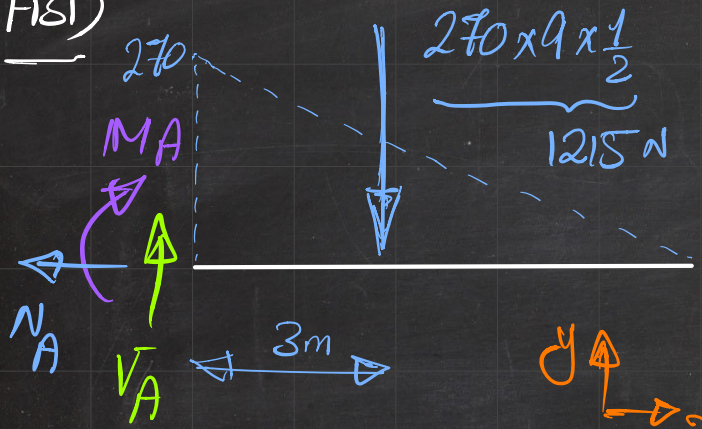
$M_A = 3645$

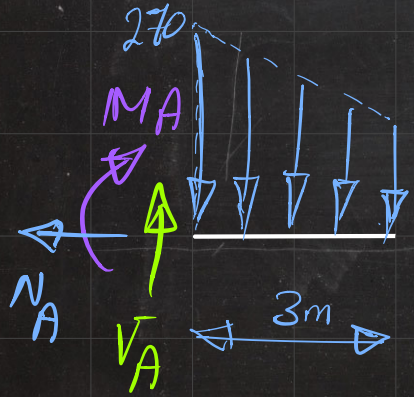
FBD





FBD

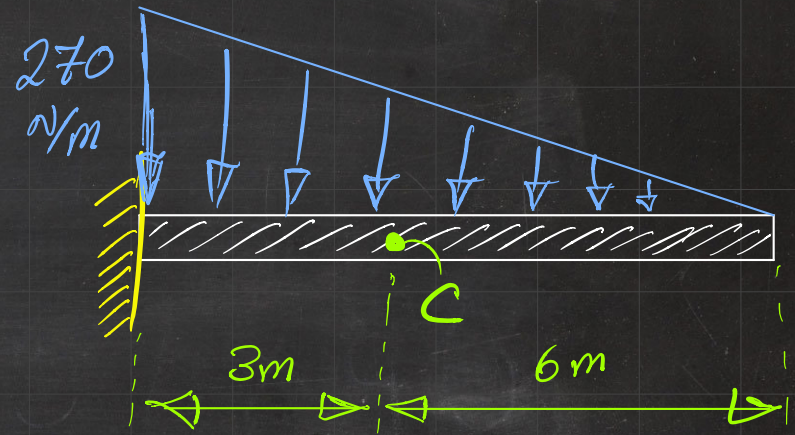




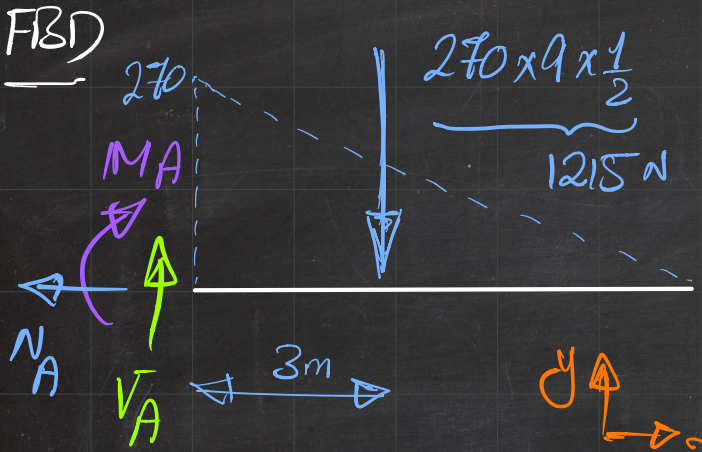
$N_A = 0$

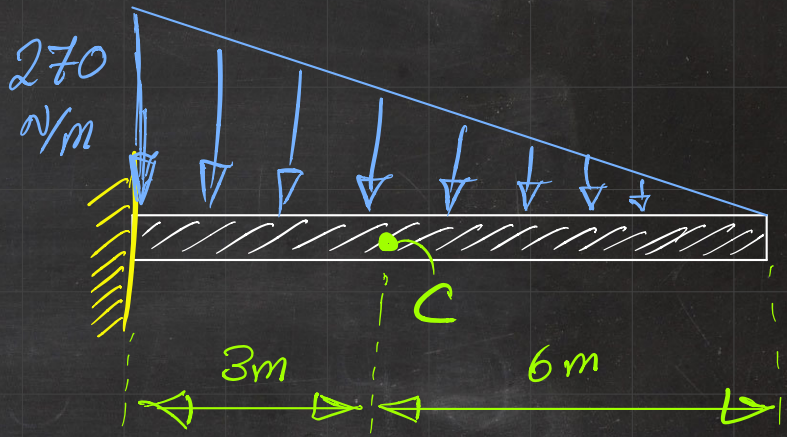
$V_A = 1215$

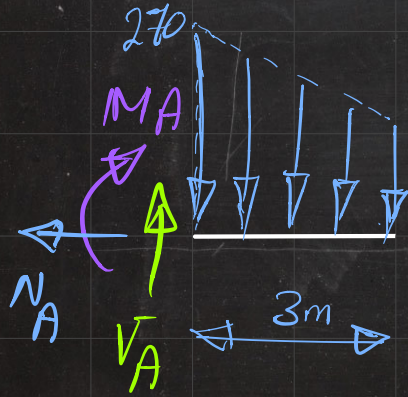
$M_A = -3645$



FBD



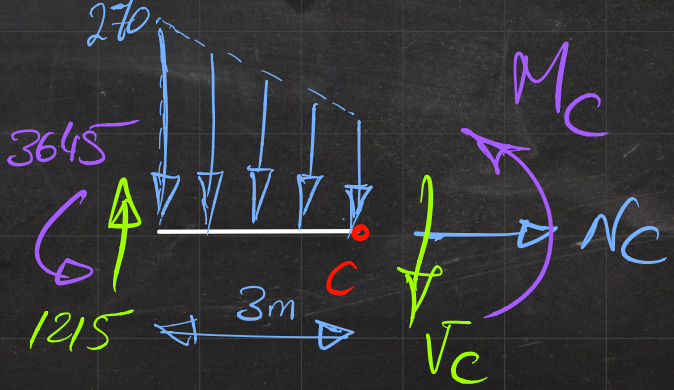


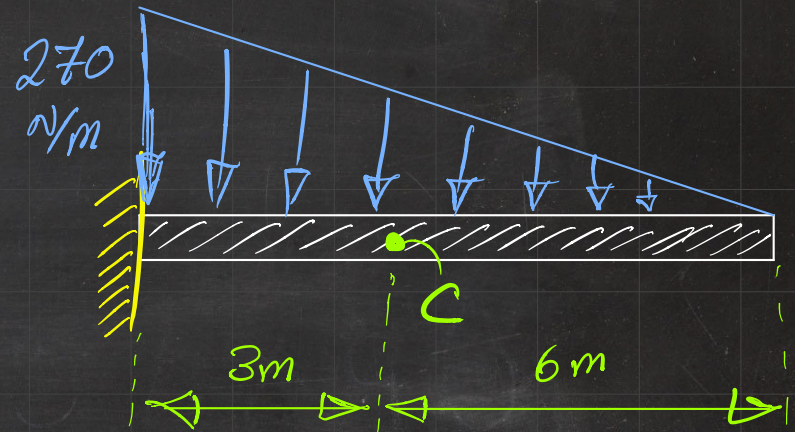


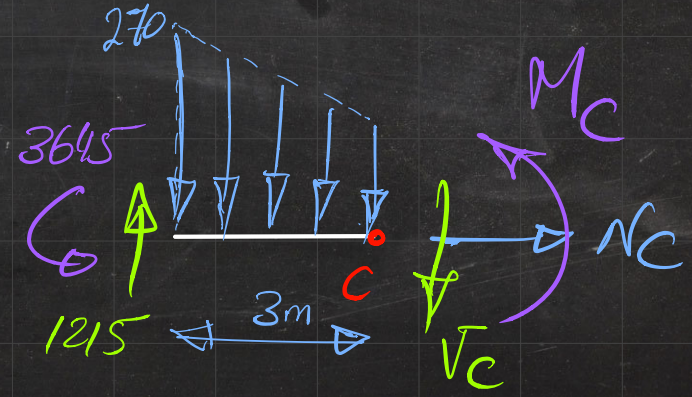
$N_A = 0$

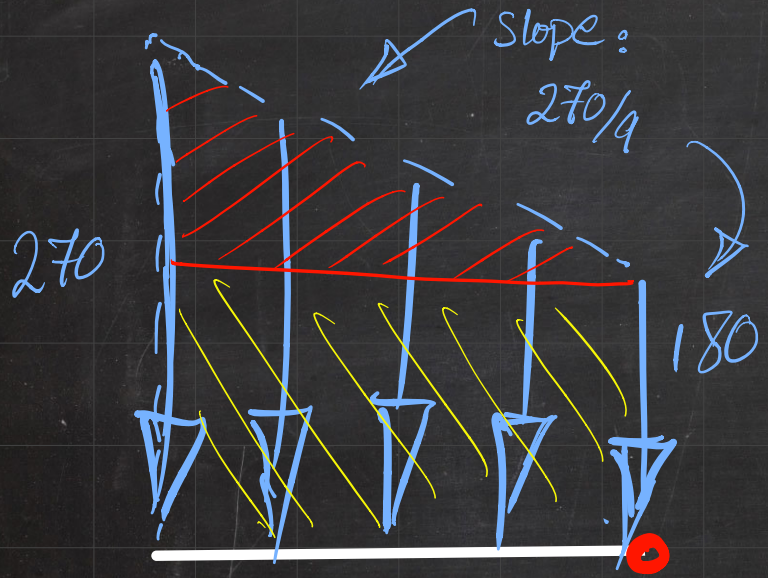
$V_A = 1215$

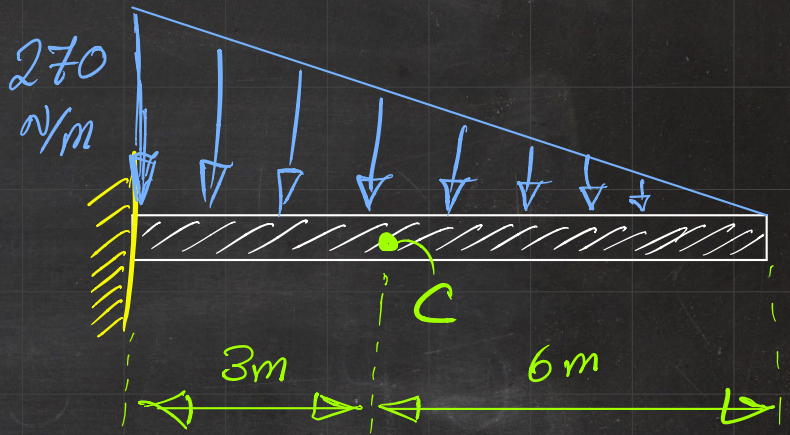
$M_A = -3645$

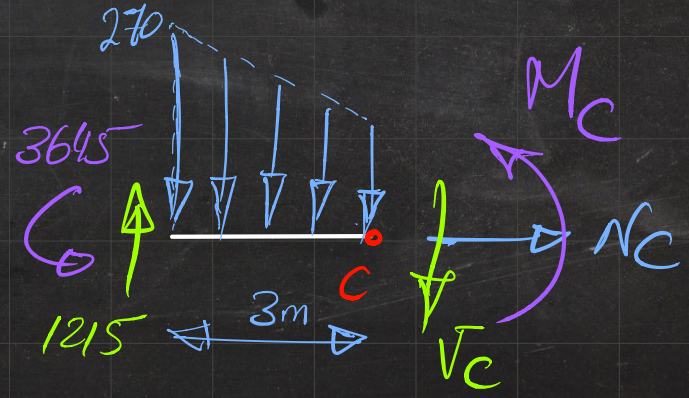


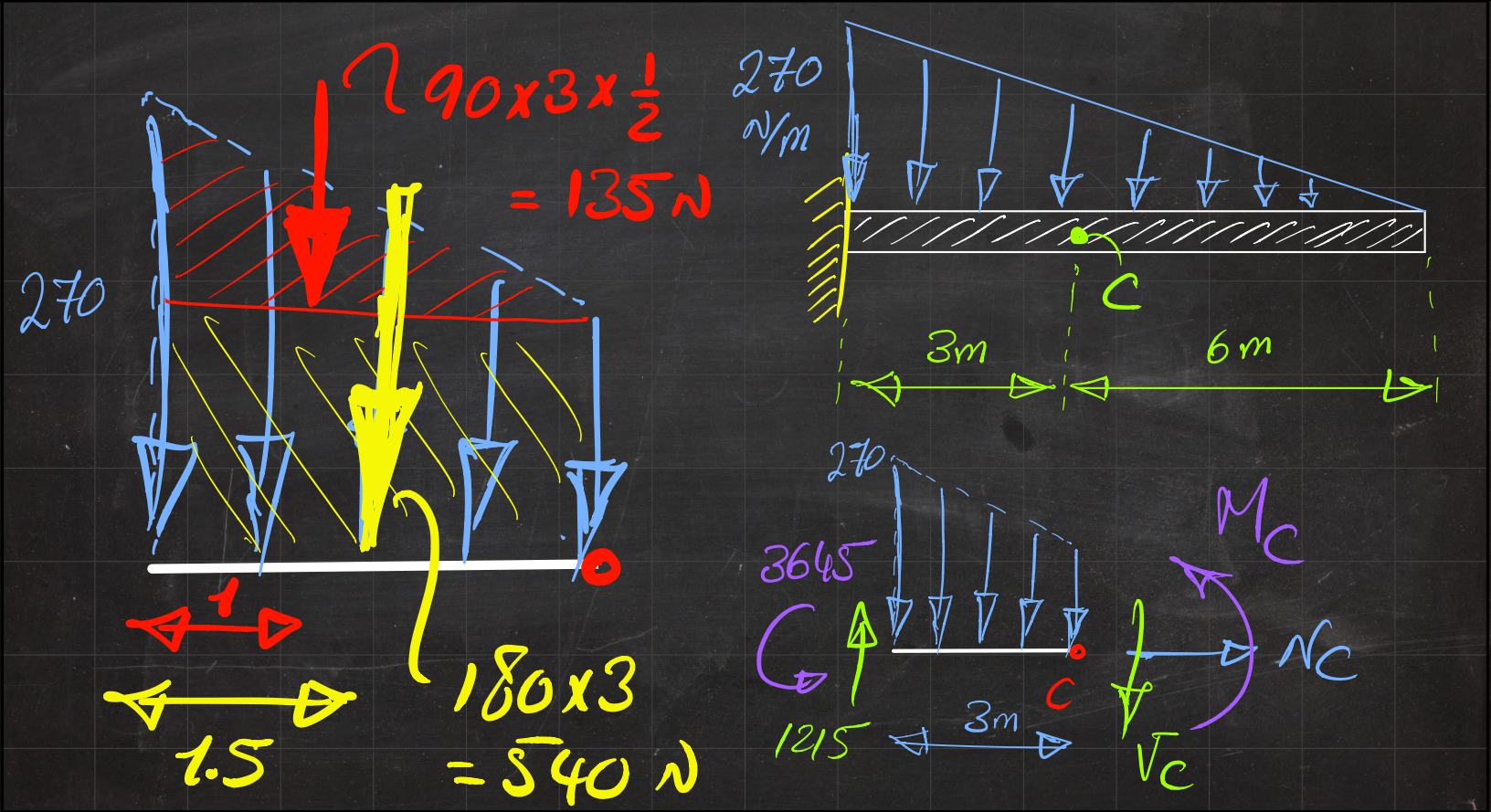

270
N/m
C
3m
6m


270
3645
C
1215
3m
C
M_C
N_C
V_C


Slope :
270/4
270
180


270
N/m
C
3m
6m

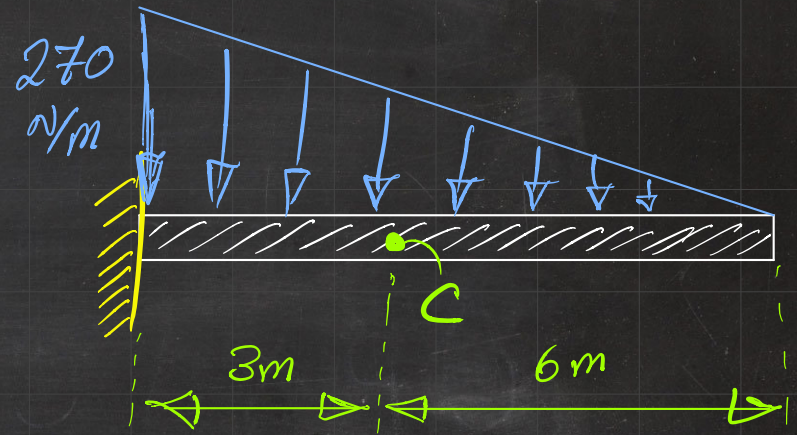

270
3645
C
1215
3m
C
M_C
N_C
V_C

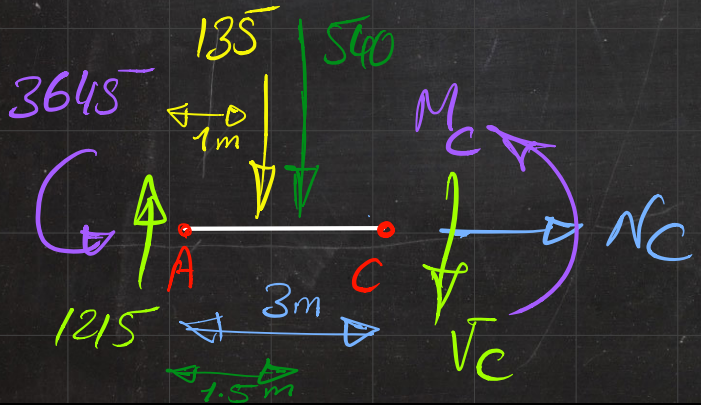

90x3x1/2
= 135 N
270
1
1.5
180x3
= 540 N
270
N/m
C
3m
6m
270
3645
1215
C
3m
Mc
Nc
Vc

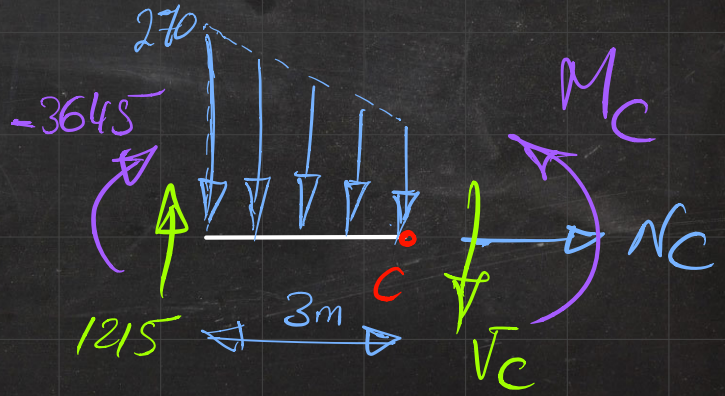
$\overset{+}{\rightarrow} \sum F_x = 0$

$+\uparrow \sum F_y = 0$

$\overset{+}{\circlearrowleft} \sum M_A = 0$



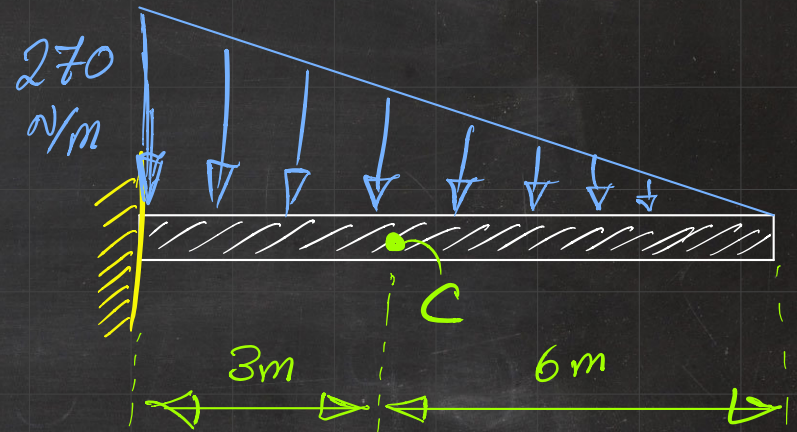


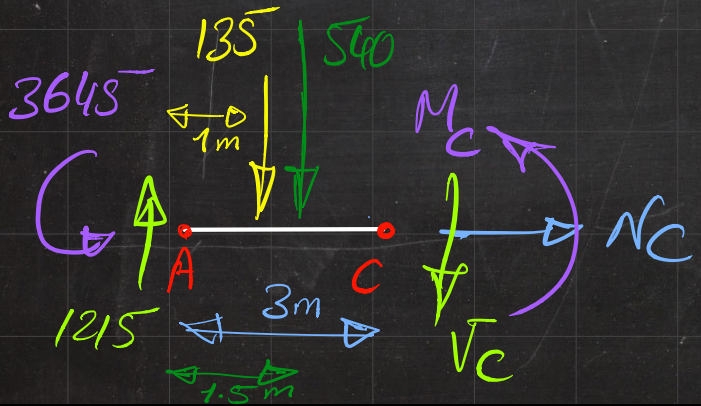


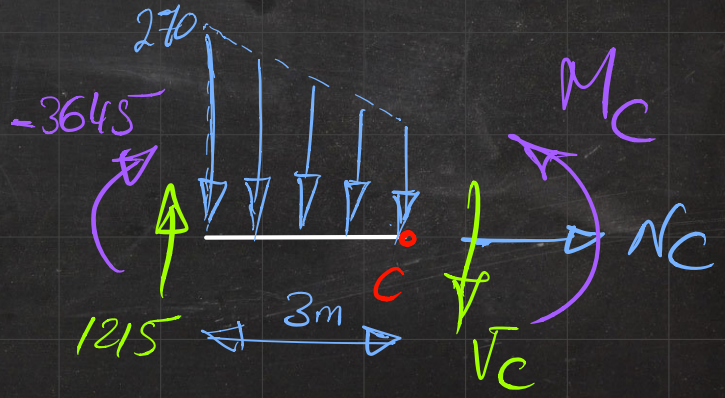
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N_C = 0$

$+\uparrow \sum F_y = 0$

$\overset{+}{\circlearrowleft} \sum M_A = 0$





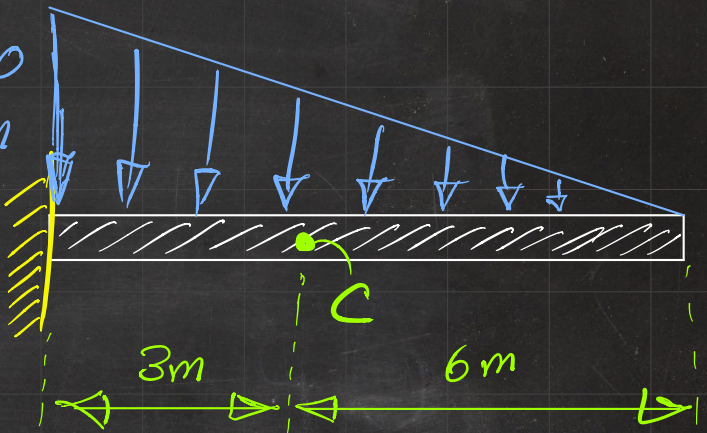


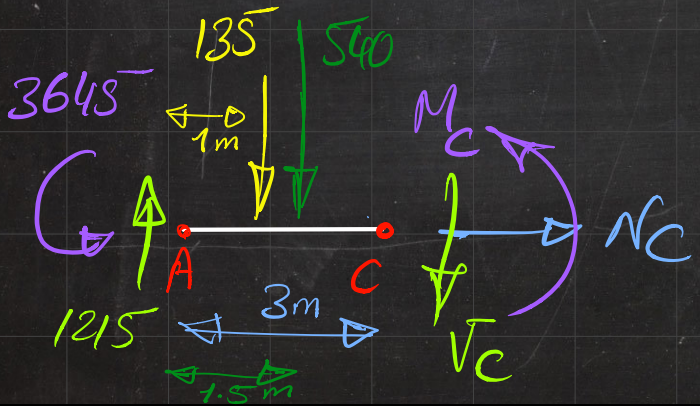
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N_C = 0$

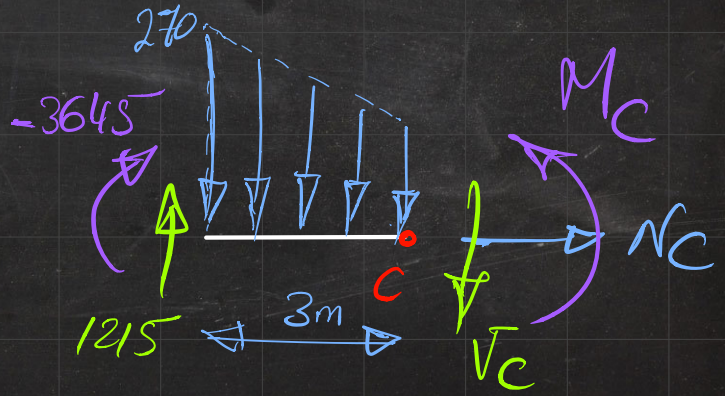
$+\uparrow \sum F_y = 0 \Rightarrow 1215 - 135 - 540 - V_C = 0$

$\Rightarrow V_C = 540\ N$

$\overset{+}{\circlearrowleft} \sum M_A = 0$







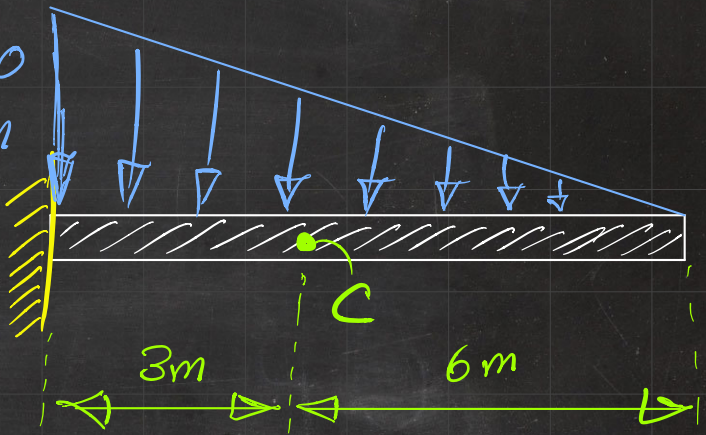
$\overset{+}{\rightarrow} \sum F_x = 0 \Rightarrow N_C = 0$

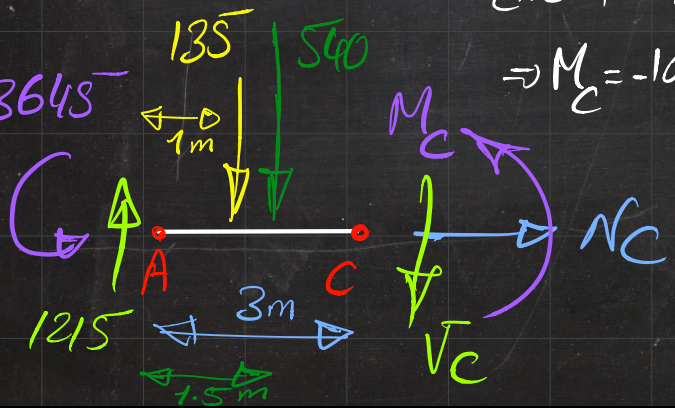
$+\uparrow \sum F_y = 0 \Rightarrow 1215 - 135 - 540 - V_C = 0$

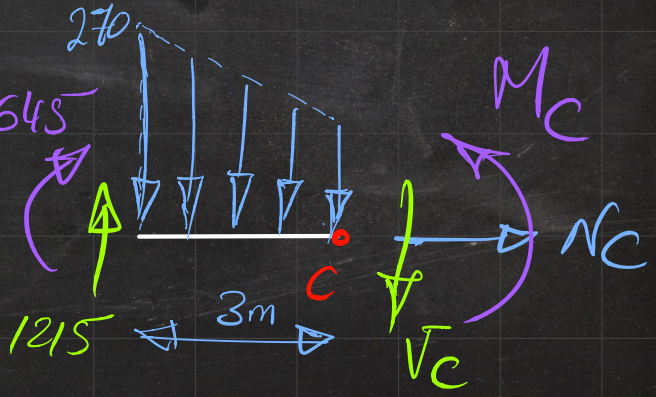
$\Rightarrow V_C = 540\ N$

$\curvearrowleft^{+} \sum M_A = 0 \Rightarrow +3645 - 135 \times 1 - 540 \times 1.5 - V_C \times 3 + M_C = 0$

$\Rightarrow M_C = -1080\ N.m$

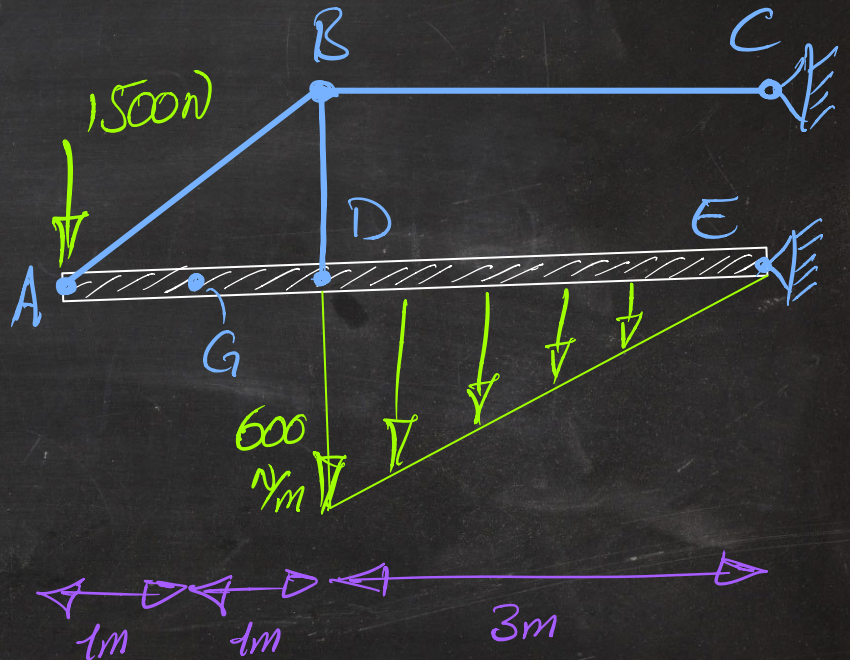


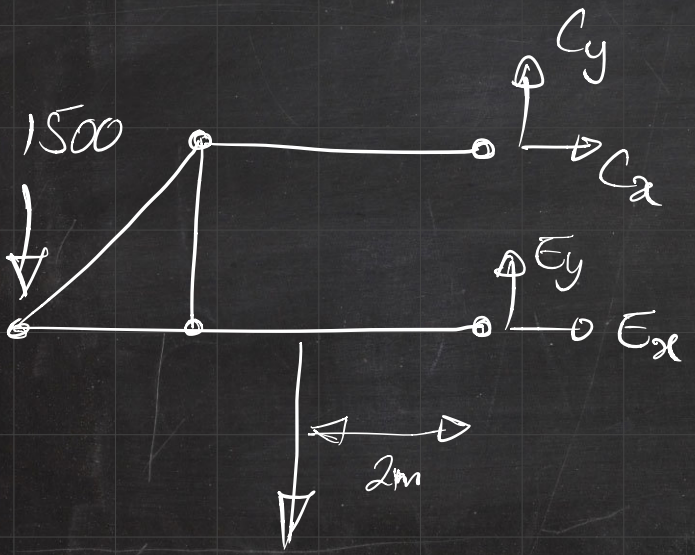




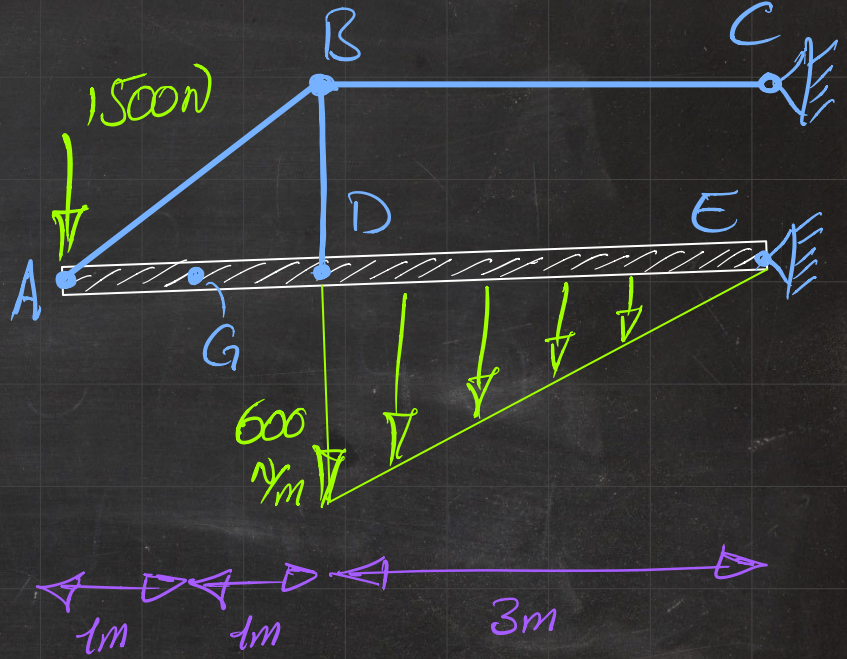
Exercise 2 . [similar to ... P. 13 ... 1.3]

DETERMINE THE RESULTANT INTERNAL LOADINGS ACTING ON THE CROSS SECTION AT G OF THE BEAM SHOWN IN THE FIGURE. EACH JOINT IS PIN-CONNECTED.



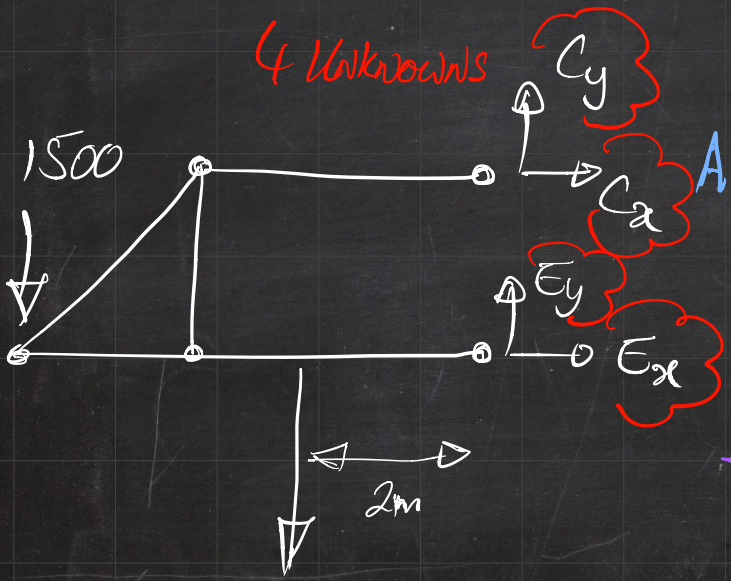


$$900 = 600 \times 3 \times 1/2$$

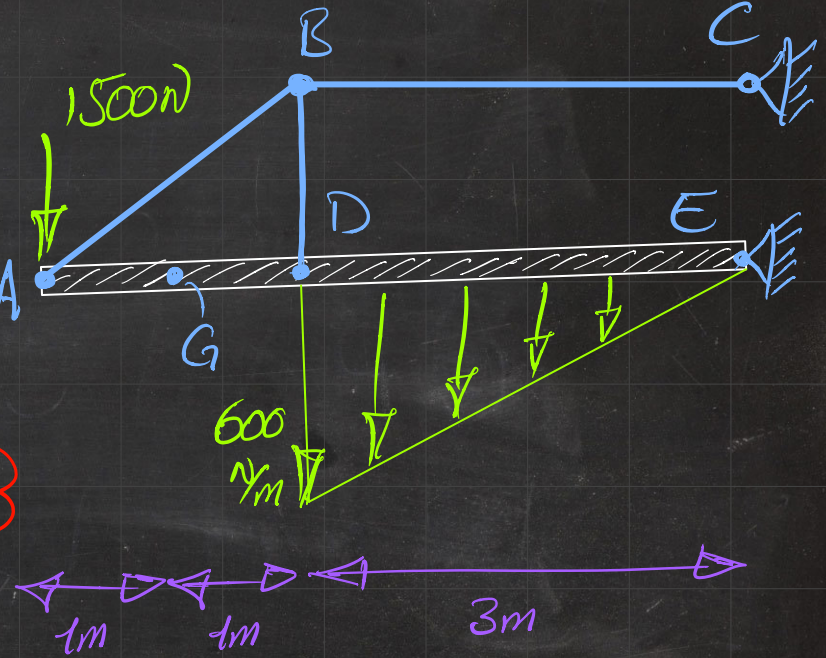


2D $\Rightarrow$ 3 EQNs

4 Unknowns

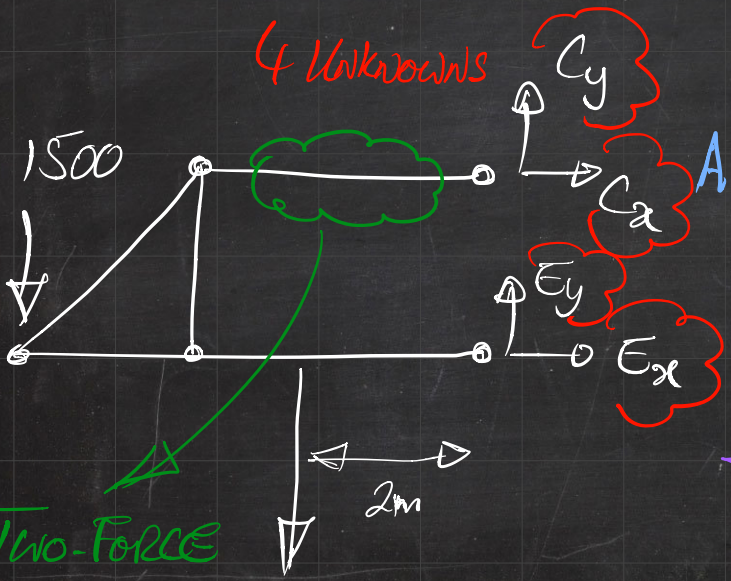


$900 = 600 \times 3 \times 1/2$



2D → 3 EQNS

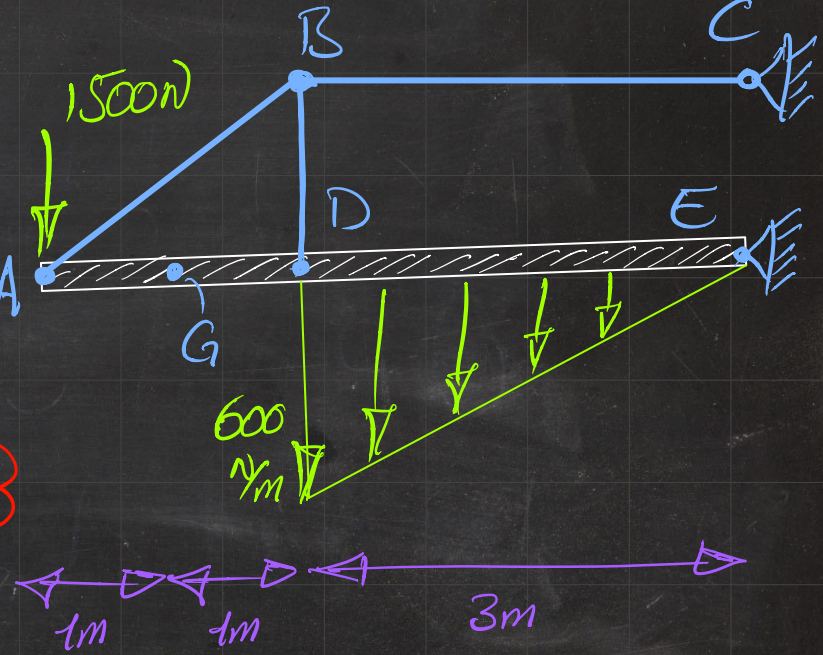
4 Unknowns



TWO-FORCE MEMBER

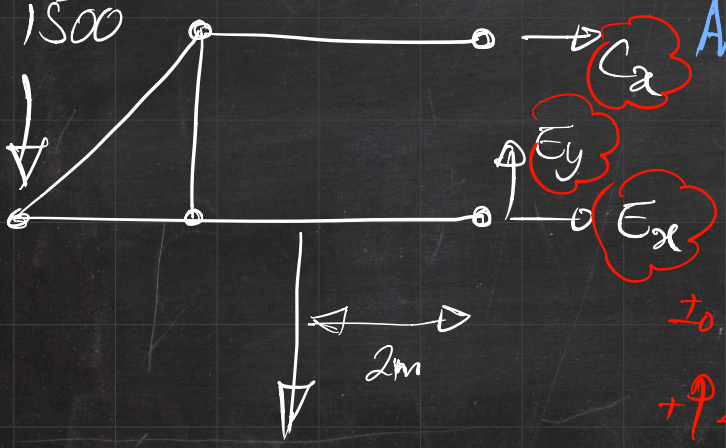
$\Rightarrow C_y = 0$

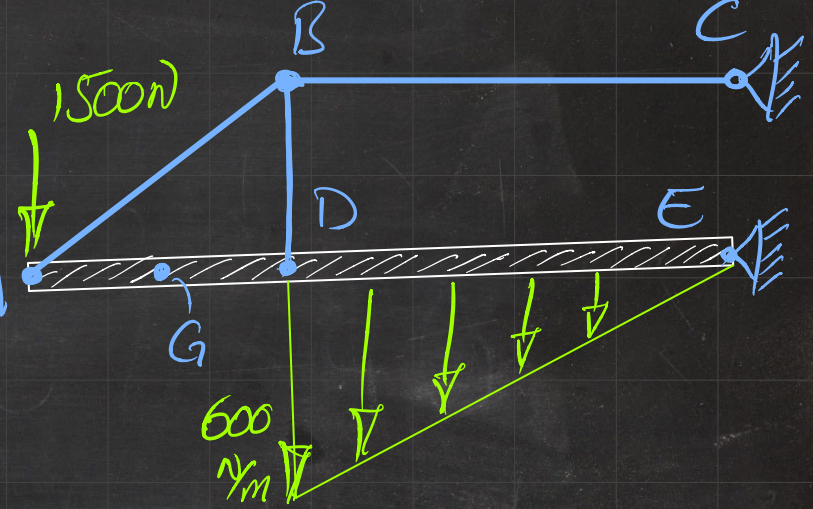
$900 = 600 \times 3 \times 1/2$



2D → 3 EQNs

3 Unknowns ✓





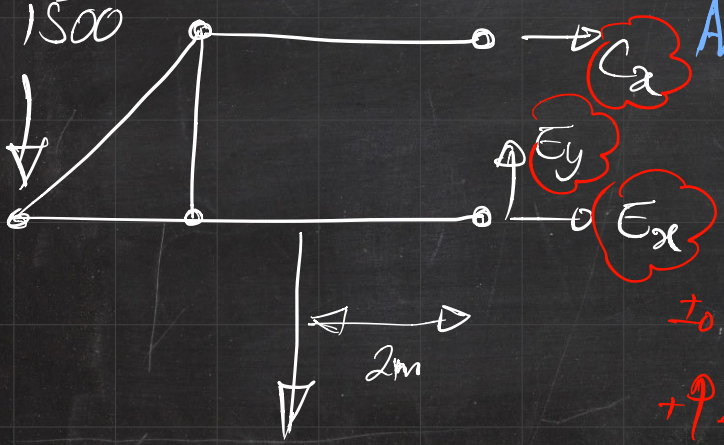
$+\rightarrow \sum F_x = 0$

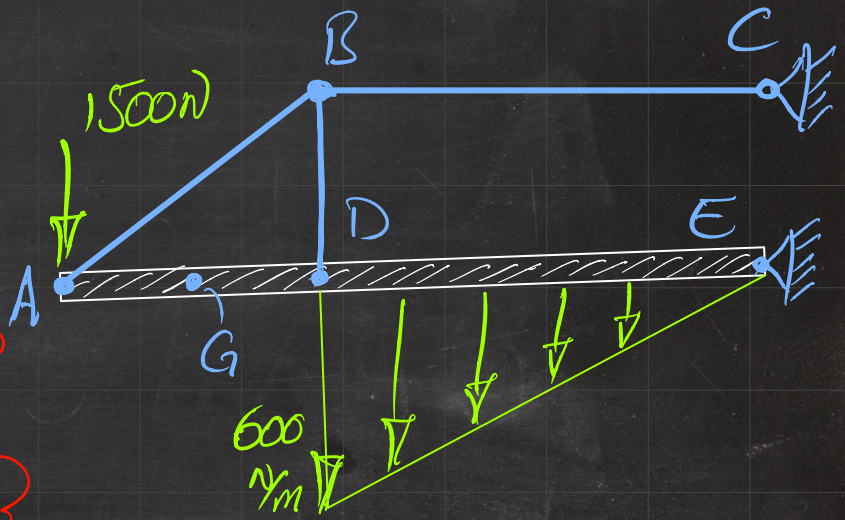
$+\uparrow \sum F_y = 0$

$+\circlearrowleft \sum M_E = 0$

2D → 3 EQNs

3 Unknowns ✓

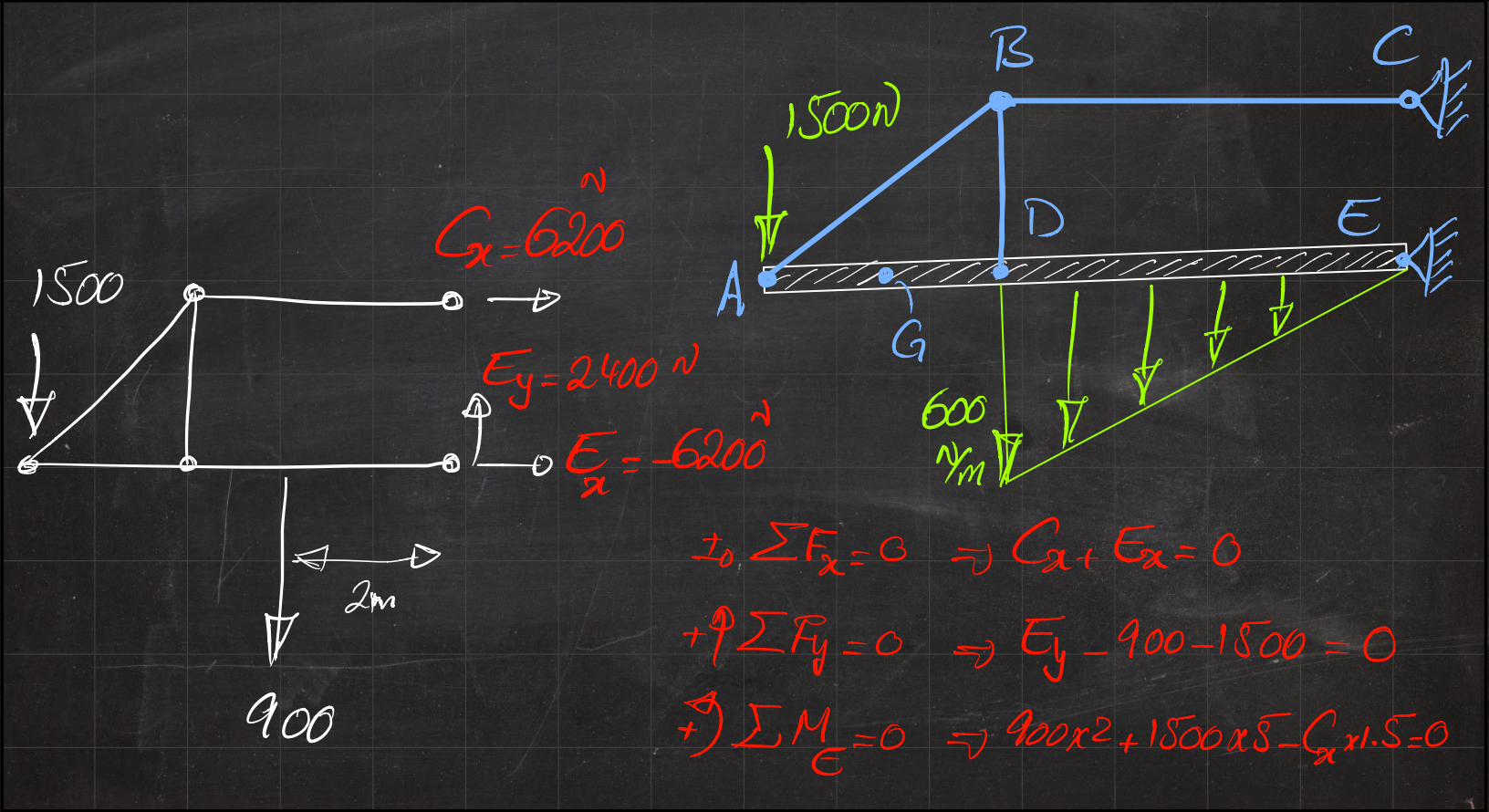




$+\rightarrow \sum F_x = 0 \Rightarrow C_x + E_x = 0$

$+\uparrow \sum F_y = 0 \Rightarrow E_y - 900 - 1500 = 0$

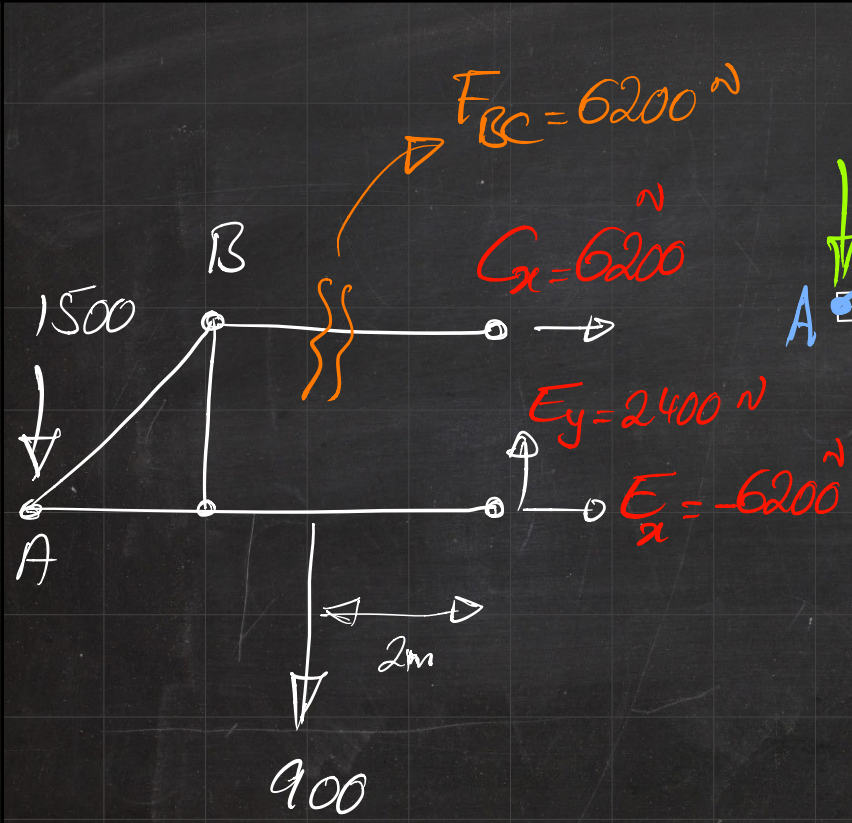
$+\circlearrowleft \sum M_E = 0 \Rightarrow 900 \times 2 + 1500 \times 3 - C_x \times 1.5 = 0$

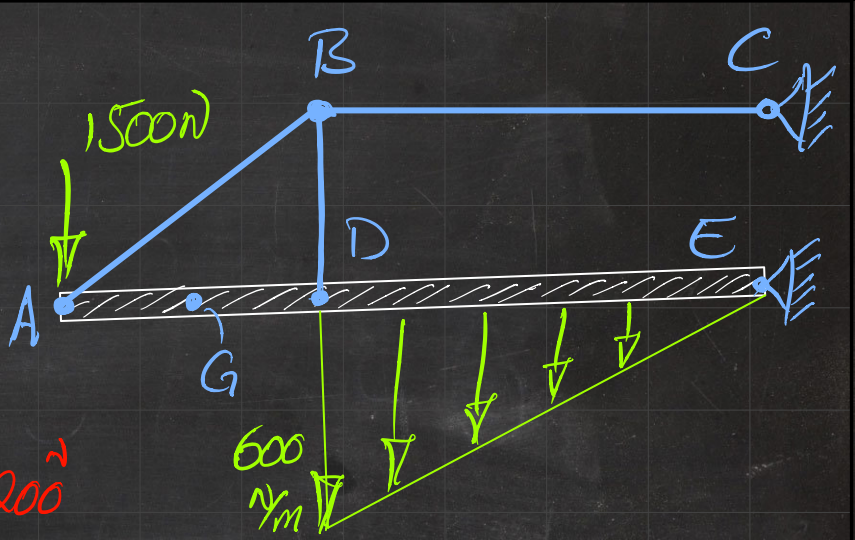


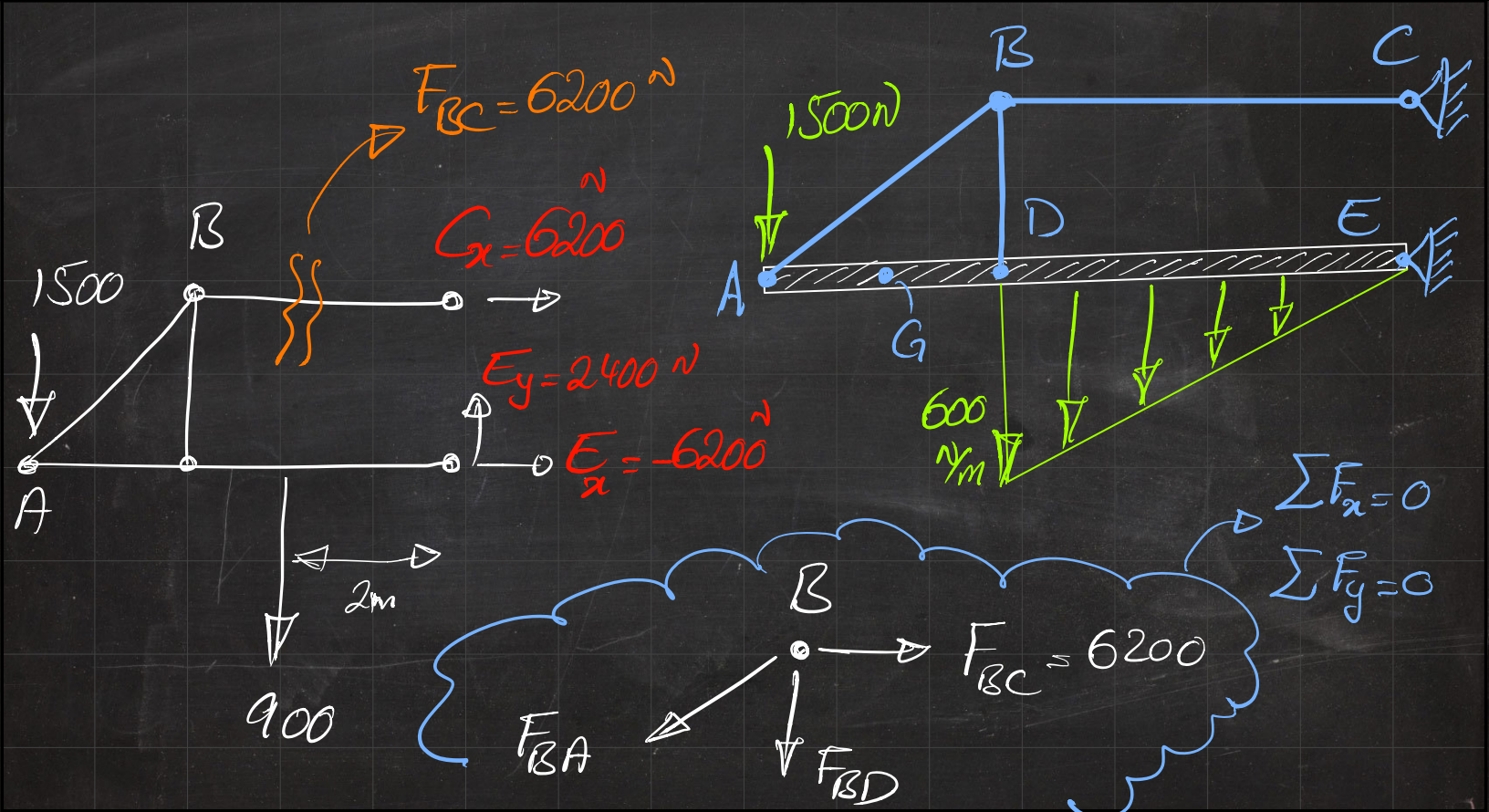
$$+\rightarrow \sum F_x = 0 \Rightarrow C_x + E_x = 0$$

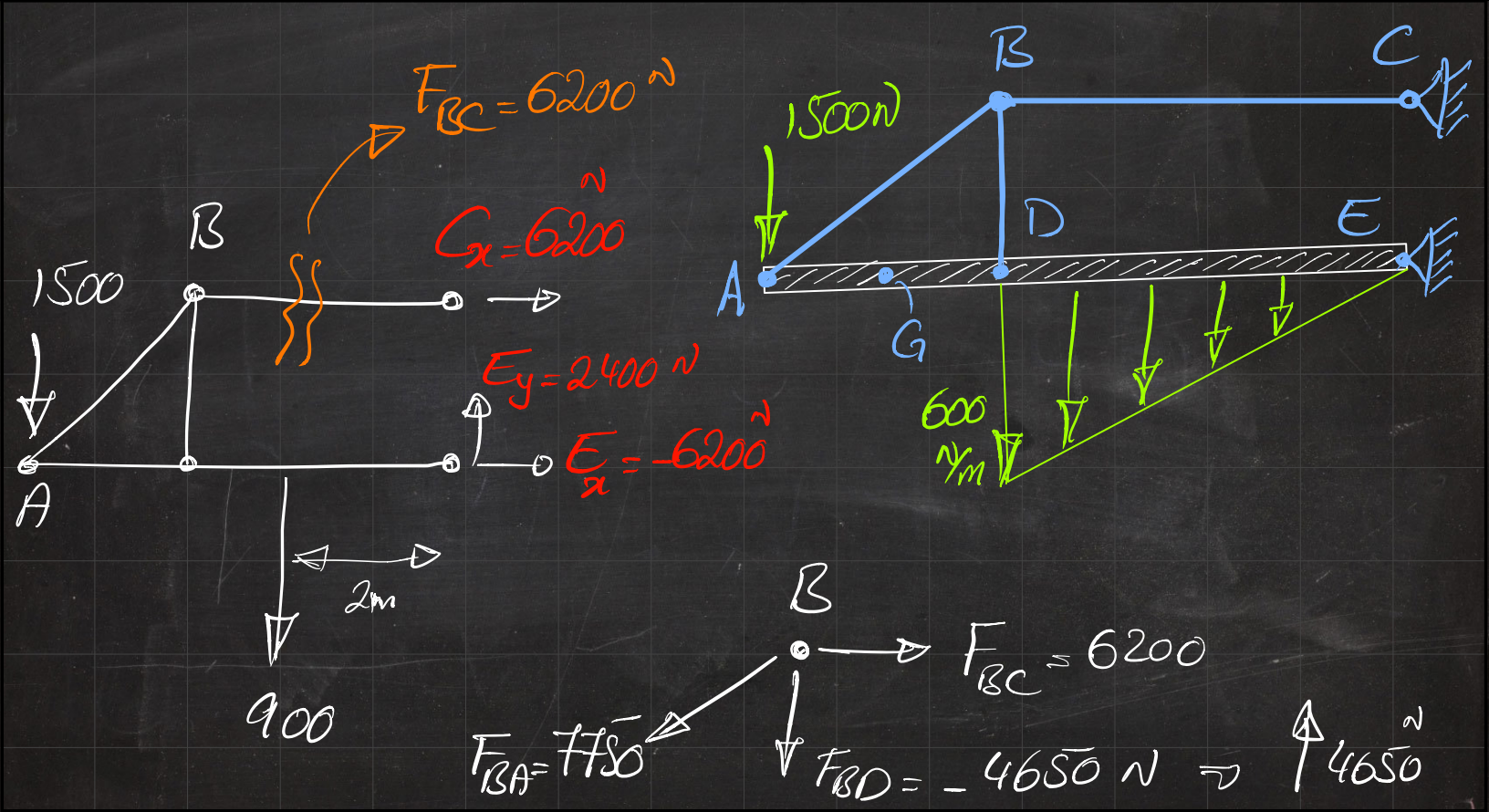
$$+\uparrow \sum F_y = 0 \Rightarrow E_y - 900 - 1500 = 0$$

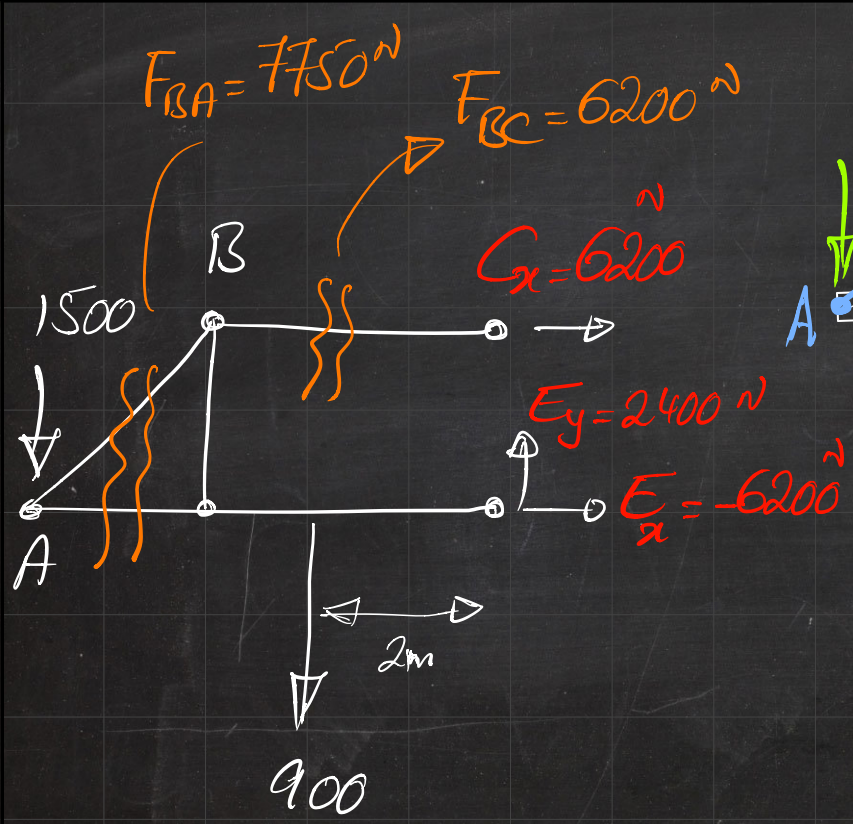
$$+\circlearrowleft \sum M_E = 0 \Rightarrow 900 \times 2 + 1500 \times 3 - C_x \times 1.5 = 0$$

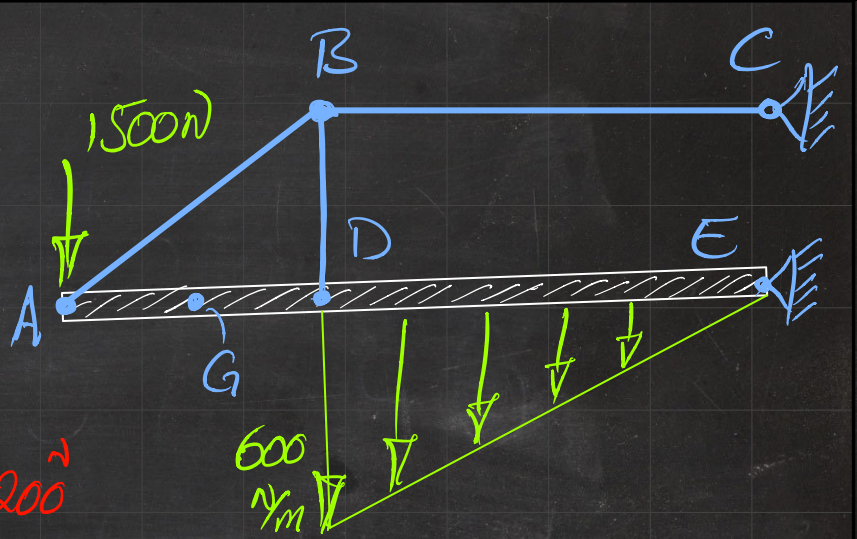

$F_{BC} = 6200\ N$
B
1500
$C_x = 6200\ N$
$C_y = 2400\ N$
$E_x = -6200\ N$
A
2m
900

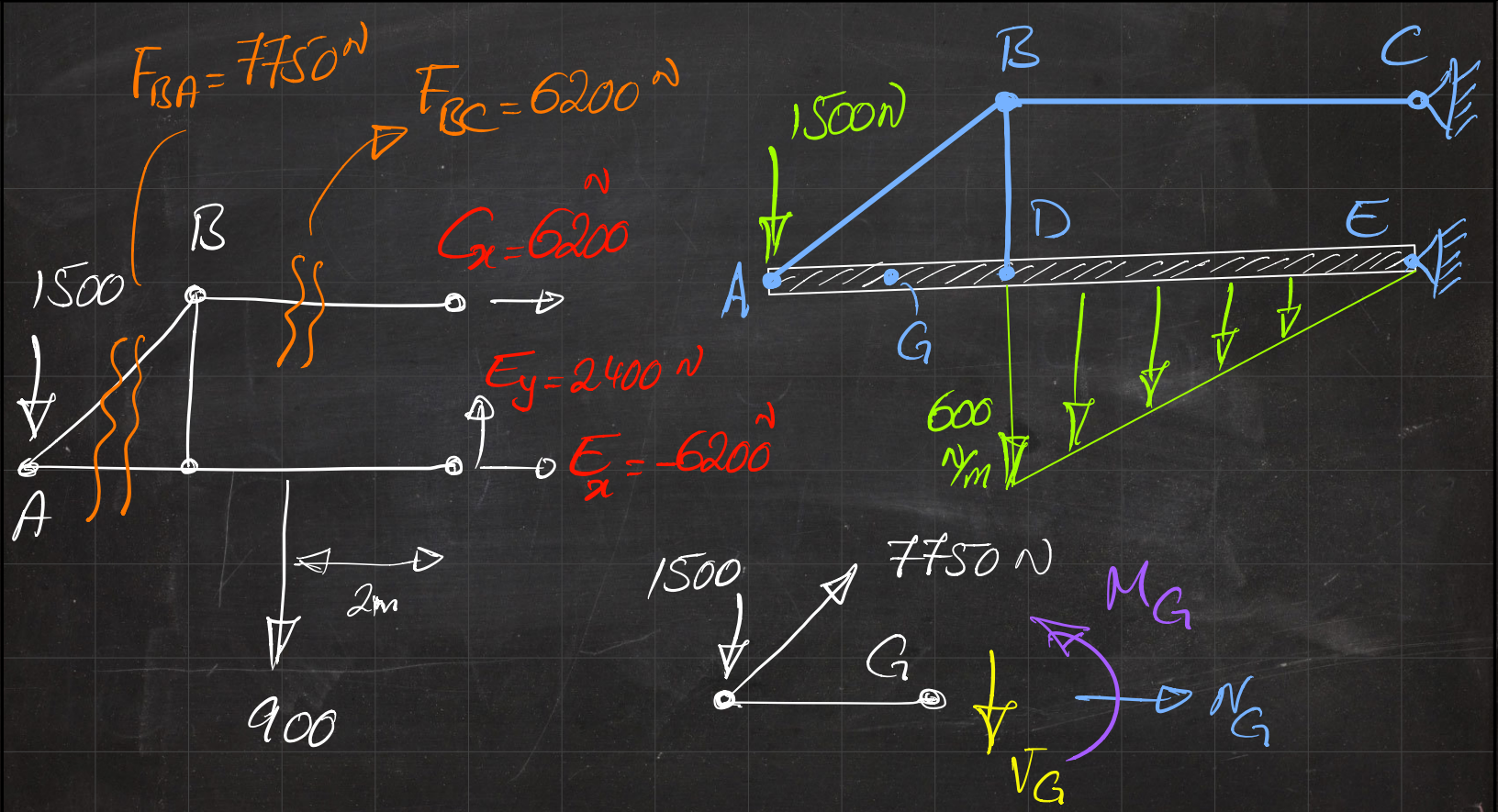

B
C
1500N
D
E
A
G
600 N/m


$F_{BC} = 6200$ N
B
1500
$C_x = 6200$ N
$E_y = 2400$ N
$E_x = -6200$ N
A
2m
900
B
C
1500N
D
E
A
G
600 N/m
$\sum F_x = 0$
$\sum F_y = 0$
B
$F_{BC} = 6200$
F_{BA}
F_{BD}


$F_{BC} = 6200$ N
B
1500
$C_x = 6200$ N
$E_y = 2400$ N
$E_x = -6200$ N
A
2m
900
1500N
B
C
D
E
A
G
600 N/m
B
$F_{BC} = 6200$
$F_{BA} = 7750$
$F_{BD} = -4650$ N
4650 N


$F_{BA} = 7750\,N$
$F_{BC} = 6200\,N$
B
1500
$C_x = 6200\,N$
$E_y = 2400\,N$
$E_x = -6200\,N$
A
2m
900

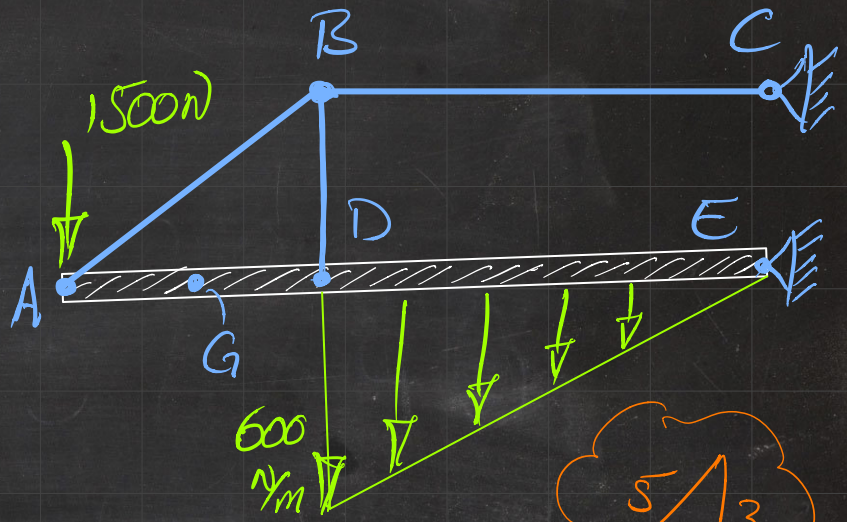

B
C
1500N
D
E
A
G
600
N/m

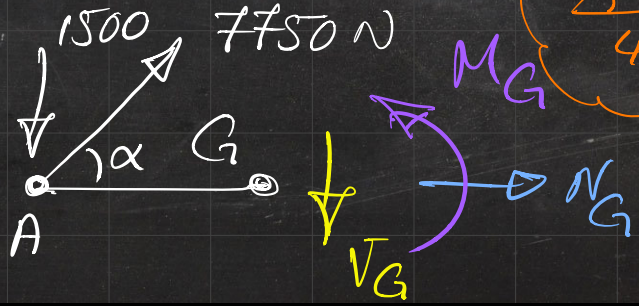

$F_{BA} = 7750\,N$
$F_{BC} = 6200\,N$
B
1500
A
$C_x = 6200\,N$
$E_y = 2400\,N$
$E_x = -6200\,N$
2m
900
B
C
1500N
D
E
A
G
600 N/m
1500
7750 N
G
M_G
N_G
V_G

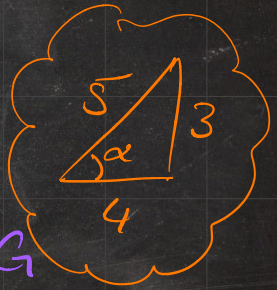
$\overset{+}{\rightarrow} \sum F_x = 0$

$+\uparrow \sum F_y = 0$

$\circlearrowleft^{+} \sum M_A = 0$







$\overset{+}{\rightarrow}\sum F_x = 0$

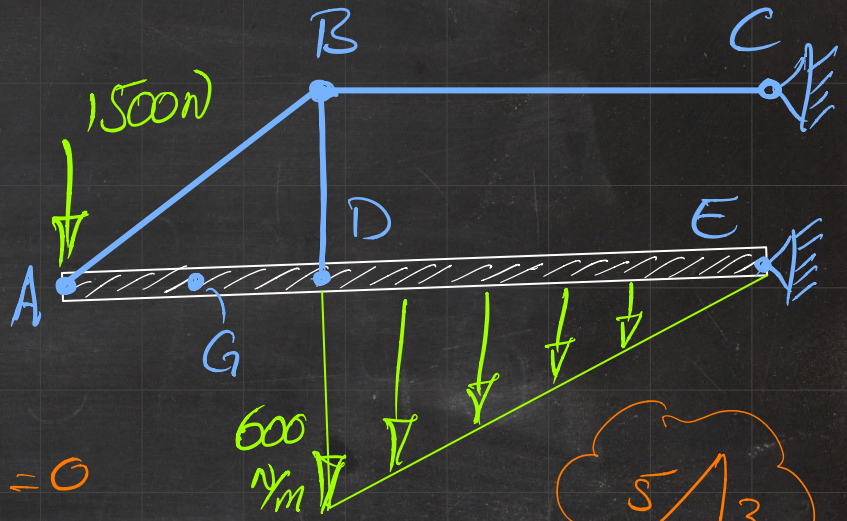
$$N_G + 7750 \times \cos\alpha = 0$$

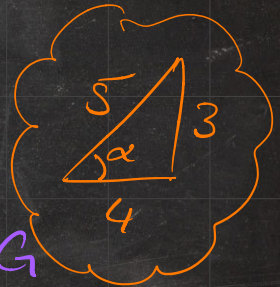
$+\uparrow \sum F_y = 0$

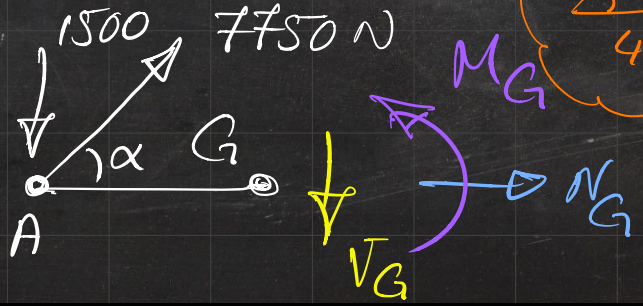
$$7750 \times \sin\alpha - V_G - 1500 = 0$$

$\circlearrowleft^{+} \sum M_A = 0$

$$M_G - V_G \times 1 = 0$$







$\overset{+}{\rightarrow}\sum F_x = 0$

$N_G + 7750 \times \cos\alpha = 0$

$\Rightarrow N_G = -6200\,N$

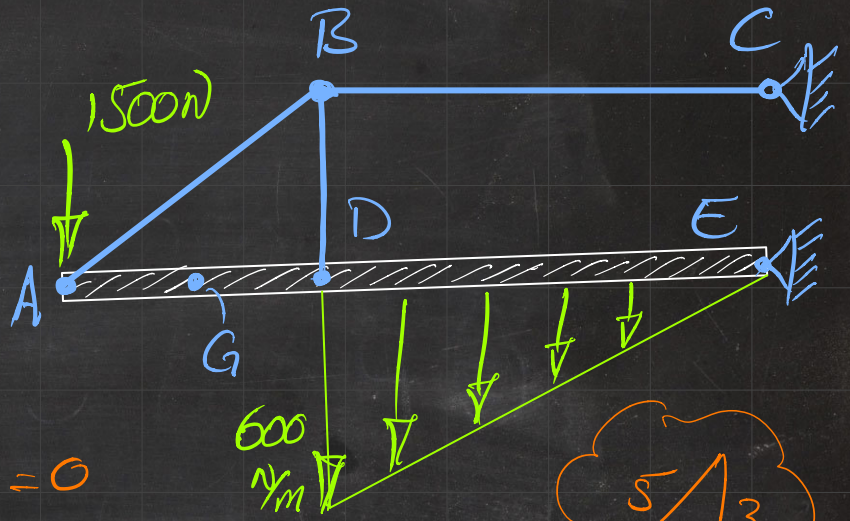
$+\uparrow \sum F_y = 0$

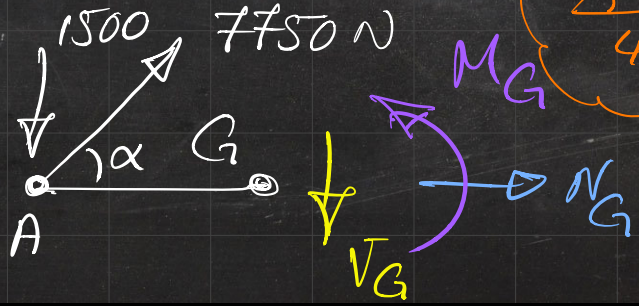
$7750 \times \sin\alpha - V_G - 1500 = 0$

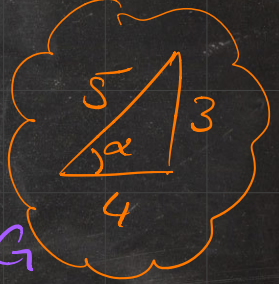
$\Rightarrow V_G = 3150\,N$

$\circlearrowleft \sum M_A = 0 \quad \Rightarrow M_G = 3150\,N.m$

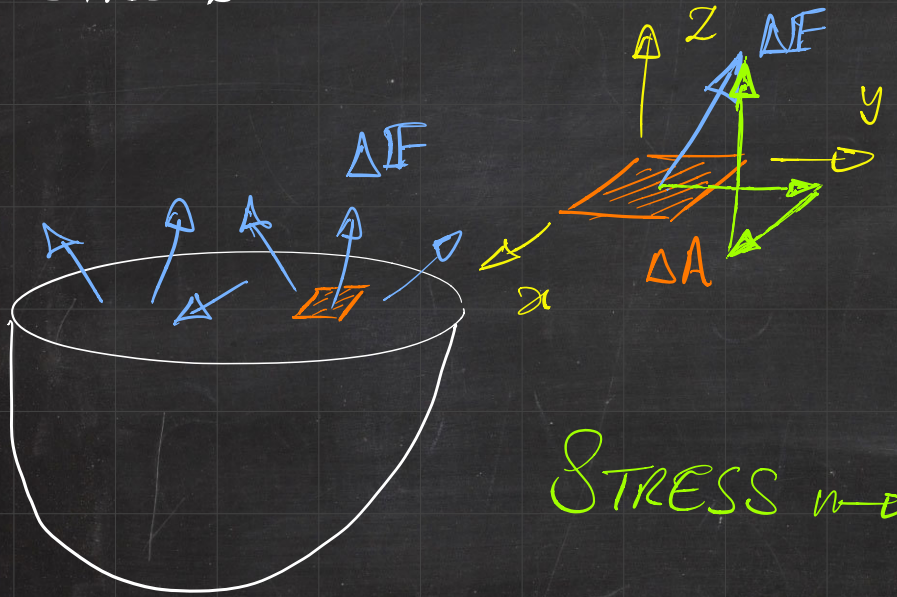
$M_G - V_G \times 1 = 0$





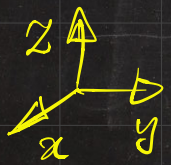


Stress

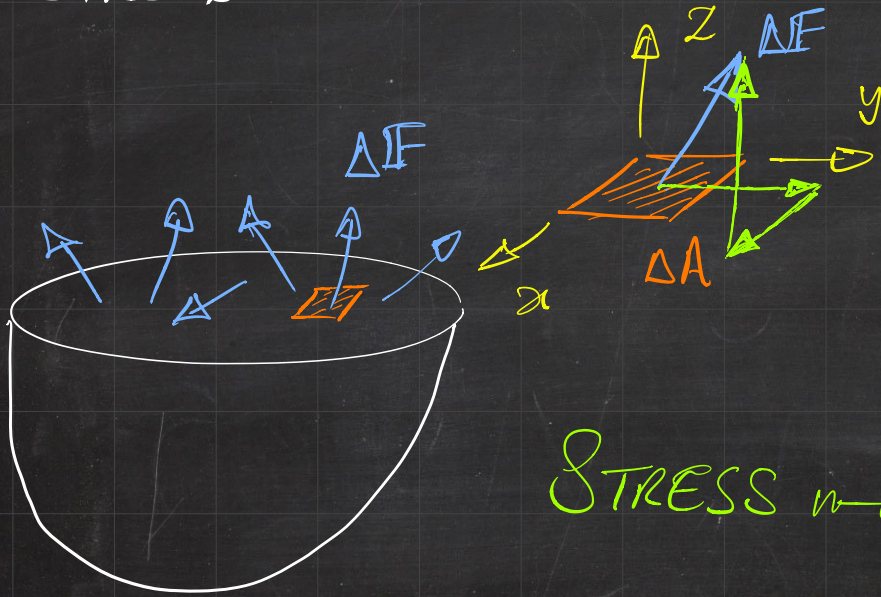


Stress → Force per Area $\left[\frac{N}{m^2}\right]$

↳ Pa



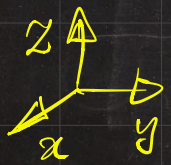
Stress



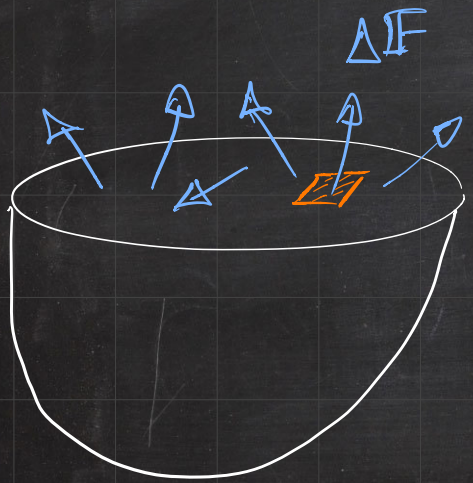
Normal Force
⇒ Normal Stress
σ

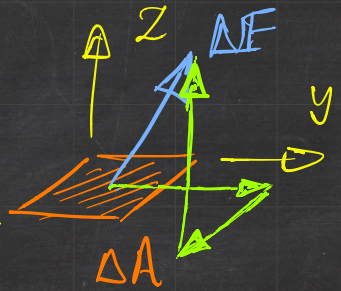
Shear Force
⇒ Shear Stress
τ

Stress ⟷ Force per Area $\left[\frac{N}{m^2}\right]$
└ Pa



Stress

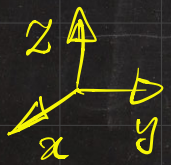




Normal Force
⇒ Normal Stress
σ

Shear Force
⇒ Shear Stress
τ

Stress ⟶ Force per Area $\left[\frac{N}{m^2}\right]$
└ Pa

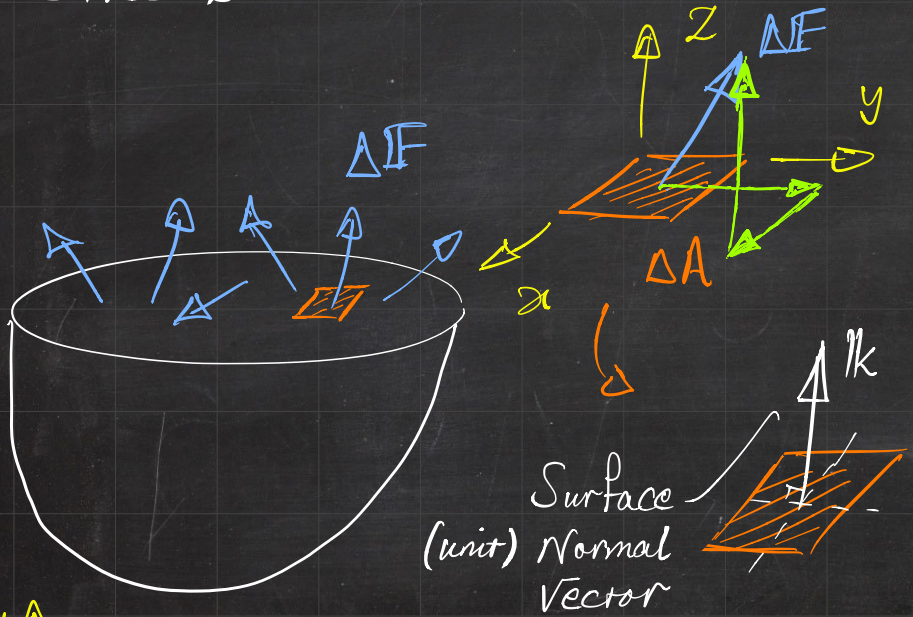


$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

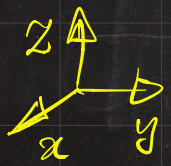
Stress



$$\sigma_z = \lim_{\Delta A_z \to 0} \frac{\Delta F_z}{\Delta A_z}$$

$$\tau_{zx} = \lim_{\Delta A_z \to 0} \frac{\Delta F_x}{\Delta A_z}$$

$$\tau_{zy} = \lim_{\Delta A_z \to 0} \frac{\Delta F_y}{\Delta A_z}$$

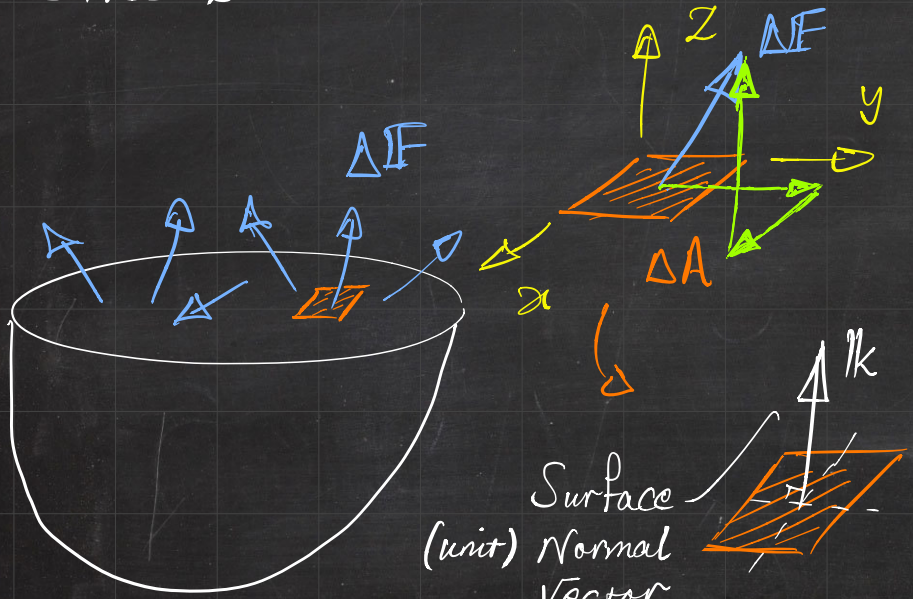


$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

Stress



$$\sigma_z = \lim_{\Delta A_z \to 0} \frac{\Delta F_z}{\Delta A_z}$$

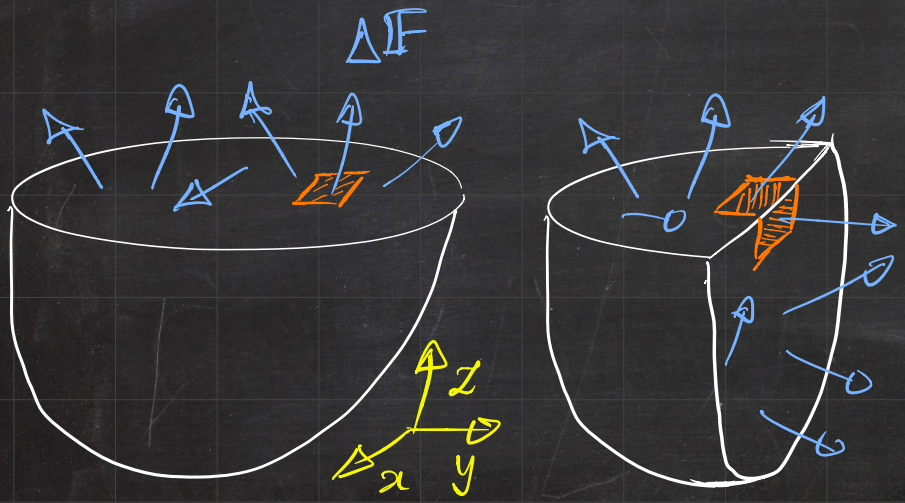
$$\tau_{zx} = \lim_{\Delta A_z \to 0} \frac{\Delta F_x}{\Delta A_z}$$

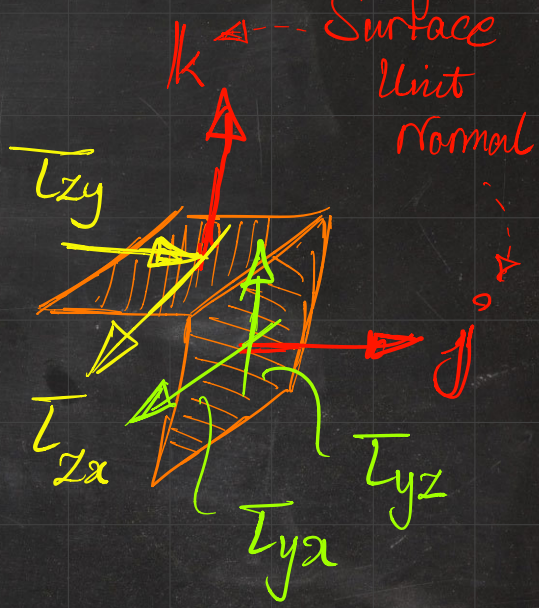
$$\tau_{zy} = \lim_{\Delta A_z \to 0} \frac{\Delta F_y}{\Delta A_z}$$

Surface Normal σ_a → Normal Force Component

Surface Normal τ_{ab} → Shear Force Component

Stress

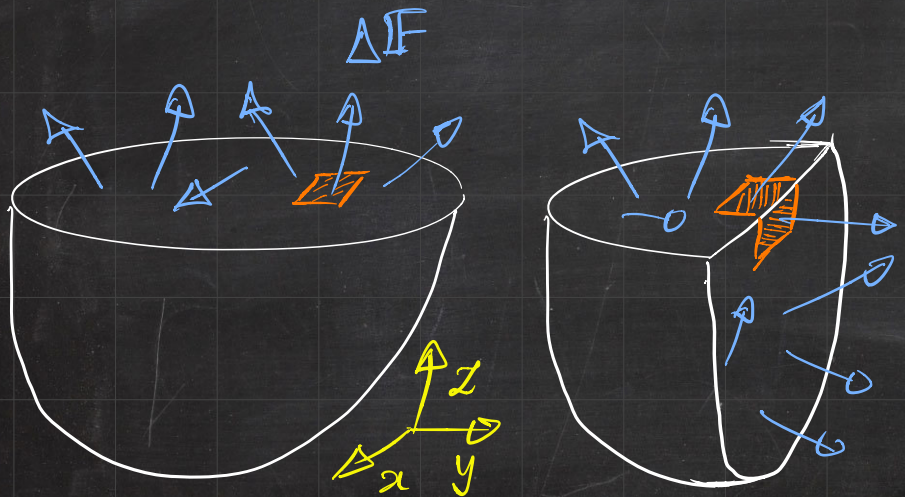


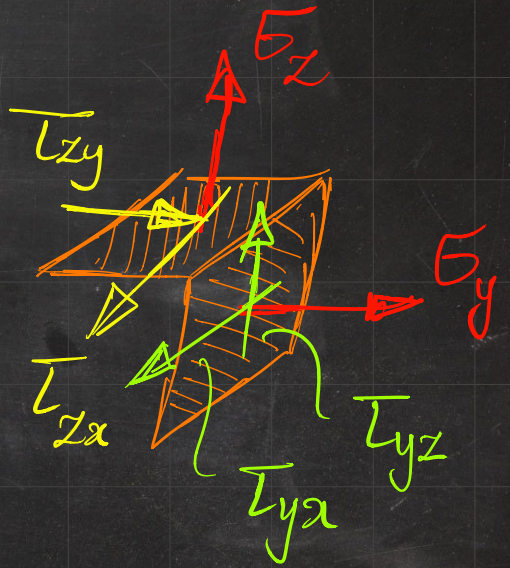


Surface Normal $\rightarrow \sigma_a$ Normal Force Component

Surface Normal $\rightarrow \tau_{ab}$ Shear Force Component

Stress

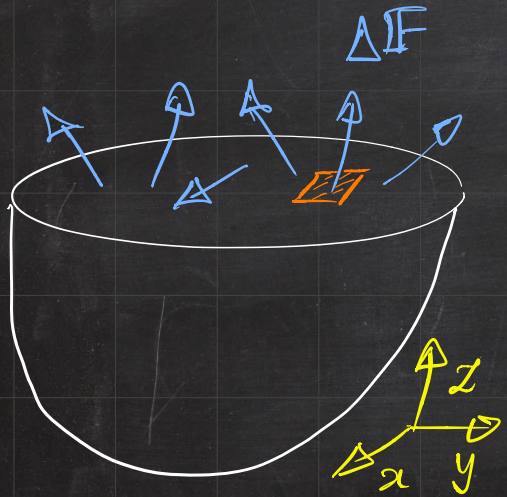


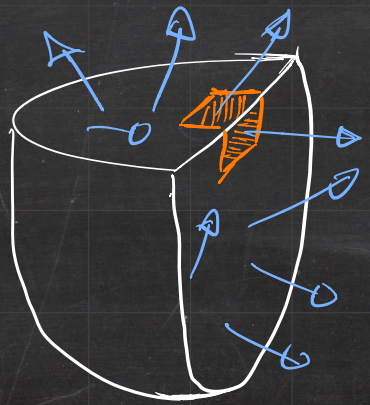


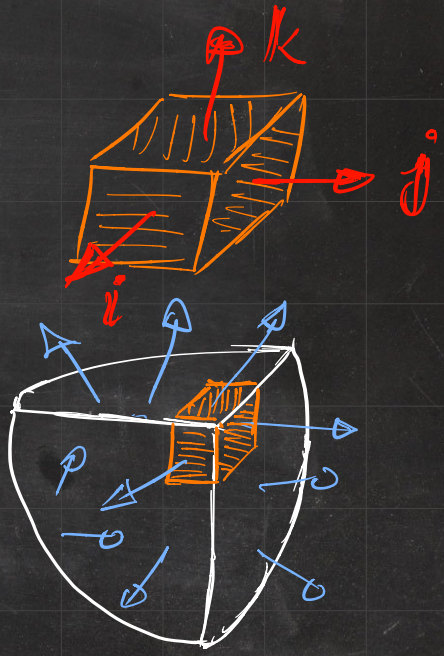
Surface Normal ← σ_a → Normal Force Component

Surface Normal ← τ_{ab} → Shear Force Component

Stress







Surface Normal → σ_a → Normal Force Component

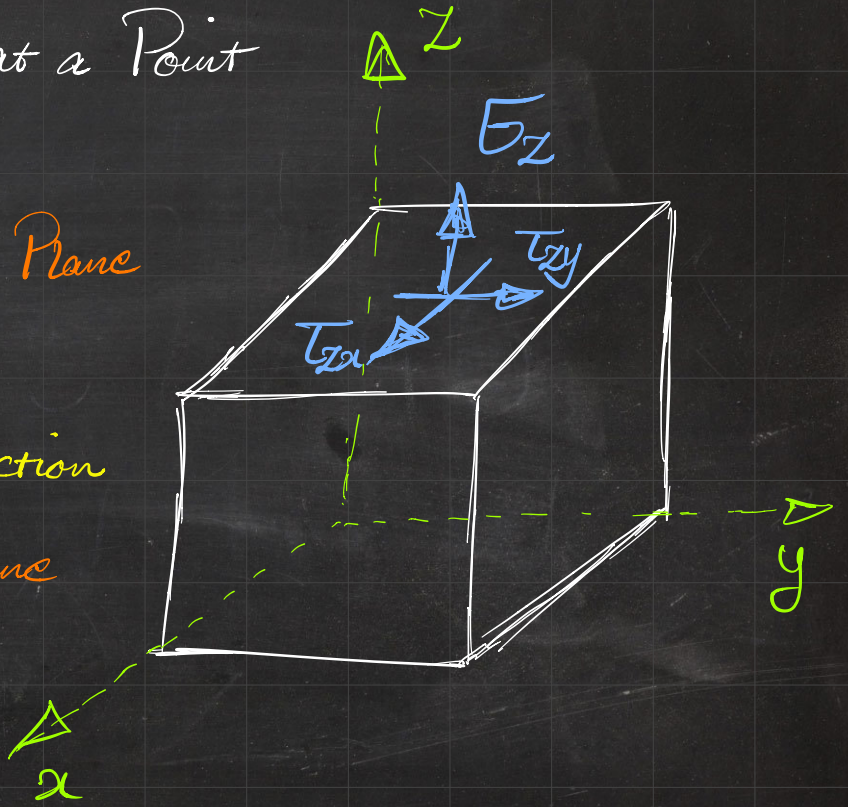
Surface Normal → τ_{ab} → Shear Force Component

General State of Stress at a Point

σ_a → Normal Axis to Plane

τ_{ab} → Shear Force Direction

τ_{ab} → Normal Axis to Plane

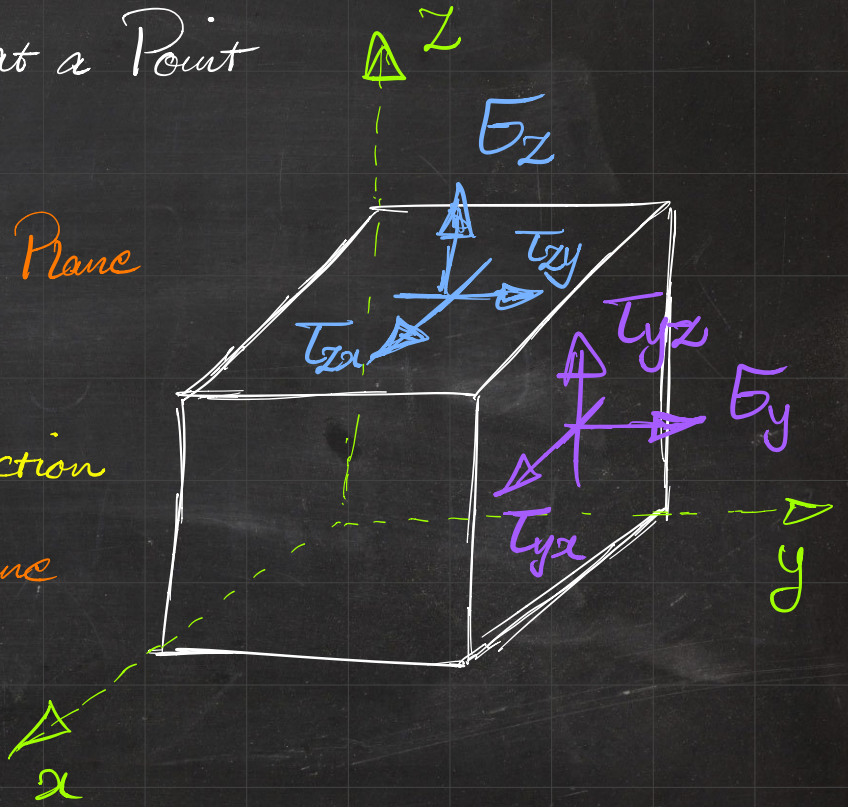


General State of Stress at a Point

σ_a → Normal Axis to Plane

τ_{ab} → Shear Force Direction

τ_{ab} → Normal Axis to Plane

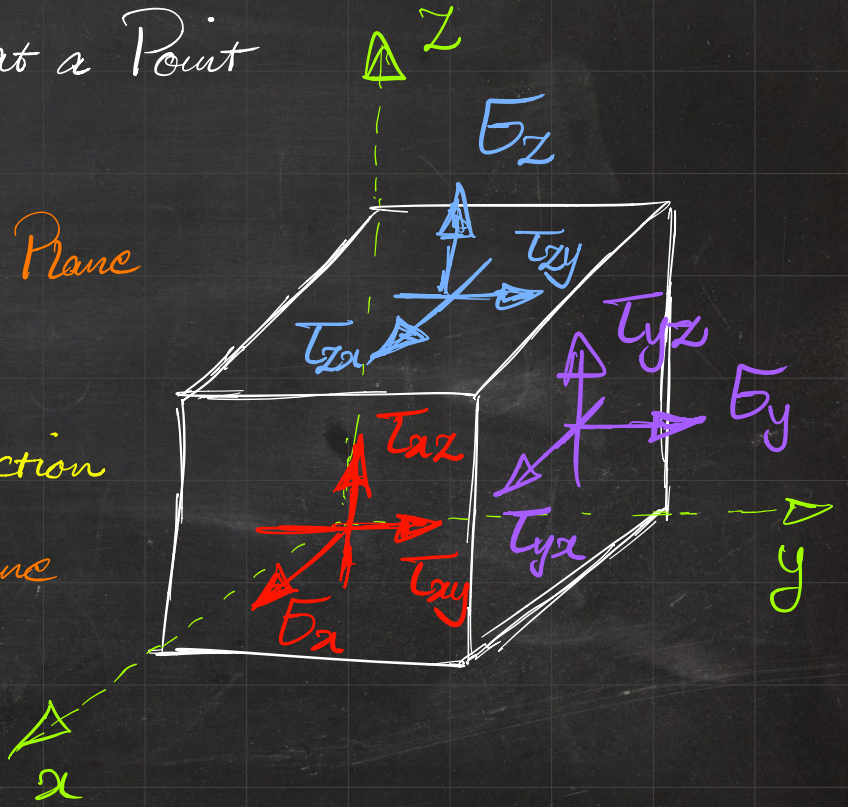


General State of Stress at a Point

σ_a → Normal Axis to Plane

τ_{ab} → Shear Force Direction

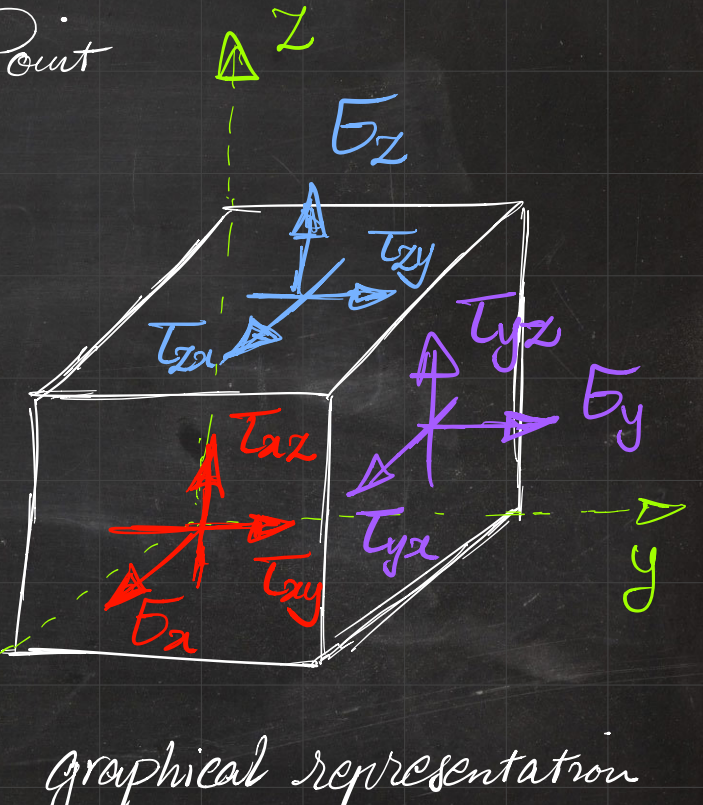
τ_{ab} → Normal Axis to Plane



General State of Stress at a Point

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

matrix representation

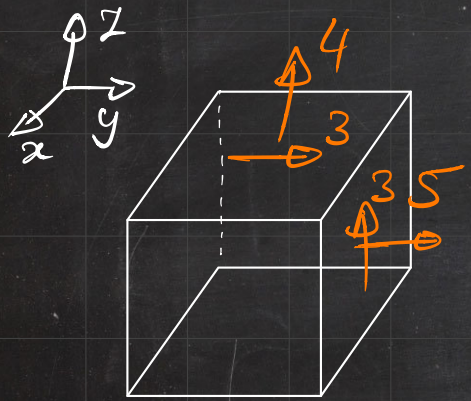


graphical representation

Examples of stress states:

$$[\mathbb{\sigma}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

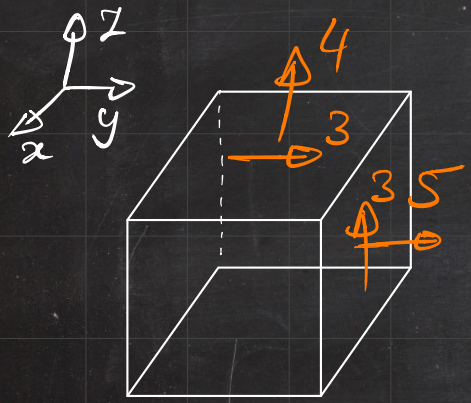
Examples of stress states:



$$[\mathbb{B}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Examples of stress states:

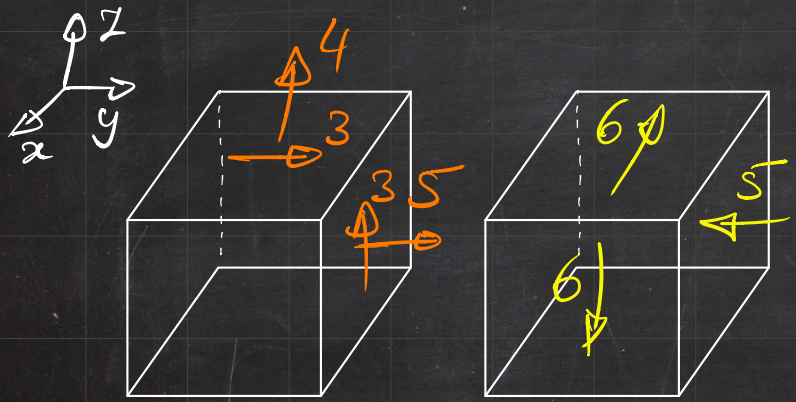
$$[\mathbb{S}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 3 \\ 0 & 3 & 4 \end{bmatrix}$$

Examples of stress states:

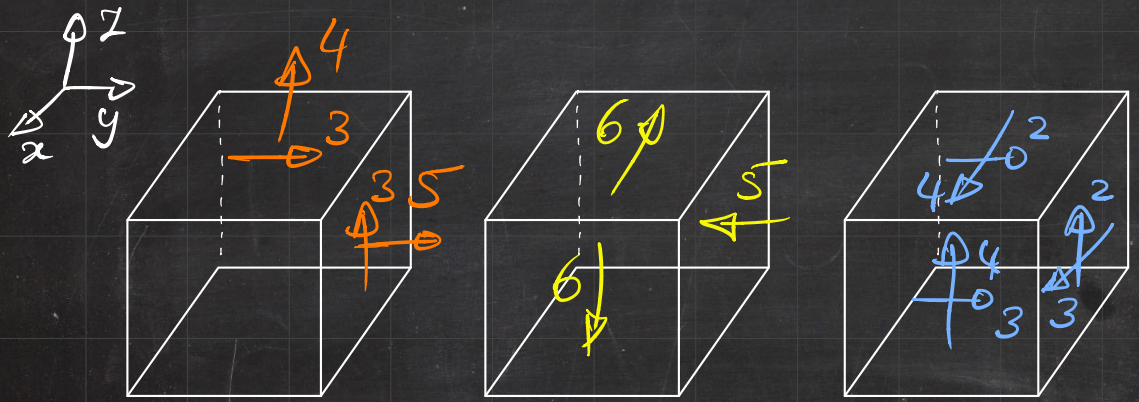
$$[\mathbb{S}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 3 \\ 0 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -6 \\ 0 & -5 & 0 \\ -6 & 0 & 0 \end{bmatrix}$$

Examples of stress states:

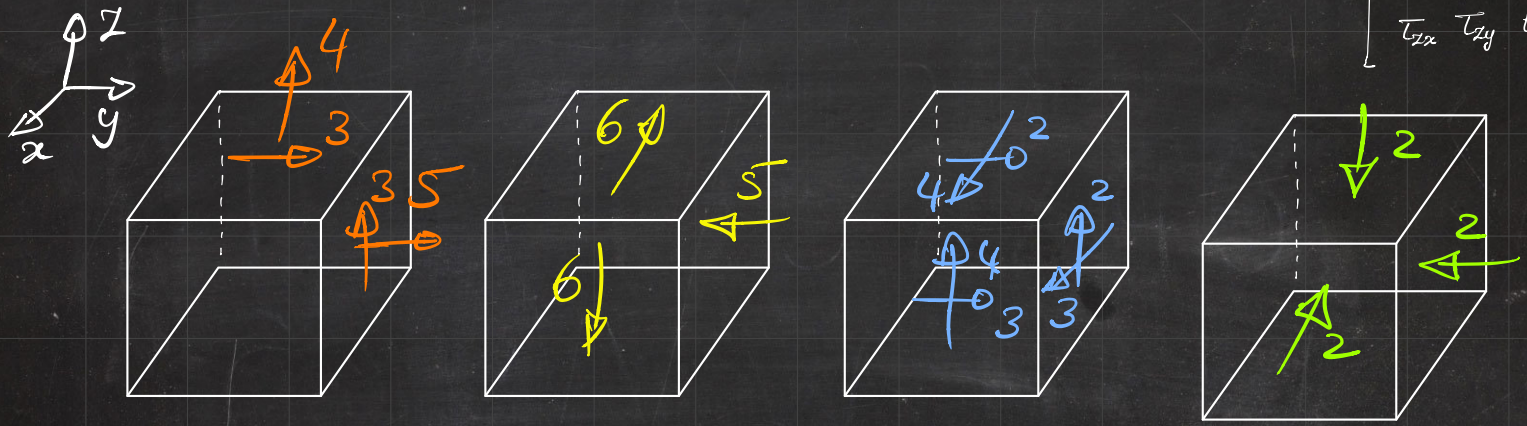
$$[\mathbb{S}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 3 \\ 0 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -6 \\ 0 & -5 & 0 \\ -6 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 3 & 4 \\ 3 & 0 & 2 \\ 4 & 2 & 0 \end{bmatrix}$$

Examples of stress states:

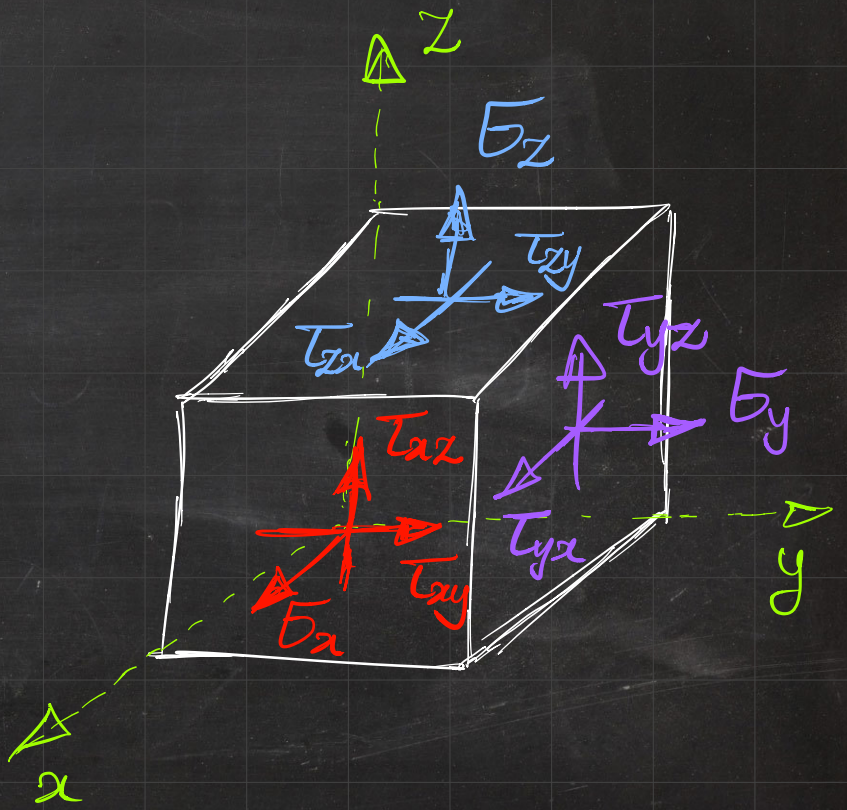
$$[\mathbb{S}] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$



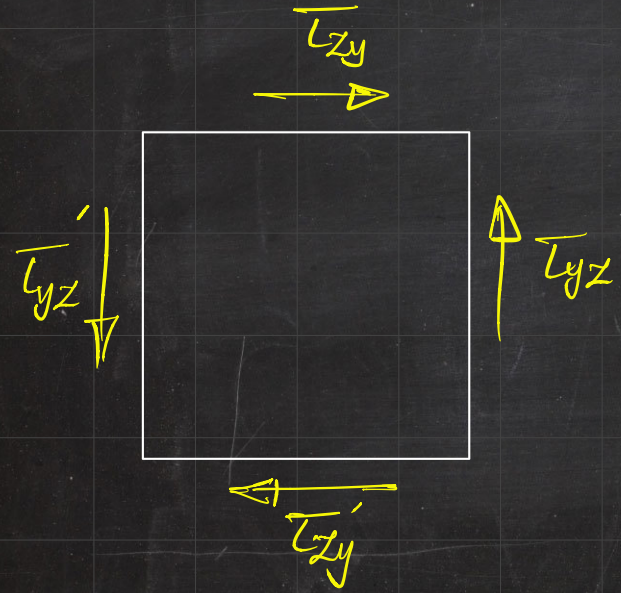
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 3 \\ 0 & 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & -6 \\ 0 & -5 & 0 \\ -6 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 3 & 4 \\ 3 & 0 & 2 \\ 4 & 2 & 0 \end{bmatrix} \qquad \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

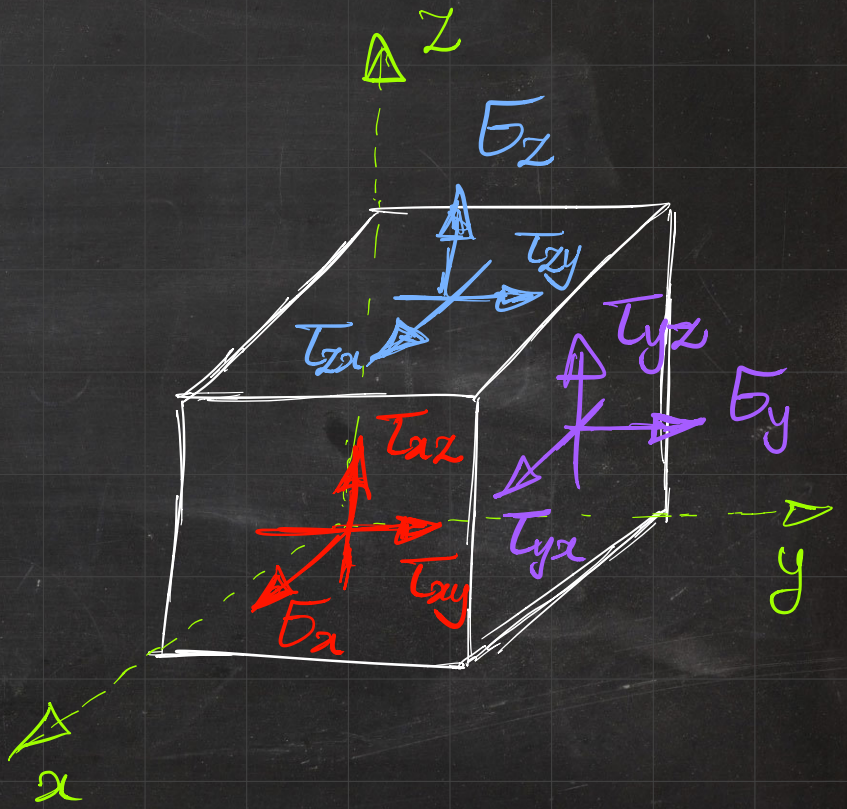
Hydrostatic Pressure

Shear Stress Equilibrium

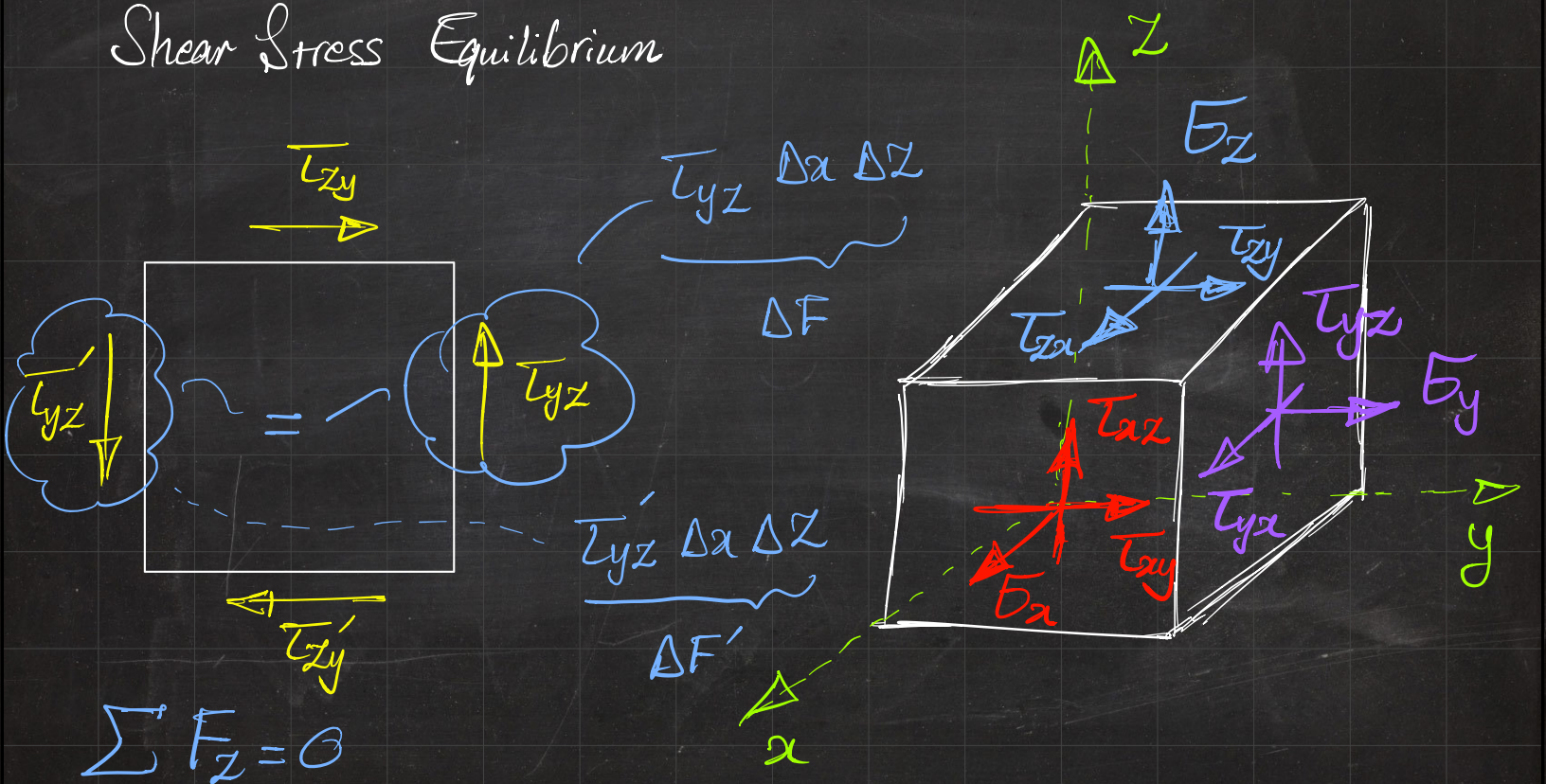


Shear Stress Equilibrium

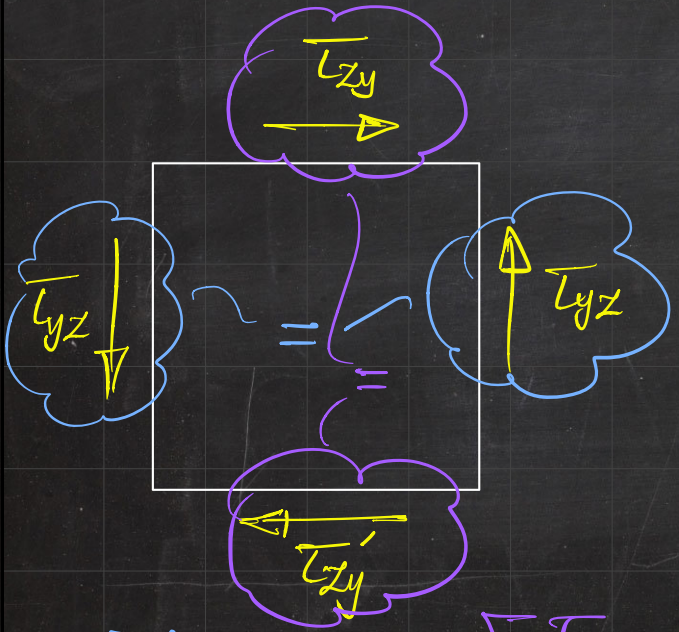




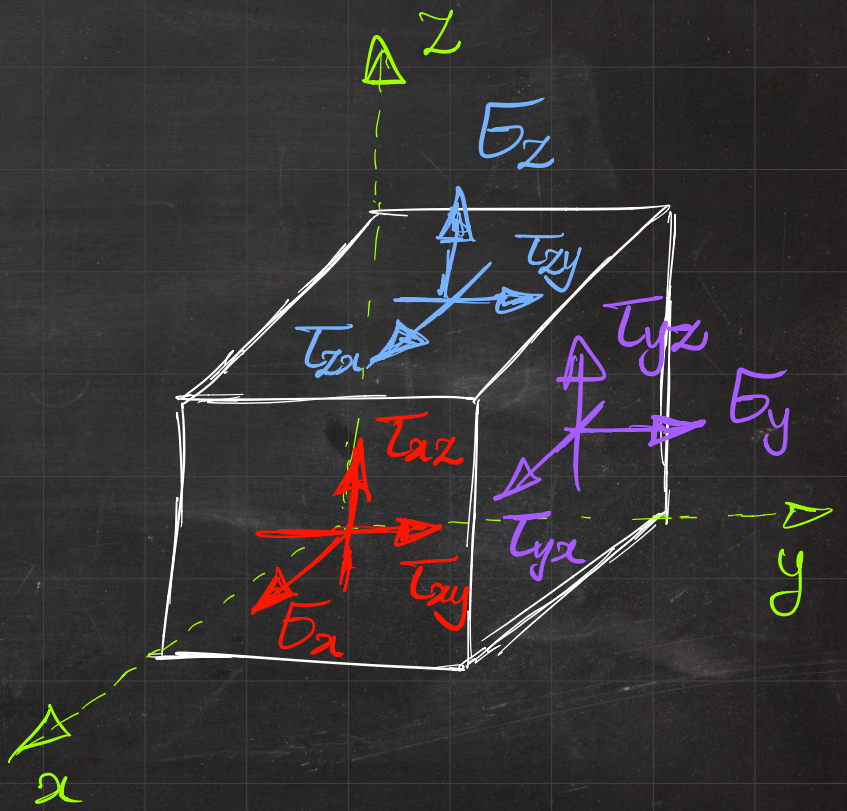
Shear Stress Equilibrium



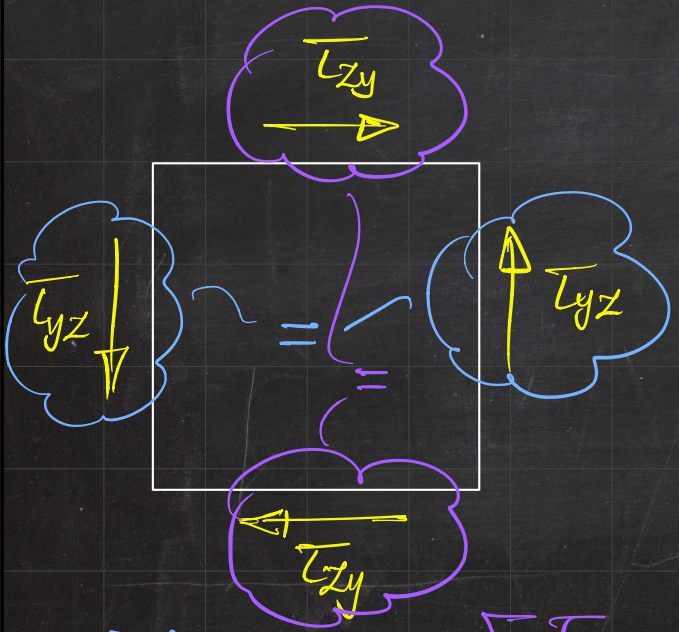
Shear Stress Equilibrium



$$\sum F_z = 0 \qquad \sum F_y = 0$$

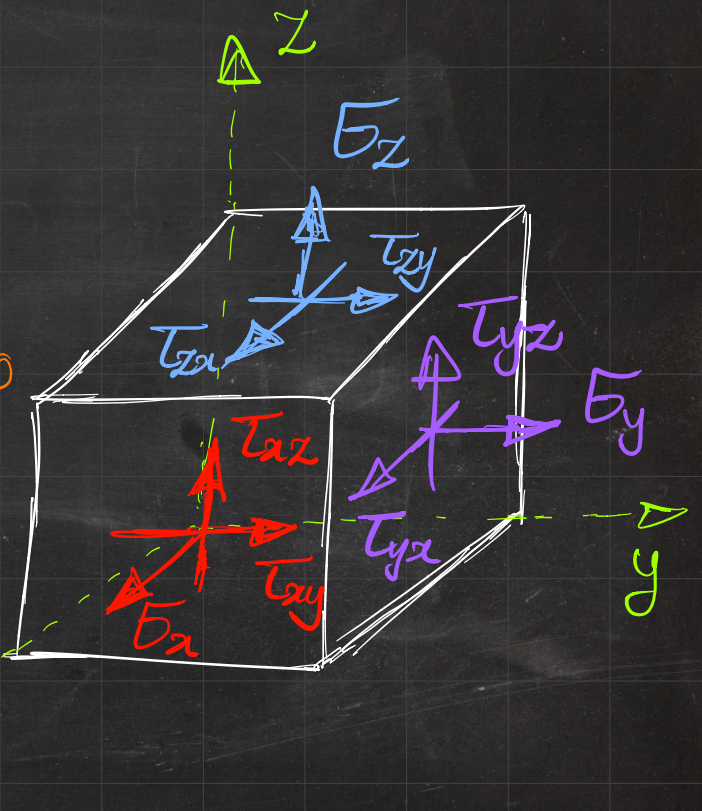


Shear Stress Equilibrium

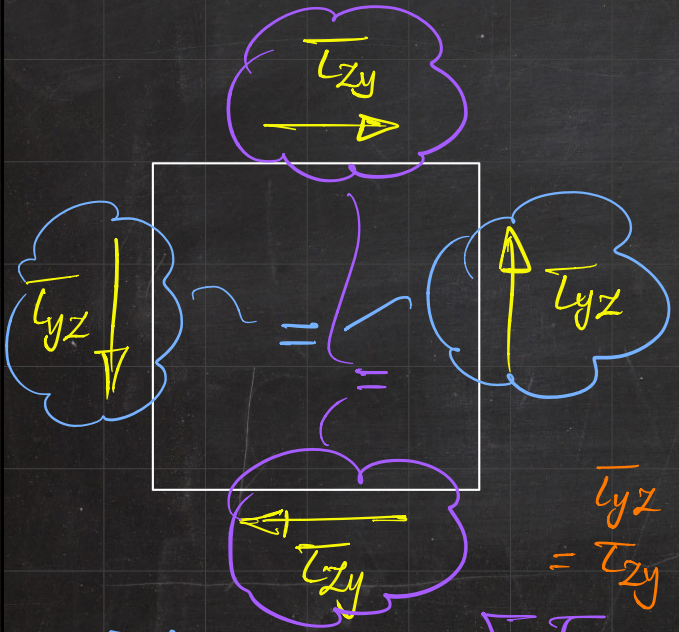


$\sum F_z = 0$ $\sum F_y = 0$

$\sum M_x = 0$



Shear Stress Equilibrium

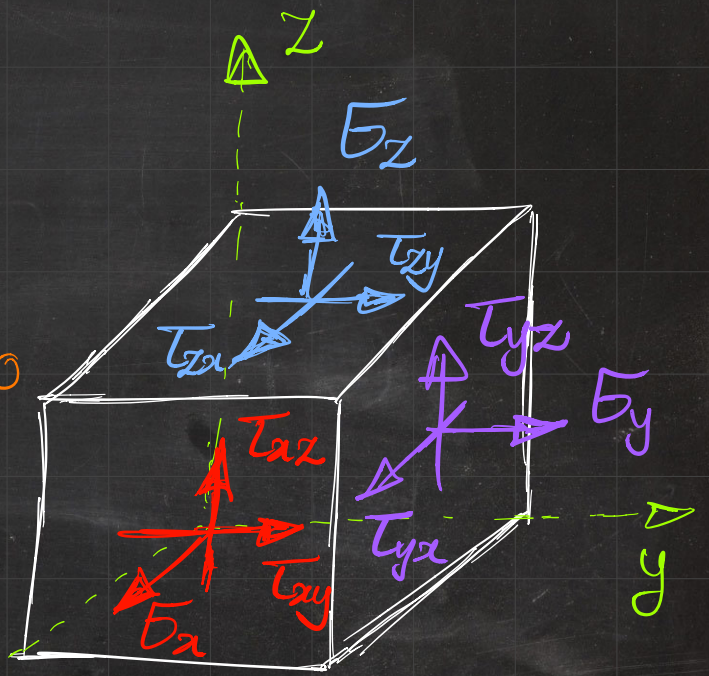


$$\sum F_z = 0 \qquad \sum F_y = 0$$

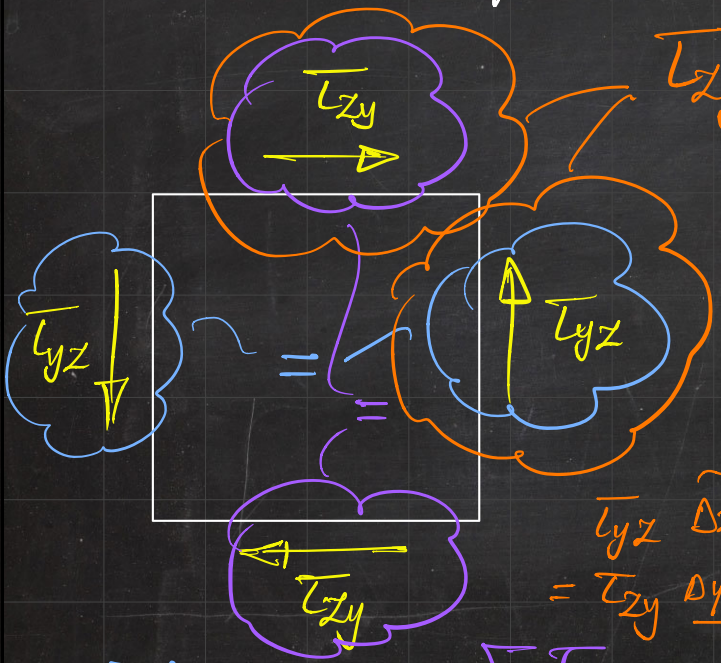
$$\sum M_x = 0$$

τ_{yz} ΔA $\Delta x\,\Delta z$ arm Δy

$= \tau_{zy}$ $\Delta y\,\Delta x\,\Delta z$



Shear Stress Equilibrium



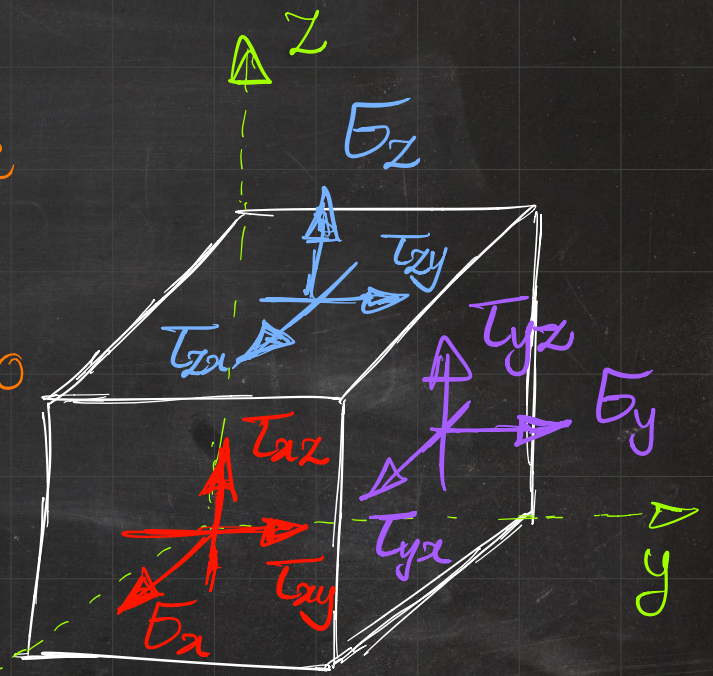
$$\tau_{zy} = \tau_{yz}$$

$$\sum M_x = 0$$

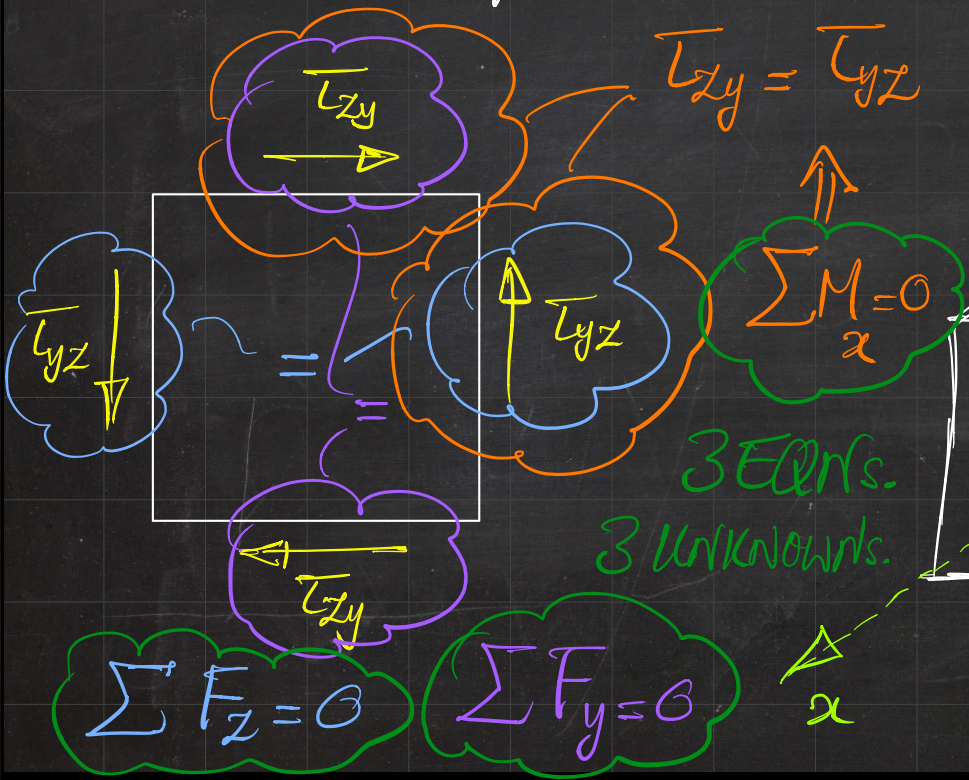
$$\tau_{yz}\ \Delta x\,\Delta z\,\Delta y = \tau_{zy}\ \Delta y\,\Delta x\,\Delta z$$

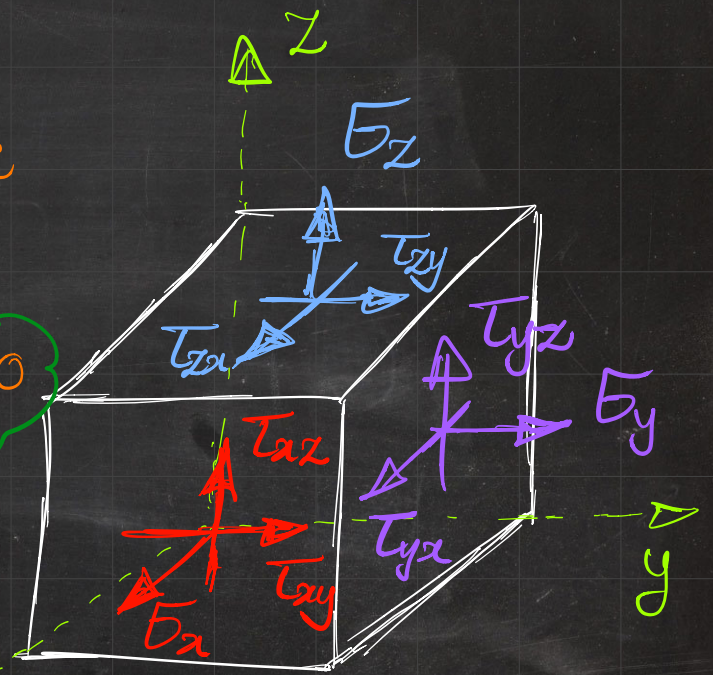
($\Delta x\,\Delta z$: ΔA, Δy: arm)

$$\sum F_z = 0 \qquad \sum F_y = 0$$



Shear Stress Equilibrium



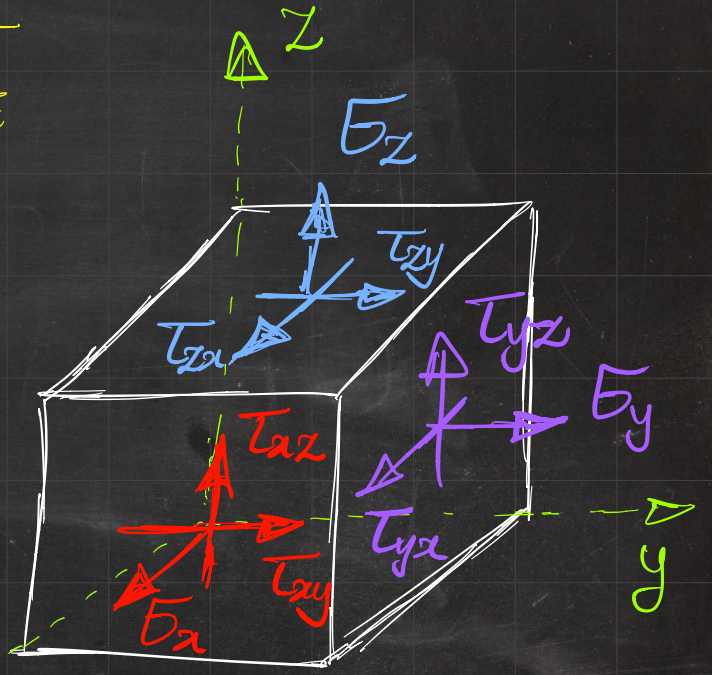


STRESS SYMMETRY $\Leftrightarrow$ MOMENT BALANCE

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

$\Downarrow$

Symmetric



$\tau_{xy} = \tau_{yx}$, $\tau_{xz} = \tau_{zx}$, $\tau_{yz} = \tau_{zy}$

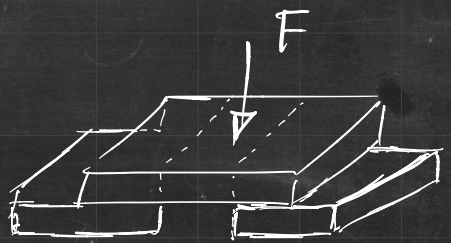
Average Stress

Average Normal Stress

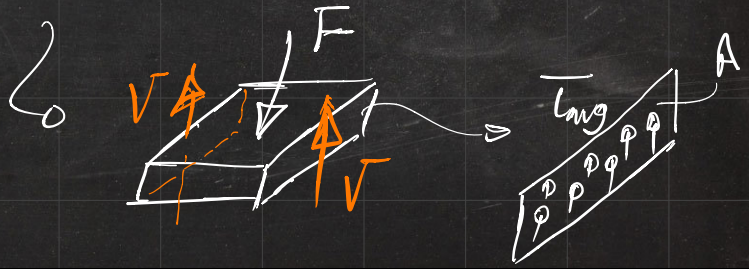


$$\sigma_{avg} = \frac{P}{A}$$

Average Shear Stress



$$\tau_{avg} = \frac{V}{A}$$



MECHANICS AND MATERIALS I

Stress i

Sections ... 1.1 – 1.5

Chap. 1

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