MECHANICS AND MATERIALS I

22

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MECHANICS AND MATERIALS I

Combined Loading

Section ... 8.2

Chap. 8

[Hibbeler 9th edition]

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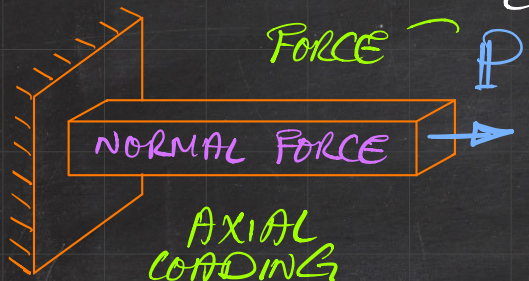
COMBINED LOADING

→ Chap. 8

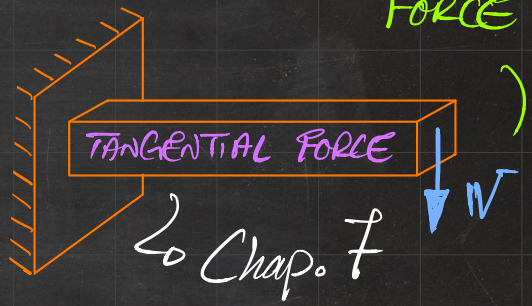
$$\tau = \frac{VQ}{It}$$



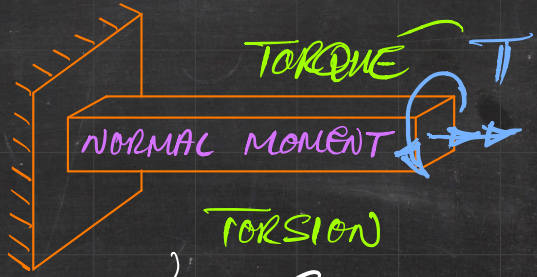
$$\delta = \frac{PL}{EA}$$

AXIAL LOADING

↳ Chap. 4



↳ Chap. 7

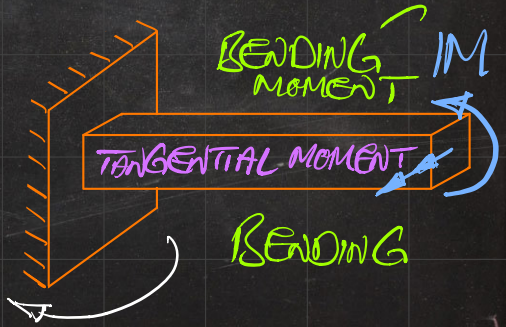


$$\phi = \frac{TL}{GJ}$$

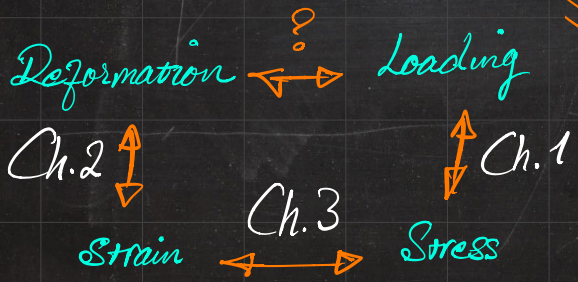
$$\theta = \frac{ML}{EI}$$

TORSION

↳ Chap. 5

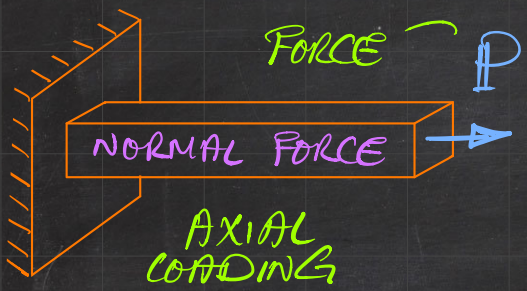


Chap. 6



Combined Loading

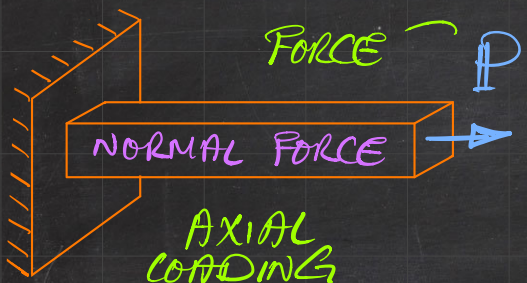
COMBINED LOADING



$$\sigma = \frac{P}{A}$$

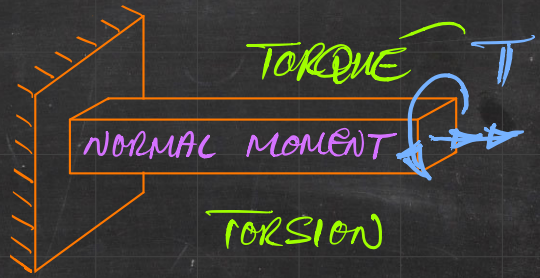
$$\delta = \frac{PL}{EA}$$

COMBINED LOADING



AXIAL LOADING

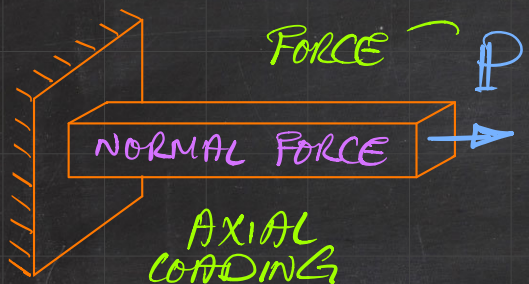
$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$



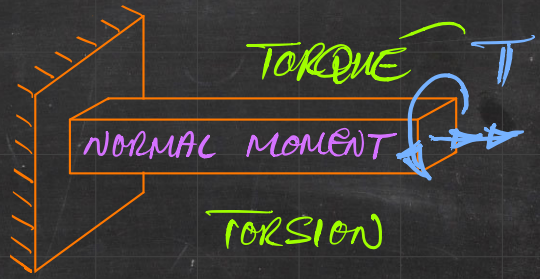
TORSION

$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$

COMBINED LOADING

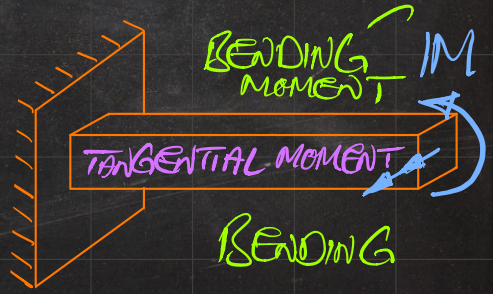


$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$

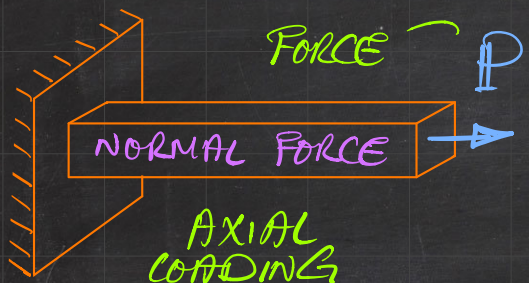


$$\sigma = -\frac{My}{I} \qquad \theta = \frac{ML}{EI}$$

$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$



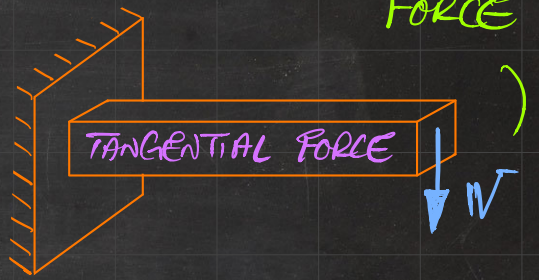
COMBINED LOADING

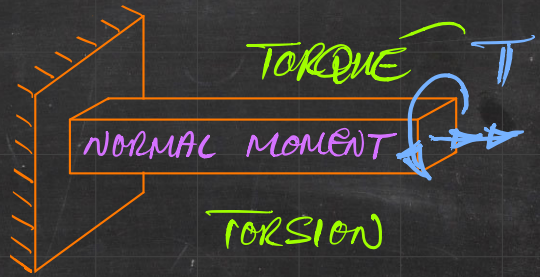


AXIAL LOADING

$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$

$$\tau = \frac{VQ}{It}$$

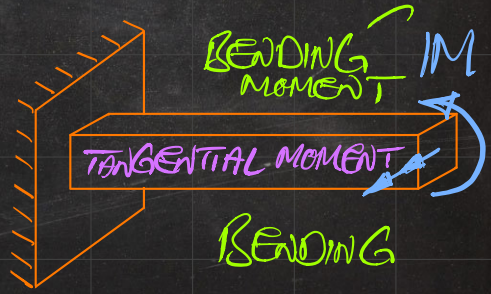




TORSION

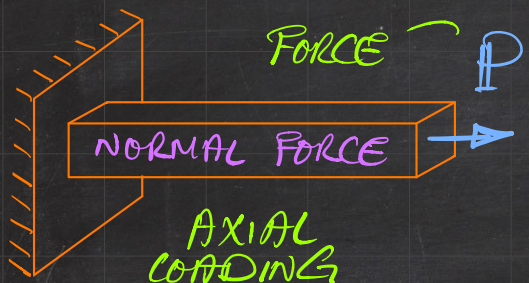
$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$

$$\sigma = -\frac{My}{I} \qquad \theta = \frac{ML}{EI}$$



BENDING

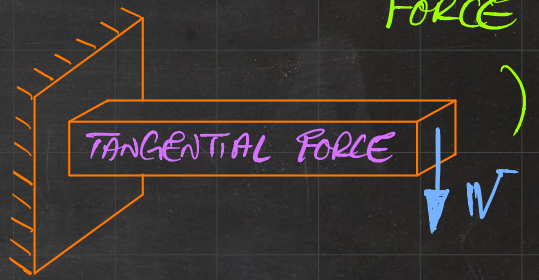
COMBINED LOADING

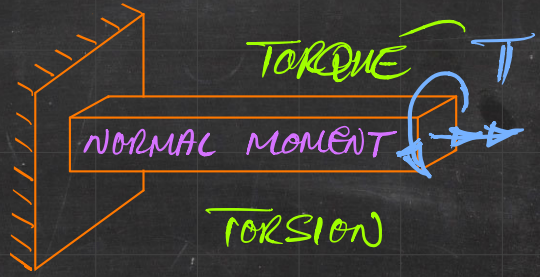


AXIAL LOADING

$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$

$$\tau = \frac{VQ}{It}$$

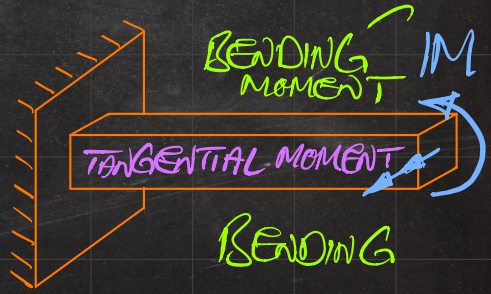




TORSION

$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$

$$\sigma = -\frac{My}{I} \qquad \theta = \frac{ML}{EI}$$



BENDING

in practice, a combination of all of these occurs!

COMBINED LOADING

* AXIAL LOADING

$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$

* TORSION

$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$

* BENDING

$$\sigma = -\frac{My}{I} \qquad \theta = \frac{ML}{EI}$$

* TRANSVERSE SHEAR

$$\tau = \frac{VQ}{It}$$

COMBINED LOADING

* AXIAL LOADING

$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$

* TORSION

$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$

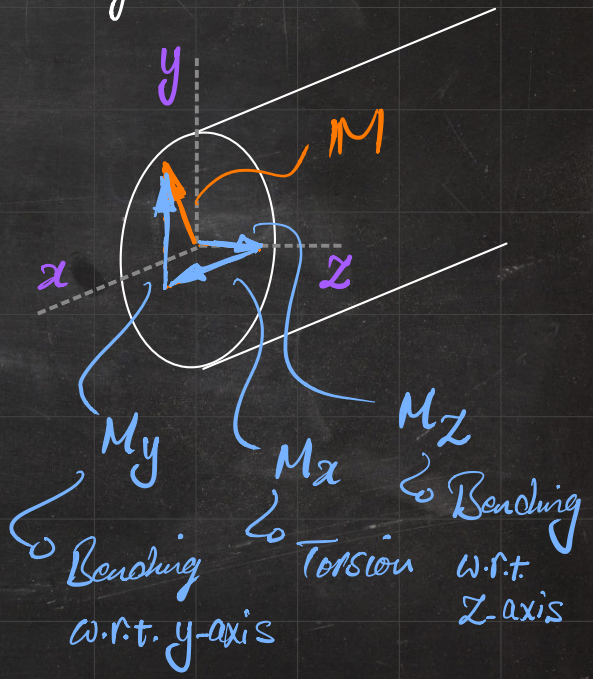
* BENDING

$$\sigma = -\frac{My}{I} \qquad \theta = \frac{ML}{EI}$$

* TRANSVERSE SHEAR

$$\tau = \frac{VQ}{It}$$

e.g.



COMBINED LOADING ↔ STRESS-STATE ?

* AXIAL LOADING

$$\sigma = \frac{P}{A} \qquad \delta = \frac{PL}{EA}$$

* TORSION

$$\tau = \frac{Tr}{J} \qquad \phi = \frac{TL}{GJ}$$

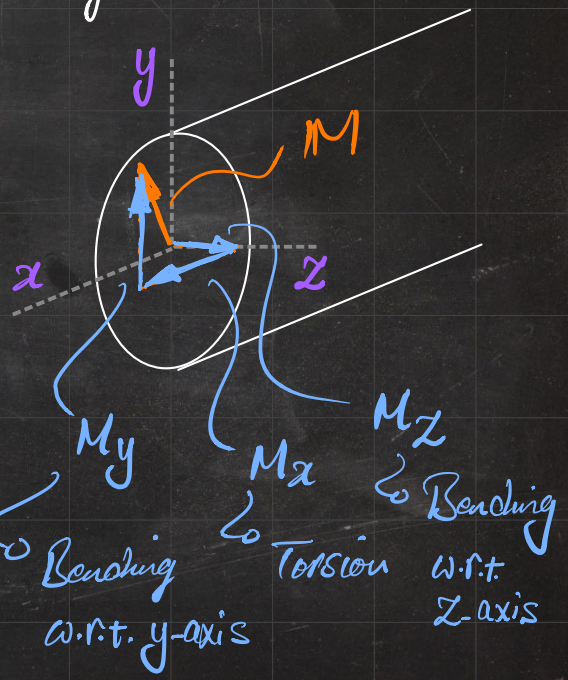
* BENDING

$$\sigma = -\frac{My}{I} \qquad \theta = \frac{ML}{EI}$$

* TRANSVERSE SHEAR

$$\tau = \frac{VQ}{It}$$

e.g.



Combined Loading

* Axial loading $\sigma = \frac{P}{A}$ $\quad \delta = \frac{PL}{EA}$
* Torsion $\tau = \frac{Tr}{J}$ $\quad \phi = \frac{TL}{GJ}$
* Bending $\sigma = -\frac{My}{I}$ $\quad \theta = \frac{ML}{EI}$
* Transverse shear $\tau = \frac{VQ}{It}$

To compute stress-state

Step 1)

Find internal forces/moments at the desired cross-section

Step 2)

Find stresses σ, τ caused due to internal reactions

Step 3)

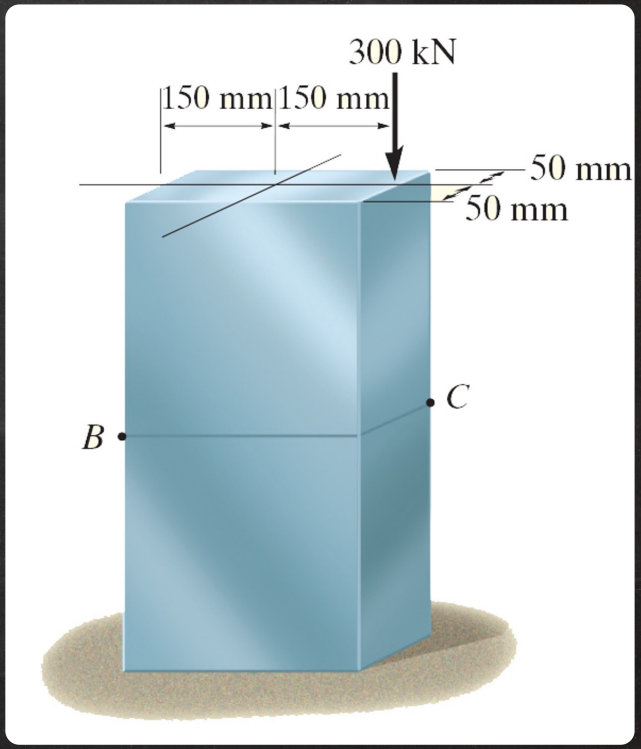
Add up the effects

(Careful with signs!)

Exercise 1 . [similar to ... P. 418 ... 8.2]

FOR THE BLOCK SHOWN IN THE FIGURE, DETERMINE THE STATE OF STRESS AT POINTS B AND C.

NEGLECT THE WEIGHT OF THE MEMBER.



STEP 1)

FIND INTERNAL FORCES/MOMENTS

AT THE DESIRED CROSS-SECTION

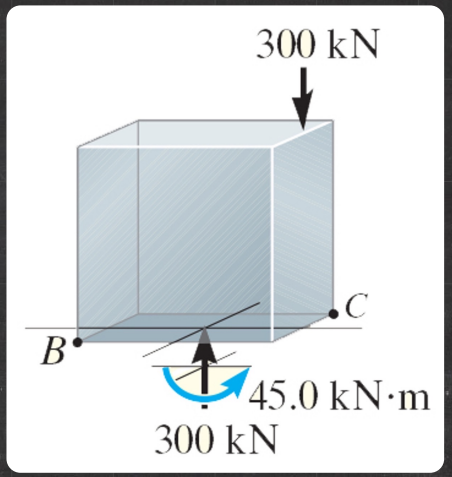
STEP 2)

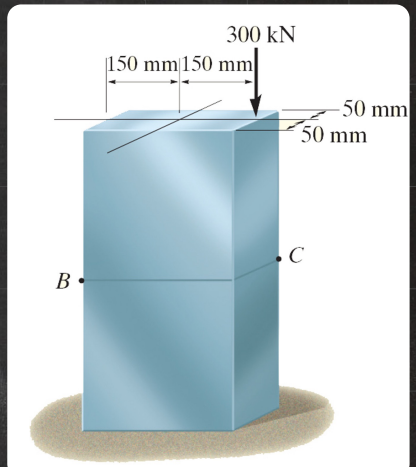
FIND STRESSES σ, τ CAUSED

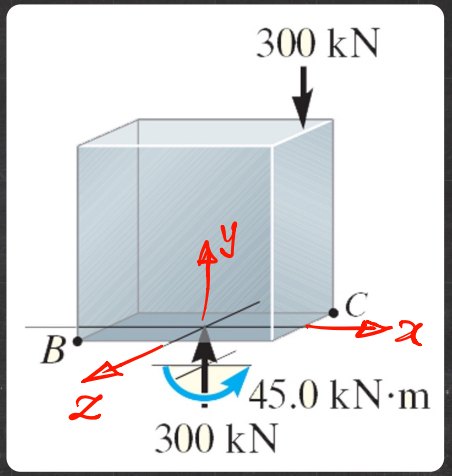
DUE TO INTERNAL REACTIONS

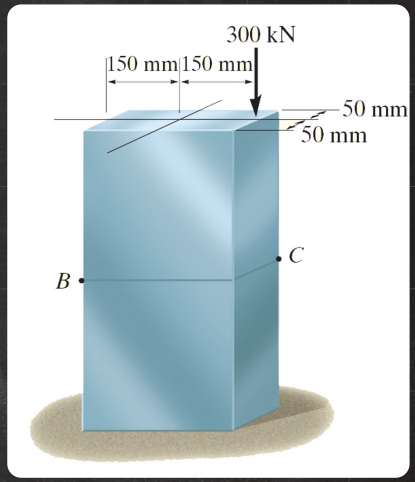
STEP 3)

ADD UP THE EFFECTS






300 kN
y
C
x
B
z
45.0 kN·m
300 kN

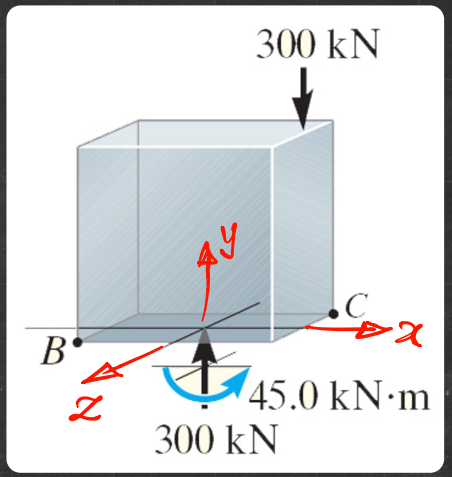

300 kN
150 mm
150 mm
50 mm
50 mm
B
C

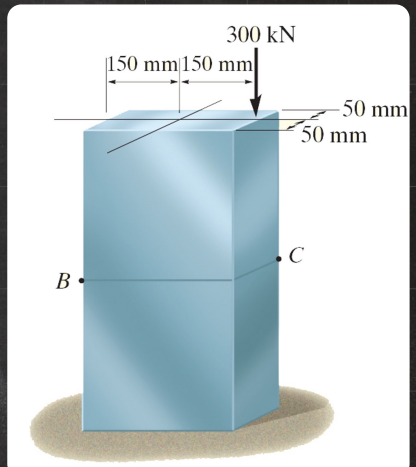
AXIAL LOADING

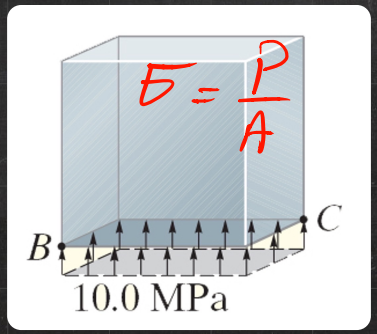
compression

$\sigma_B = \sigma_C = -10$ MPa

$P = -300 \times 10^3$, $A = 0.03$







AXIAL LOADING

$\sigma_B = \sigma_C = -10$ MPa (compression)

$P = -300 \times 10^3$, $A = 0.03$

BENDING

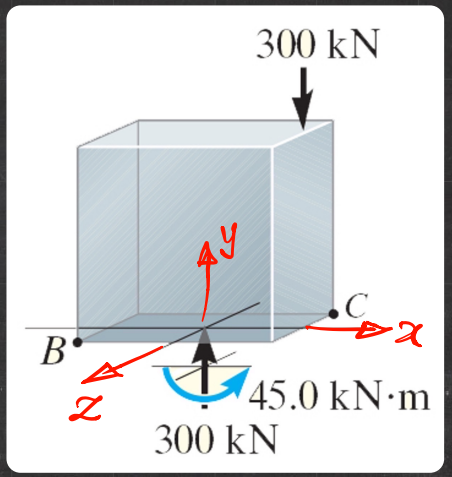
$M_z = 45 \times 10^3$

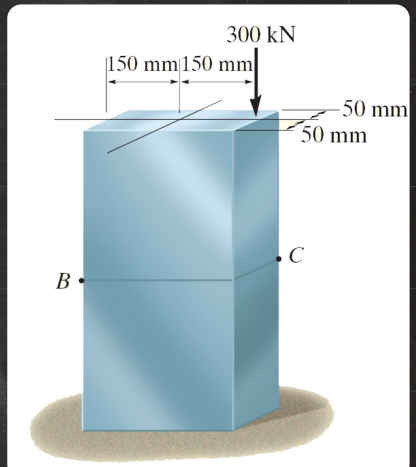
$\sigma = -\frac{Mx}{I}$ $I_z = \frac{1}{12} bh^3$ (100 mm, 300 mm)

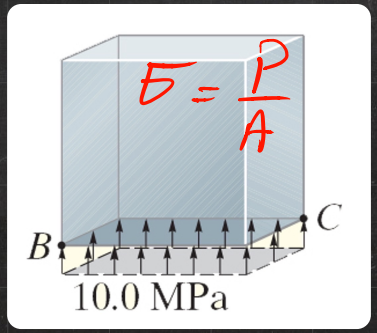
$-200x$ MPa

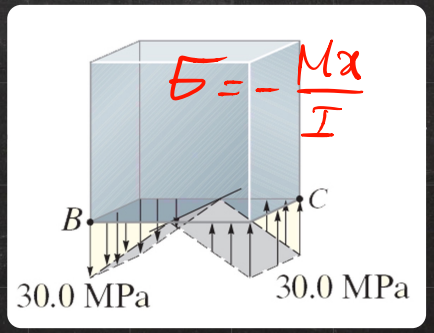
$225 \times 10^{-6}\ m^4$

$\sigma_B = +30$ MPa, $\sigma_C = -30$ MPa









AXIAL LOADING

$$\sigma_B = \sigma_C = -10 \text{ MPa}$$ (compression)

$P = -300 \times 10^3$, $A = 0.03$

BENDING

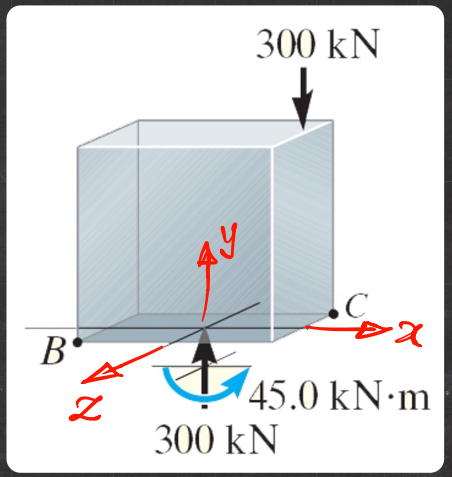
$$\sigma = -\frac{M_x}{I}$$ ($M_x = 45 \times 10^3$, 100 mm)

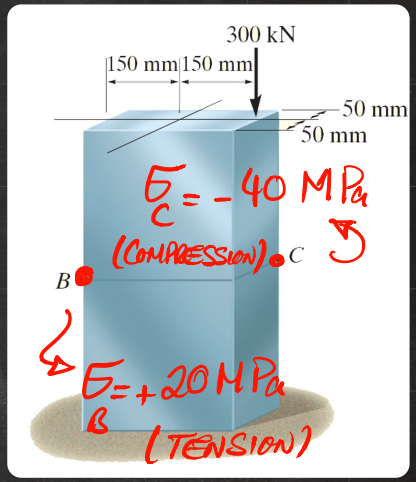
$I_z = \frac{1}{12} bh^3$ (300 mm)

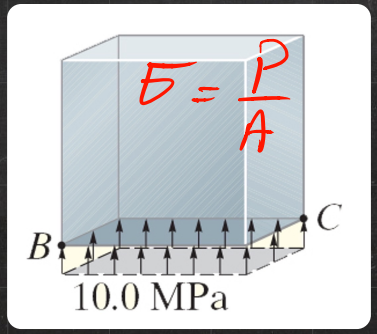
$225 \times 10^{-6} \text{ m}^4$

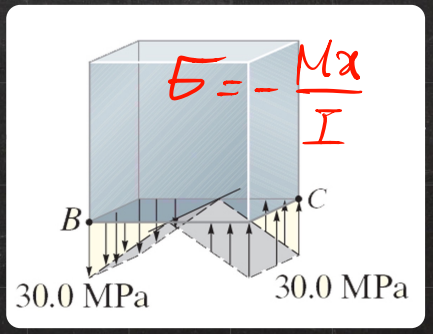
$-200x$ MPa

$$\sigma_B = +30 \text{ MPa}, \quad \sigma_C = -30 \text{ MPa}$$



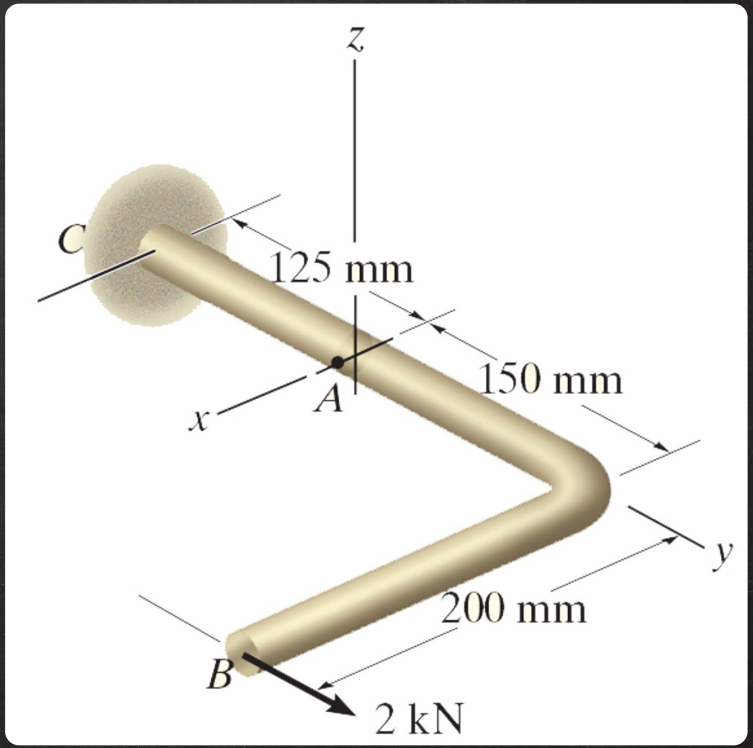






Exercise 2 . [similar to ... P. 422 ... 8.5]

Determine the state of stress at point A on the solid rod shown in the figure. The radius of the shaft is 20 mm.



STEP 1)

FIND INTERNAL FORCES/MOMENTS

AT THE DESIRED CROSS-SECTION

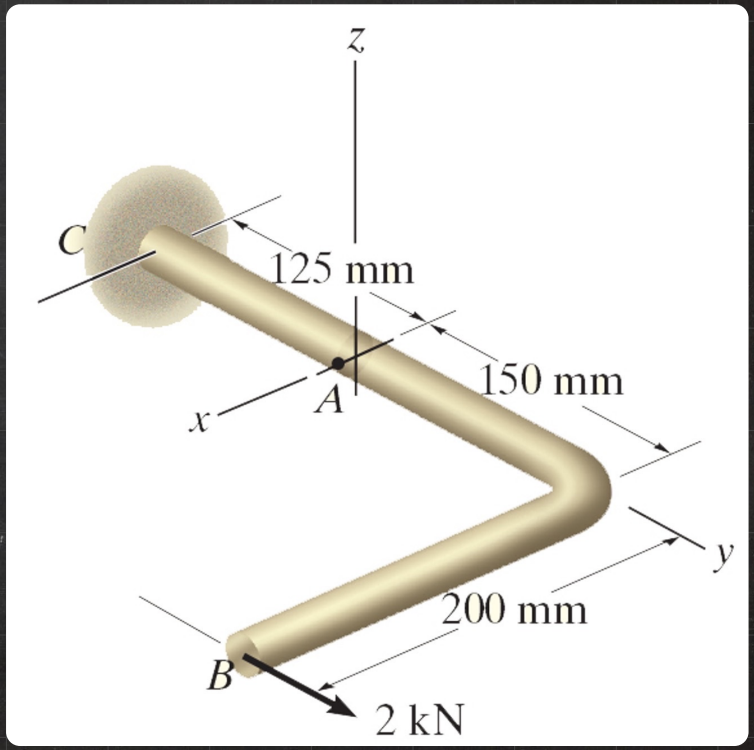
STEP 2)

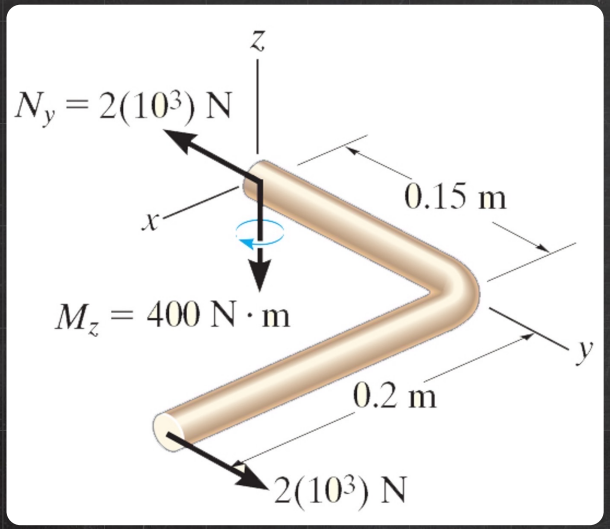
FIND STRESSES σ, τ CAUSED

DUE TO INTERNAL REACTIONS

STEP 3)

ADD UP THE EFFECTS

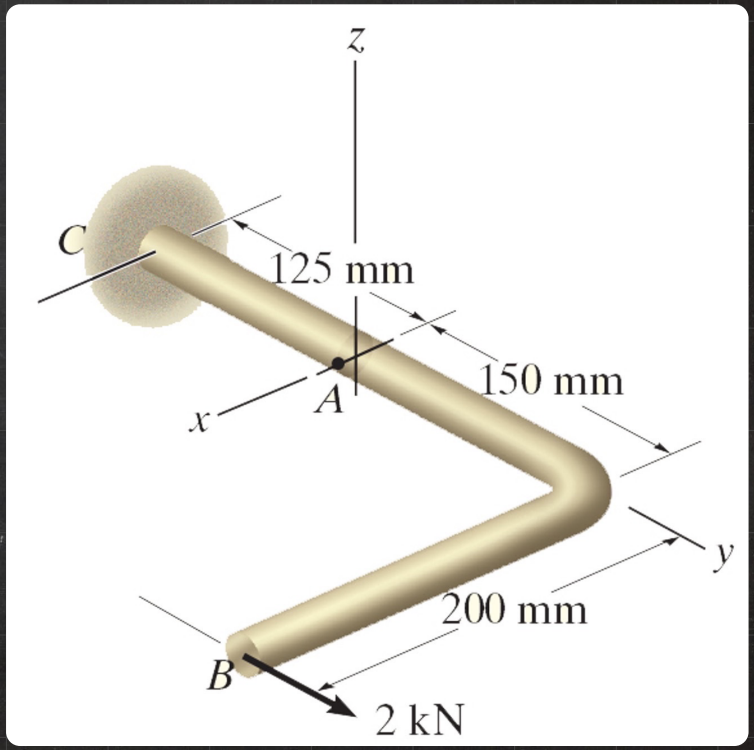


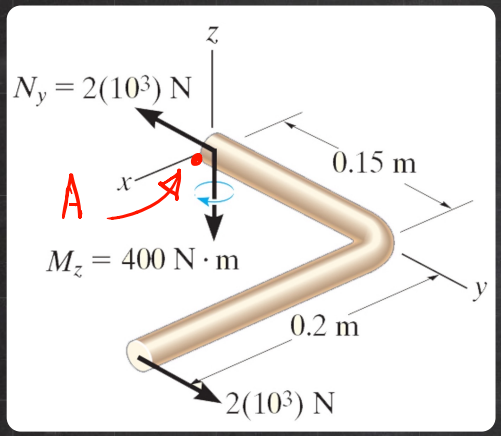


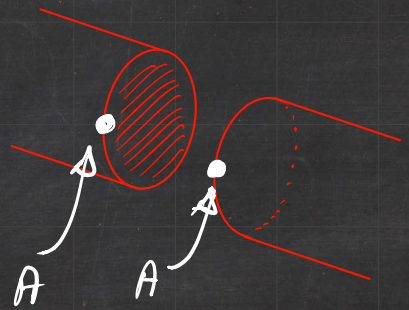
$\Sigma F_y = 0 \Rightarrow N \checkmark$

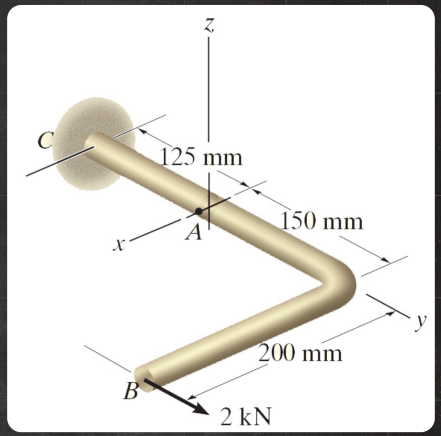
$\Sigma M = 0 \Rightarrow M \checkmark$

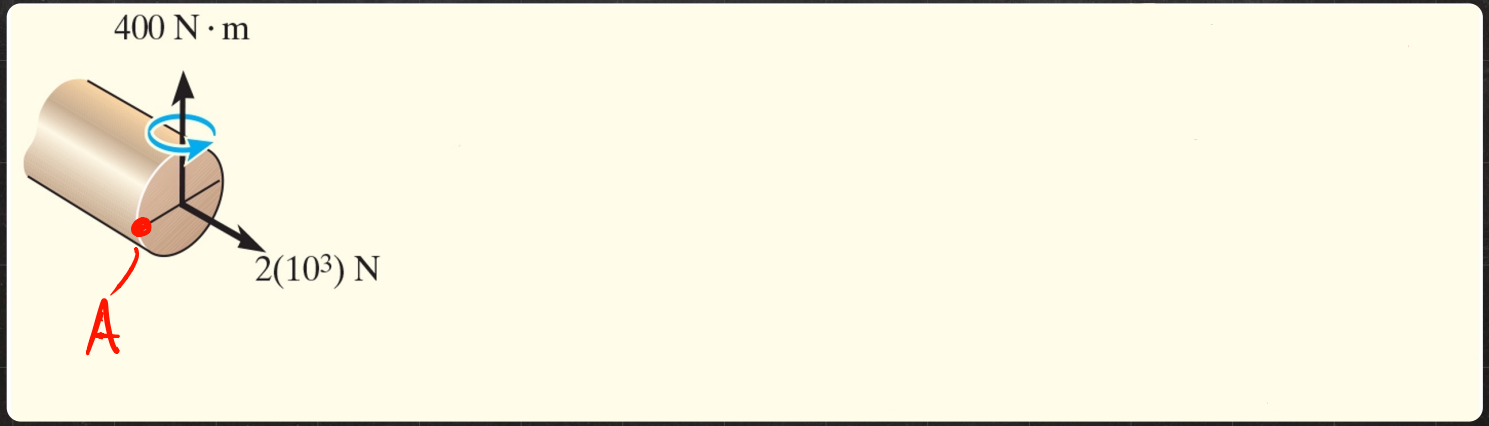
CAREFUL WITH DIRECTIONS

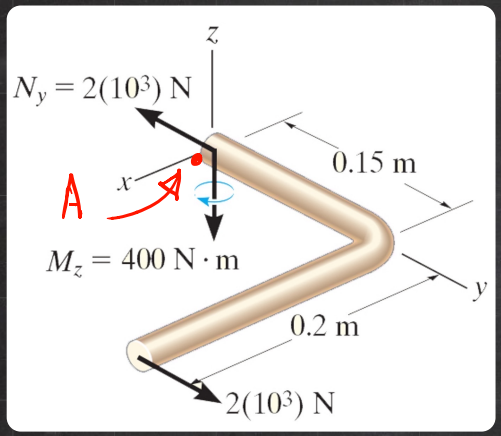


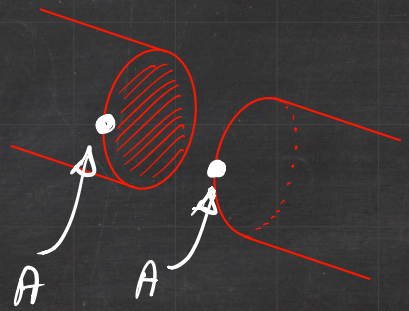

z
$N_y = 2(10^3)$ N
A
x
0.15 m
$M_z = 400$ N · m
y
0.2 m
$2(10^3)$ N

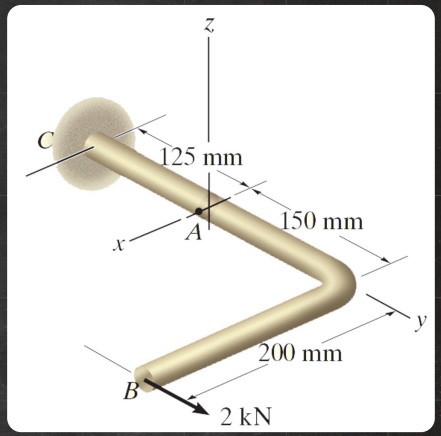

A
A

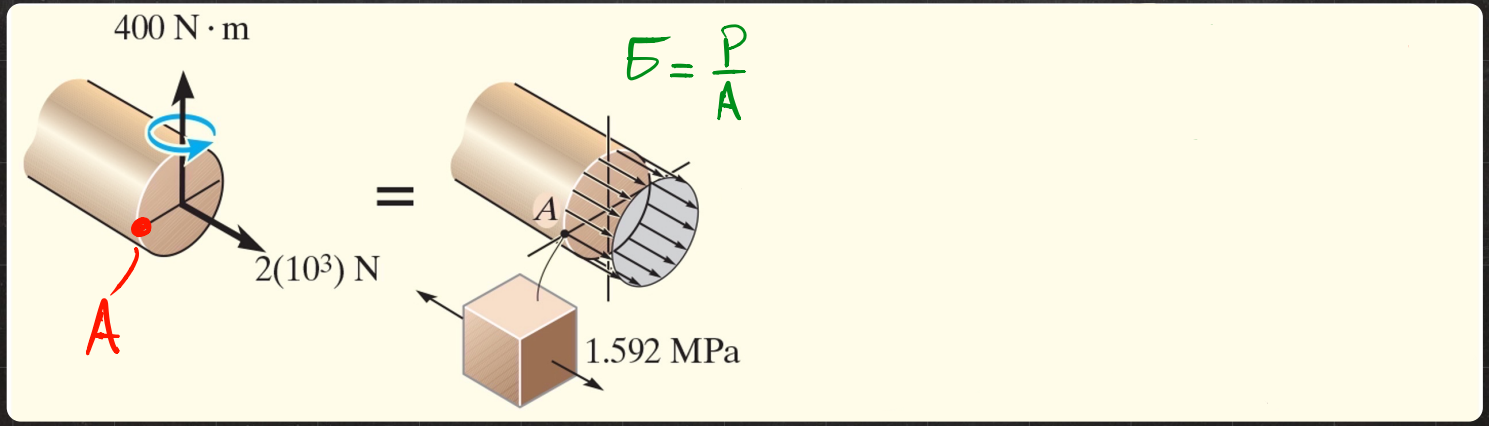

z
C
125 mm
150 mm
x
A
y
200 mm
B
2 kN

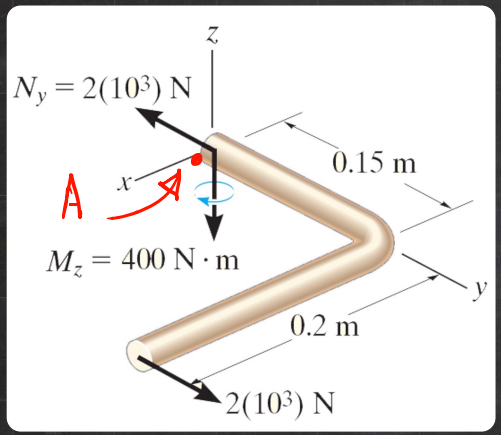

400 N · m
$2(10^3)$ N
A

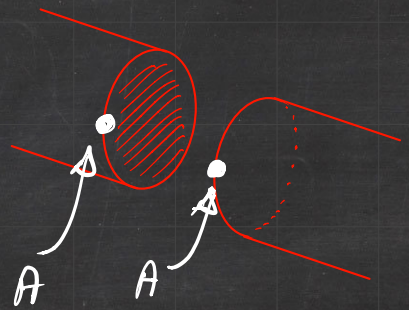

z
$N_y = 2(10^3)$ N
A
x
0.15 m
$M_z = 400$ N · m
y
0.2 m
$2(10^3)$ N

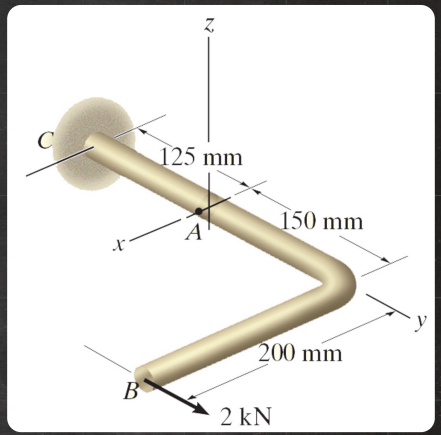

A
A

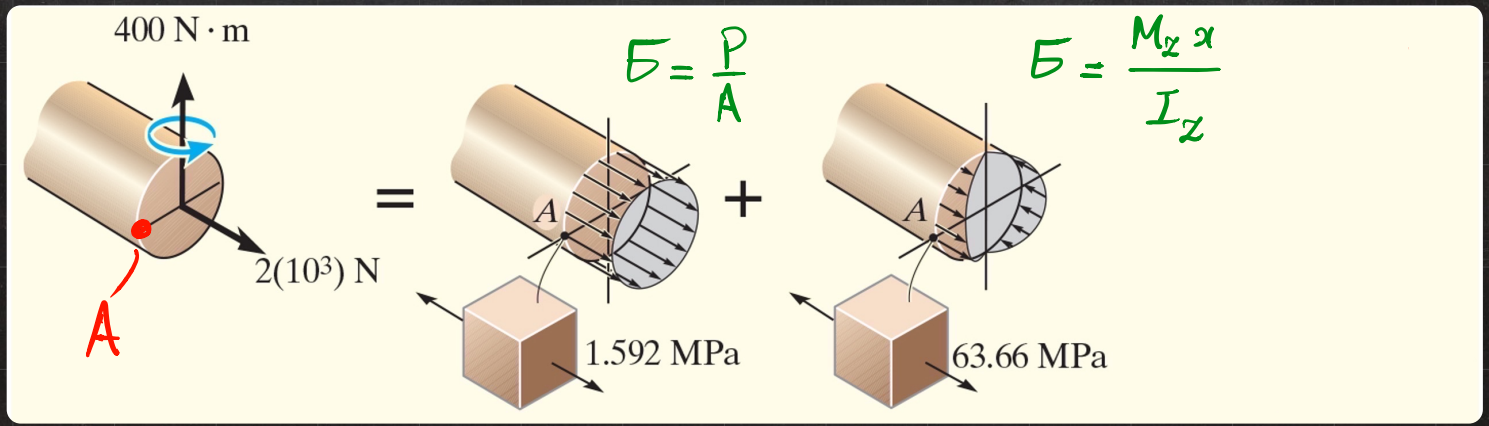

z
C
125 mm
150 mm
x
A
y
200 mm
B
2 kN

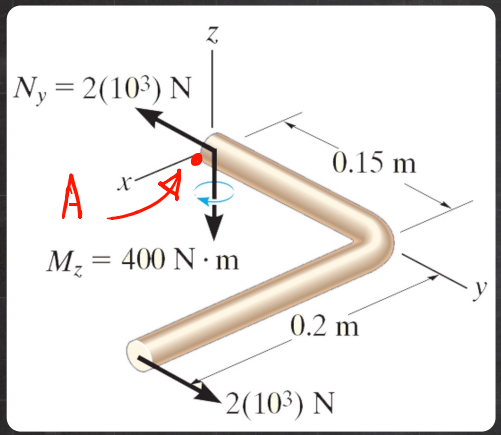

400 N · m
$\sigma = \frac{P}{A}$
A
$2(10^3)$ N
=
A
1.592 MPa

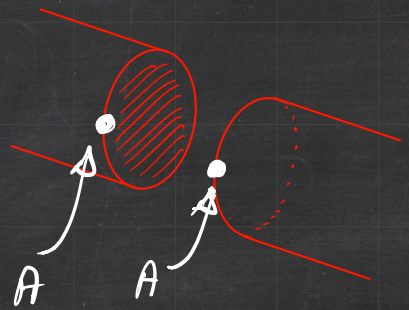

z
$N_y = 2(10^3)$ N
A
x
0.15 m
$M_z = 400$ N · m
y
0.2 m
$2(10^3)$ N

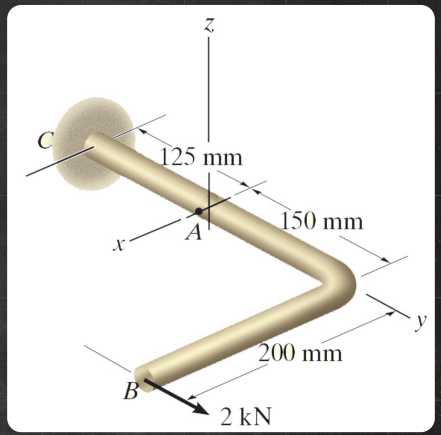

A
A

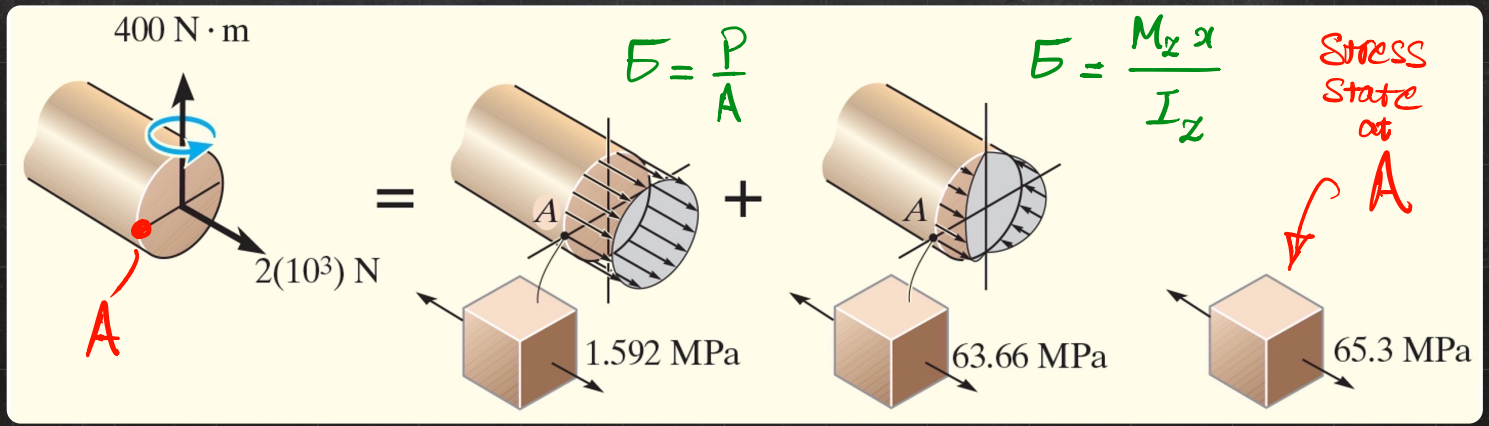

z
C
125 mm
150 mm
x
A
y
200 mm
B
2 kN


400 N · m
$2(10^3)$ N
A
=
A
1.592 MPa
+
A
63.66 MPa
$\sigma = \frac{P}{A}$
$\sigma = \frac{M_z x}{I_z}$


z
$N_y = 2(10^3)$ N
A
x
0.15 m
$M_z = 400$ N · m
y
0.2 m
$2(10^3)$ N

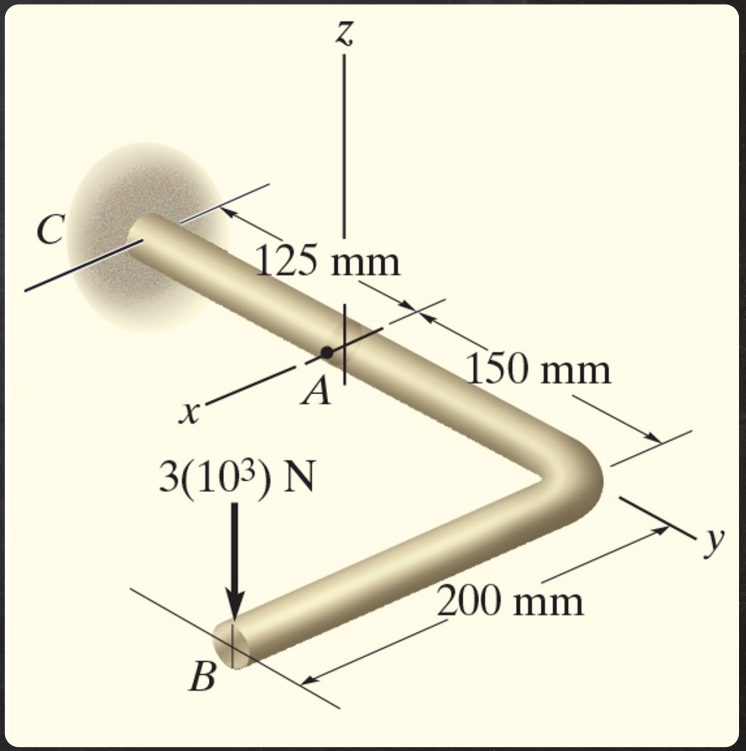

A
A


z
C
125 mm
150 mm
x
A
y
200 mm
B
2 kN


400 N · m
$2(10^3)$ N
A
=
A
1.592 MPa
+
A
63.66 MPa
$\sigma = \frac{P}{A}$
$\sigma = \frac{M_z x}{I_z}$
Stress State at A
65.3 MPa

Exercise 3 . [similar to ... P. 423 ... 8.6]

DETERMINE THE STATE OF STRESS AT POINT A ON THE SOLID ROD SHOWN IN THE FIGURE. THE RADIUS OF THE SHAFT IS 20 mm.



STEP 1)

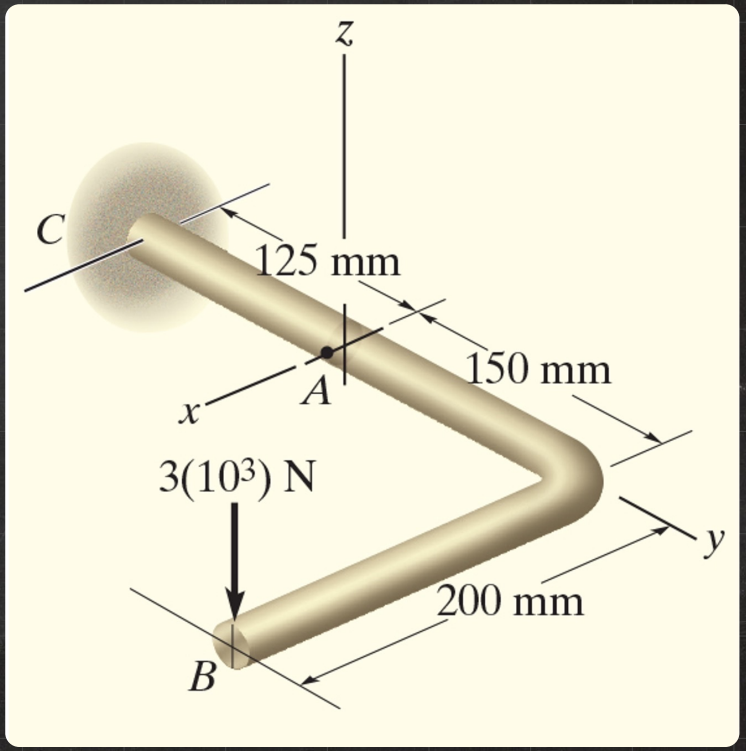
FIND INTERNAL FORCES/MOMENTS AT THE DESIRED CROSS-SECTION

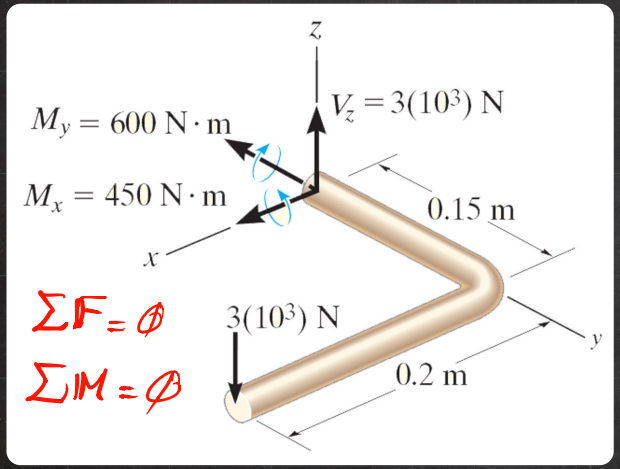
STEP 2)

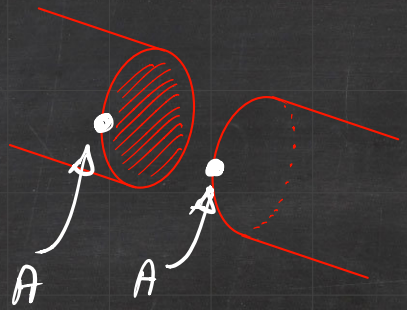
FIND STRESSES σ, τ CAUSED DUE TO INTERNAL REACTIONS

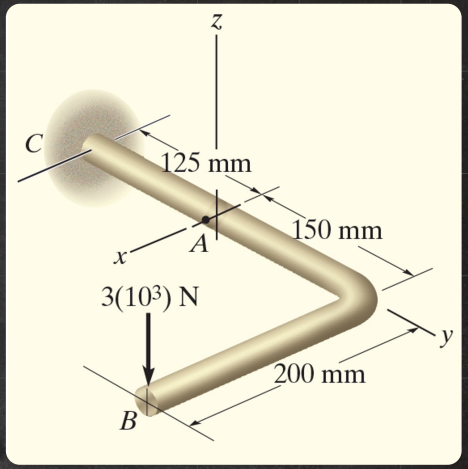
STEP 3)

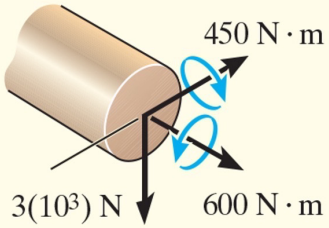
ADD UP THE EFFECTS

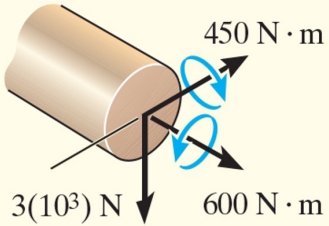



z
$V_z = 3(10^3)$ N
$M_y = 600$ N · m
$M_x = 450$ N · m
0.15 m
x
$\Sigma F = \emptyset$
$3(10^3)$ N
$\Sigma M = \emptyset$
0.2 m
y

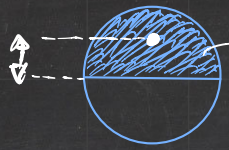

A
A


z
C
125 mm
150 mm
A
x
$3(10^3)$ N
y
200 mm
B


450 N · m
$3(10^3)$ N
600 N · m


450 N · m
3(10³) N
600 N · m

$$\bar{y}' = \frac{4c}{3\pi}$$



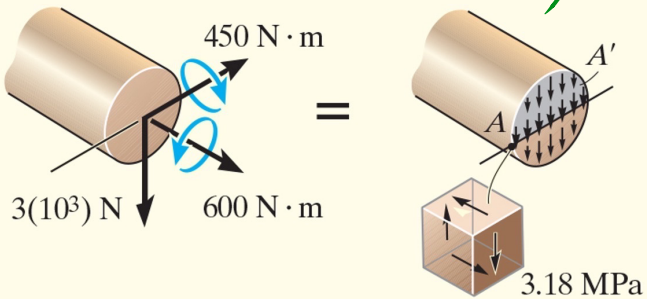
$$A' = \frac{\pi c^2}{2}$$

$$Q = \bar{y}' A' = \frac{4c}{3\pi}\left(\frac{1}{2}\pi c^2\right)$$

thickness at A

$$t = 2c, \quad I = \frac{\pi}{4}c^4 \qquad c = 0.02$$

$$\tau = \frac{VQ}{It}$$



$\bar{y}' = \frac{4c}{3\pi}$

$A' = \frac{\pi c^2}{2}$

$Q = \bar{y}' A' = \frac{4c}{3\pi}\left(\frac{1}{2}\pi c^2\right)$

$\tau = \frac{VQ}{It}$

thickness at A

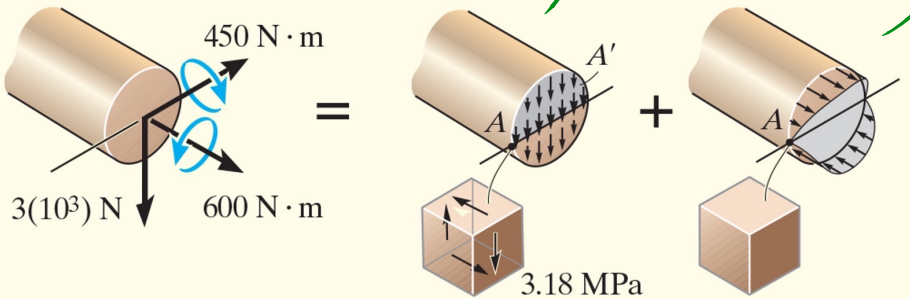
$t = 2c, \quad I = \frac{\pi}{4}c^4$

$c = 0.02$

A IS ON NEUTRAL AXIS

$\sigma_A = 0$

$\sigma = \frac{M_x z}{I_x}$



$\bar{y}' = \frac{4c}{3\pi}$

$A' = \frac{\pi c^2}{2}$

$Q = \bar{y}' A' = \frac{4c}{3\pi}\left(\frac{1}{2}\pi c^2\right)$

thickness at A

$t = 2c, \quad I = \frac{\pi}{4}c^4$

$c = 0.02$

$\tau = \frac{VQ}{It}$

A IS ON NEUTRAL AXIS

$\sigma_A = 0$

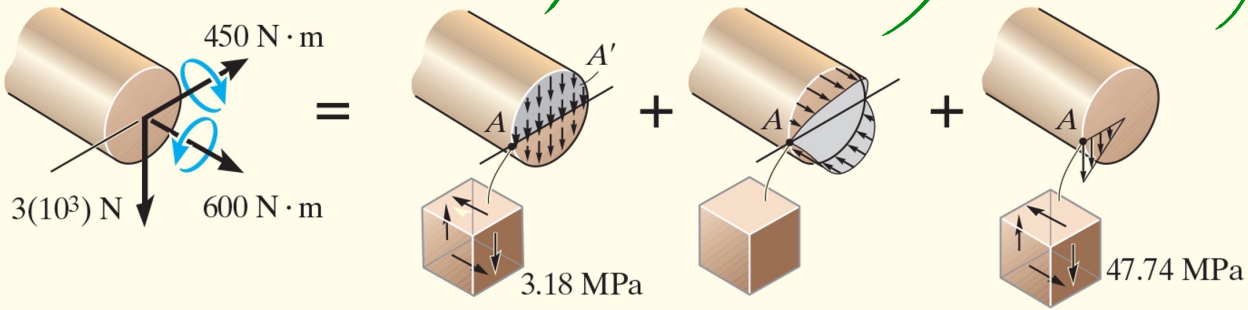
$\sigma = \frac{M_x z}{I_x}$

$\tau = M_y = 600\ \text{N.m}$

$\tau = \frac{Tr}{J}$

$r = c$ AT A

$J = \frac{\pi}{2}c^4$



$\bar{y}' = \frac{4c}{3\pi}$

$A' = \frac{\pi c^2}{2}$

$Q = \bar{y}' A' = \frac{4c}{3\pi}\left(\frac{1}{2}\pi c^2\right)$

thickness at A

$t = 2c, \quad I = \frac{\pi}{4}c^4$ ($c = 0.02$)

$\tau = \frac{VQ}{It}$

A IS ON NEUTRAL AXIS

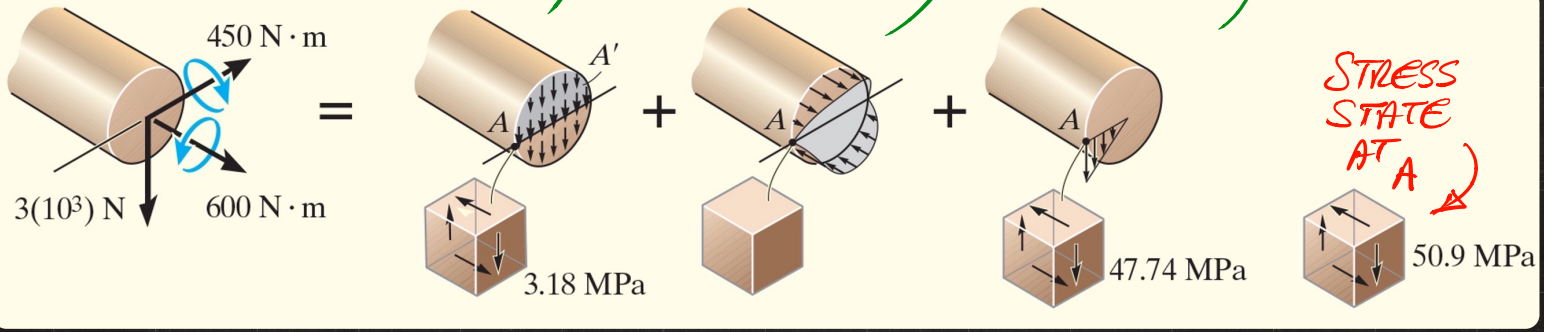
$\sigma_A = 0$

$\sigma = \frac{M_x z}{I_x}$

$\tau = \frac{Tr}{J}$ ($r = c$ AT A)

$T = M_y = 600\ \text{N.m}$

$J = \frac{\pi}{2}c^4$



COMBINED LOADING

* AXIAL LOADING $\sigma = \frac{P}{A}$ $\quad \delta = \frac{PL}{EA}$
* TORSION $\tau = \frac{Tr}{J}$ $\quad \phi = \frac{TL}{GJ}$
* BENDING $\sigma = -\frac{My}{I}$ $\quad \theta = \frac{ML}{EI}$
* TRANSVERSE SHEAR $\tau = \frac{VQ}{It}$

STEP 1)

FIND INTERNAL FORCES/MOMENTS AT THE DESIRED CROSS-SECTION

STEP 2)

FIND STRESSES σ, τ CAUSED DUE TO INTERNAL REACTIONS

STEP 3)

ADD UP THE EFFECTS

(Careful with signs!)

MECHANICS AND MATERIALS I

Combined Loading

Section ... 8.2

Chap. 8

[Hibbeler 9th edition]

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MECHANICS AND MATERIALS I

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