MECHANICS AND MATERIALS I

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18

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MECHANICS AND MATERIALS I

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Bending iii

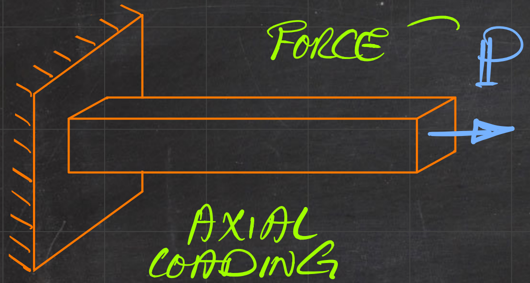
Sections … 6.3 – 6.4

Chap. 6

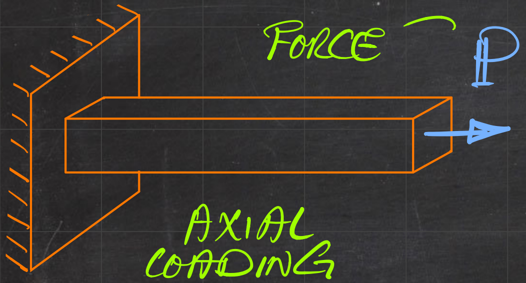
[Hibbeler 9th edition]

BENDING DEFORMATION OF A STRAIGHT MEMBER

Bending Deformation of a Straight Member



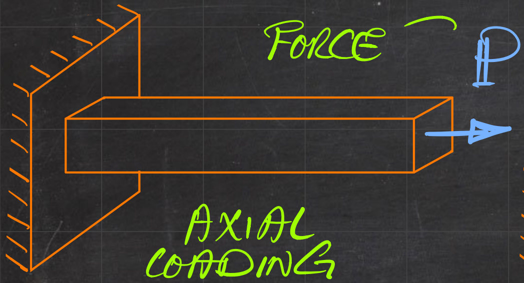
BENDING DEFORMATION OF A STRAIGHT MEMBER

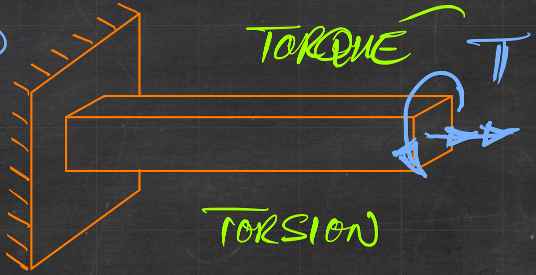


APPLIED LOADING — (NORMAL) FORCE

STRESS/STRAIN — (NORMAL) σ, ε

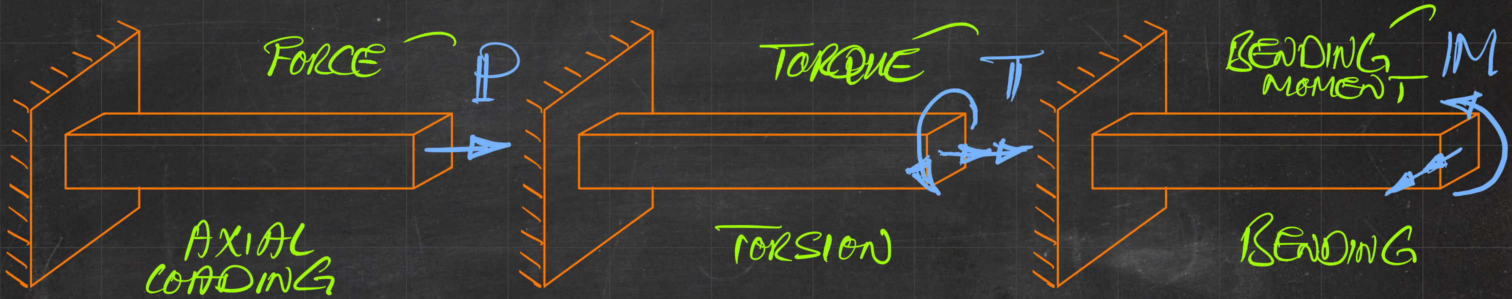
BENDING DEFORMATION OF A STRAIGHT MEMBER





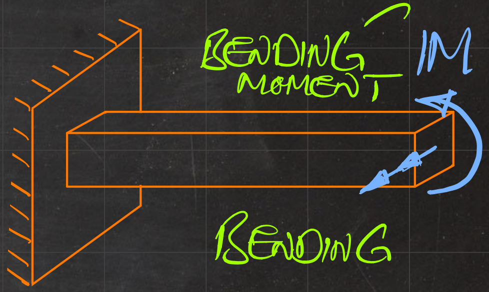
APPLIED LOADING	(NORMAL) FORCE	(NORMAL) MOMENT
STRESS/STRAIN	(NORMAL) σ, ε	(SHEAR) τ, γ

Bending Deformation of a Straight Member



Applied loading	(normal) Force	(normal) Moment	(tangential) Moment
Stress/Strain	(normal) σ, ε	(shear) τ, γ	(normal) σ, ε

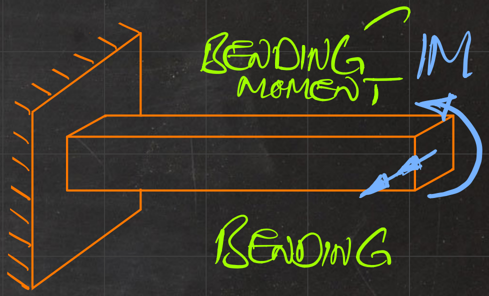
BENDING DEFORMATION OF A STRAIGHT MEMBER



BENDING DEFORMATION OF A STRAIGHT MEMBER

↳ STRAIGHT, PRISMATIC BEAM

↳ identical cross section



BENDING DEFORMATION OF A STRAIGHT MEMBER

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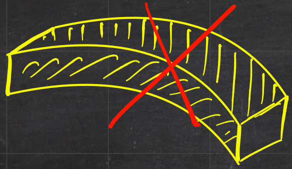
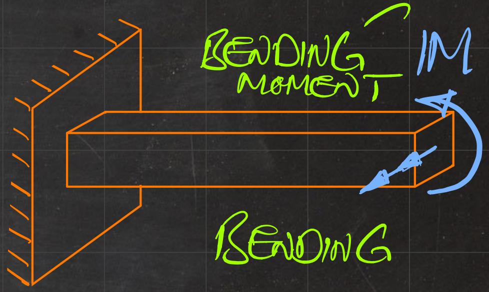


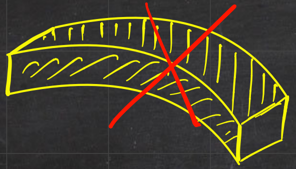
↳ identical cross section

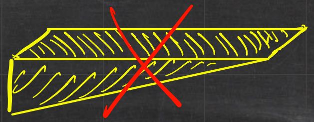


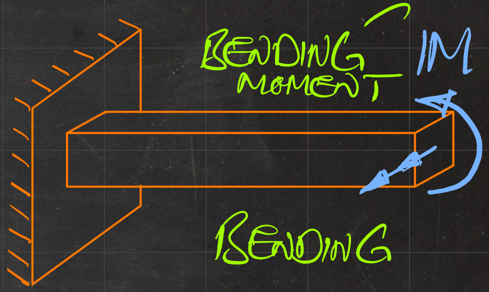
BENDING DEFORMATION OF A STRAIGHT MEMBER

→ STRAIGHT, PRISMATIC BEAM

↳ identical cross section







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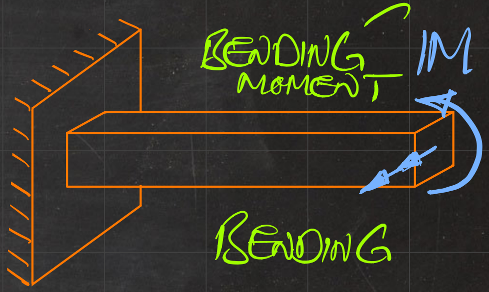
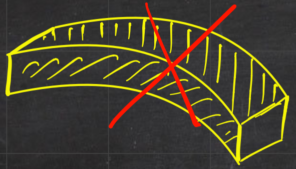
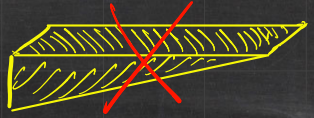
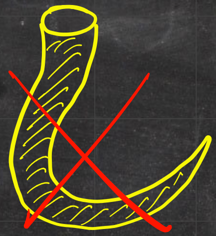


↳ STRAIGHT, PRISMATIC BEAM

└ identical cross section







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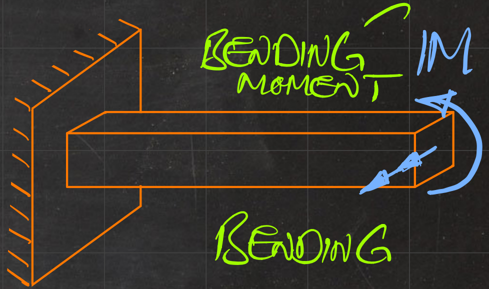

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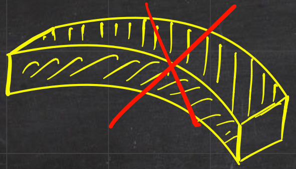
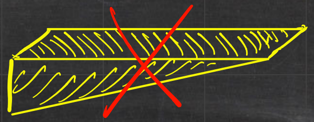
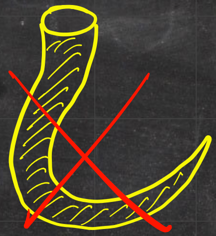
↳ identical cross section

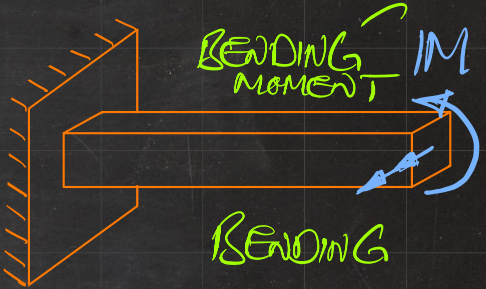






↳ SYMMETRICAL WITH RESPECT TO AN AXIS

BENDING DEFORMATION OF A STRAIGHT MEMBER



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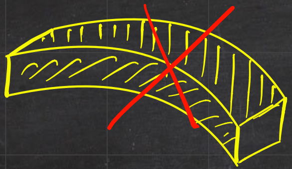
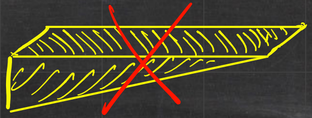


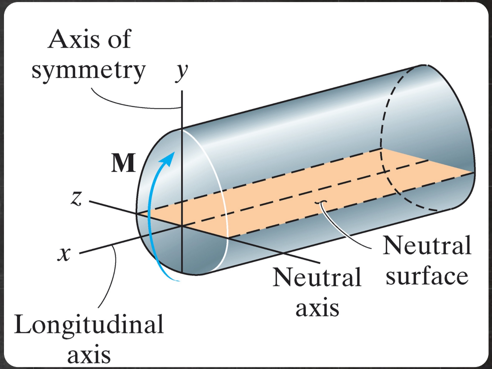


↳ SYMMETRICAL WITH RESPECT TO AN AXIS

↳ MOMENT APPLIED ABOUT AN AXIS PERPENDICULAR TO THE AXIS OF SYMMETRY

BENDING DEFORMATION OF A STRAIGHT MEMBER

- STRAIGHT, PRISMATIC BEAM
- SYMMETRICAL WITH RESPECT TO AN AXIS
- MOMENT APPLIED ABOUT AN AXIS PERPENDICULAR TO THE AXIS OF SYMMETRY



BENDING DEFORMATION OF A STRAIGHT MEMBER

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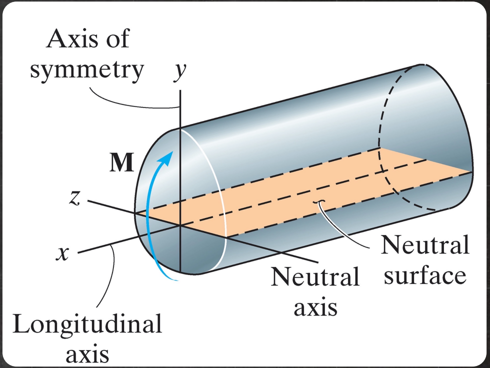
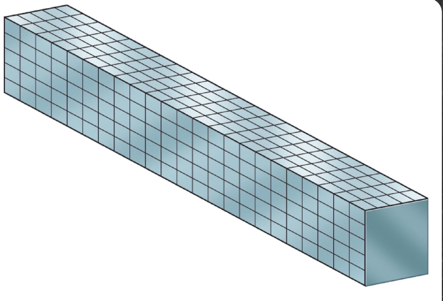
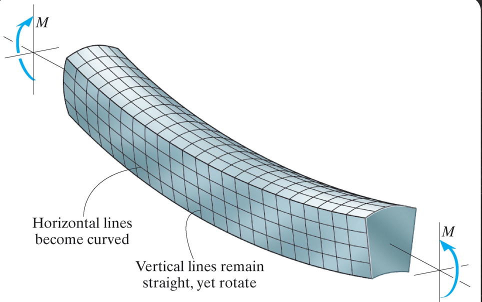


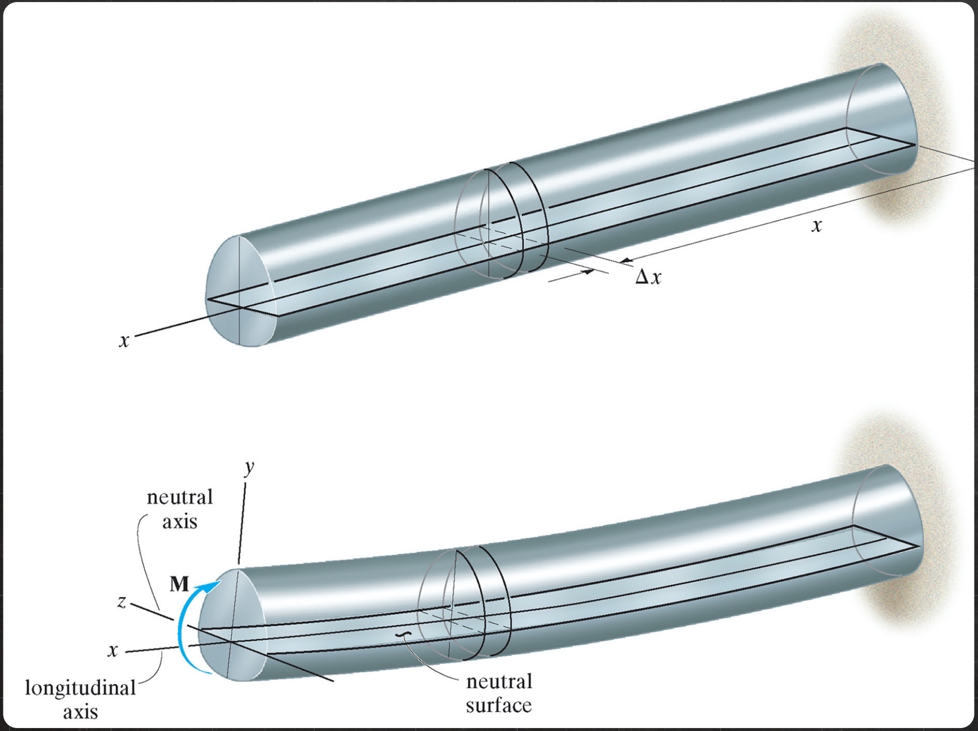
- Horizontal lines become curved!
- Vertical lines remain straight but rotate!
- Plane cross sections remain planes perpendicular to longitudinal axis!

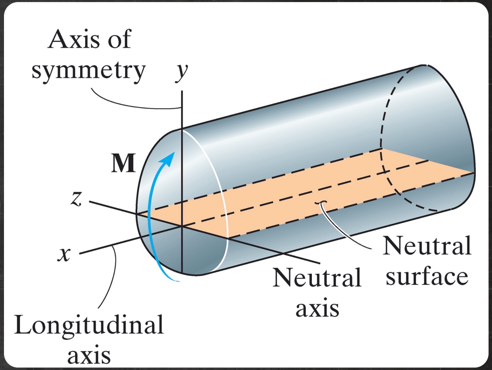

Before deformation

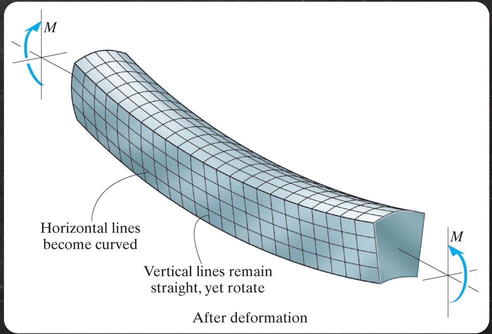


After deformation

Bending Deformation of a Straight Member







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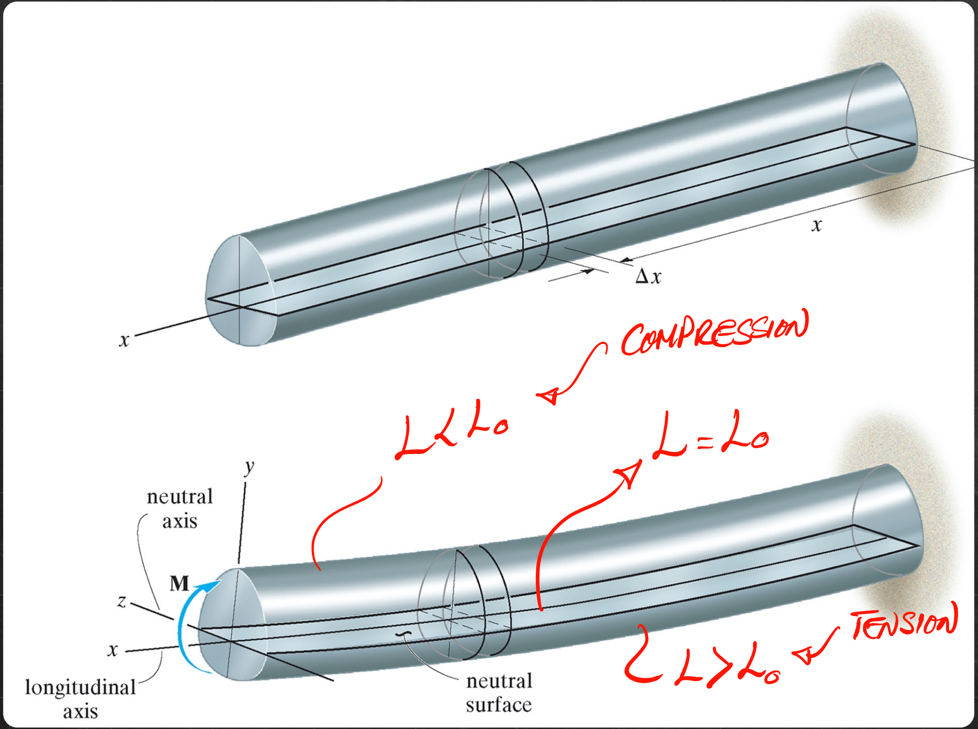

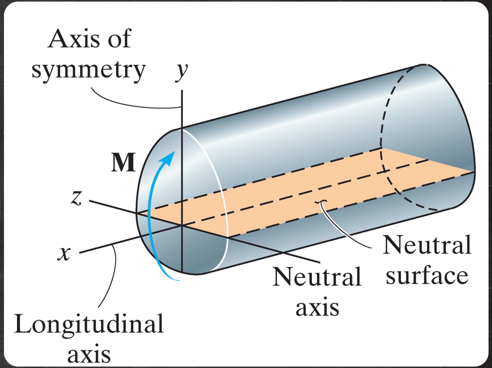

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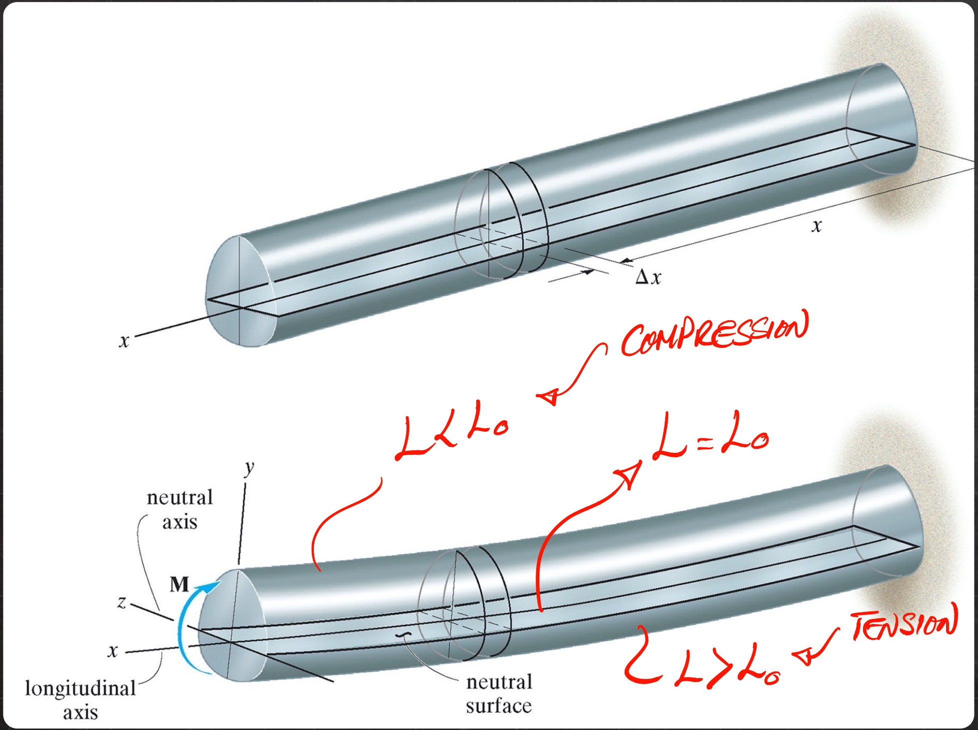
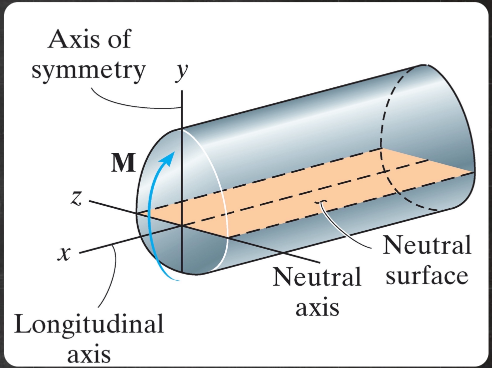
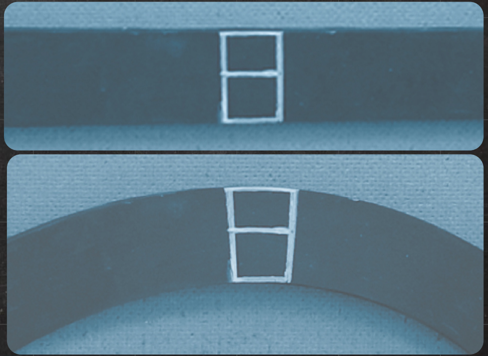
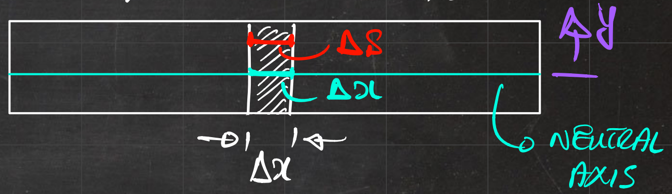
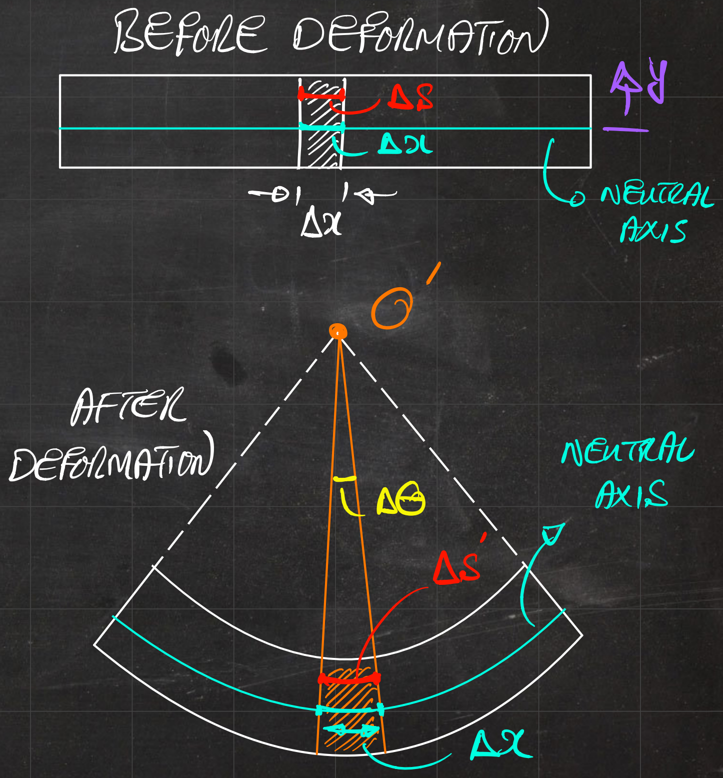






BEFORE DEFORMATION



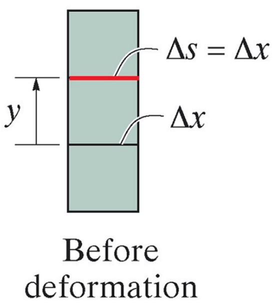


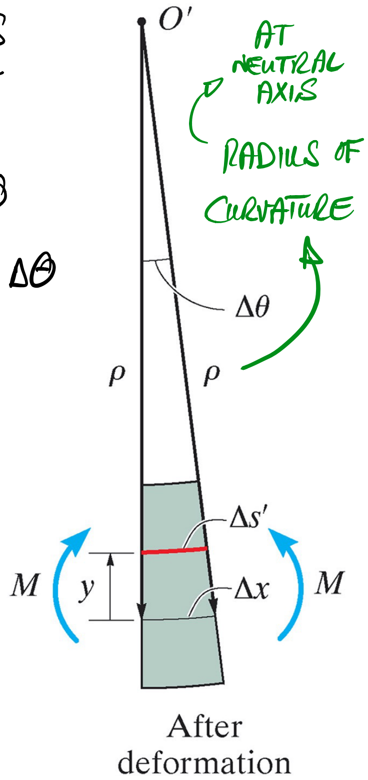
BEFORE DEFORMATION
ΔS
Δx
Δx
y
NEUTRAL AXIS
O'
AFTER DEFORMATION
Δθ
ΔS'
NEUTRAL AXIS
Δx

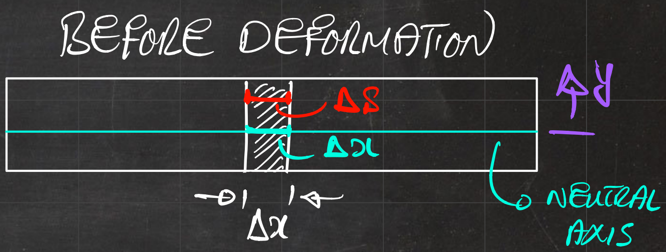
$$\varepsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

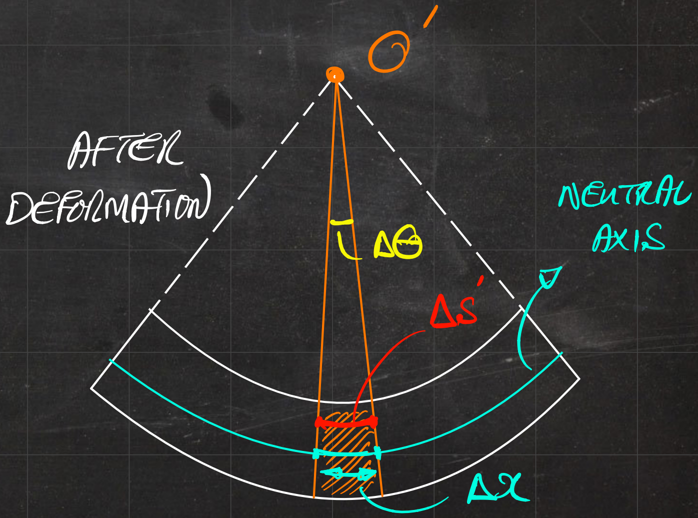
$$\begin{cases} \Delta s = \rho \, \Delta\theta \\ \Delta s' = (\rho - y) \, \Delta\theta \end{cases}$$

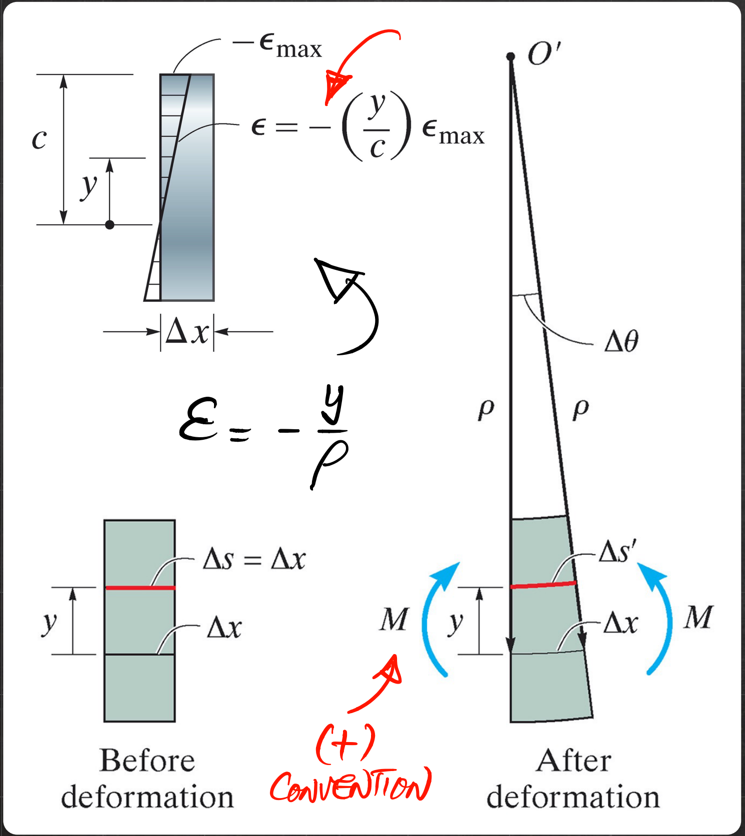
$$\Rightarrow \varepsilon = -\frac{y}{\rho}$$



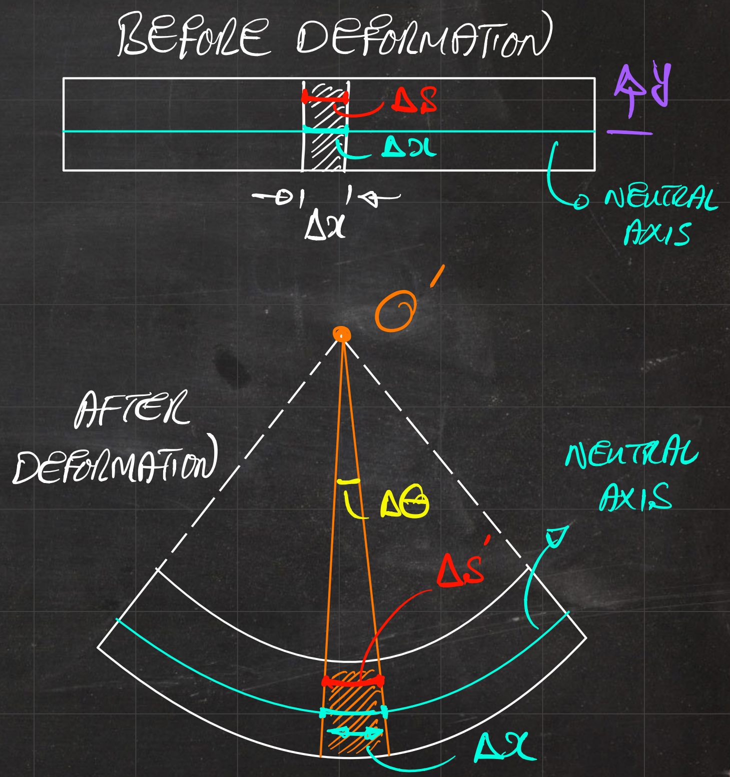






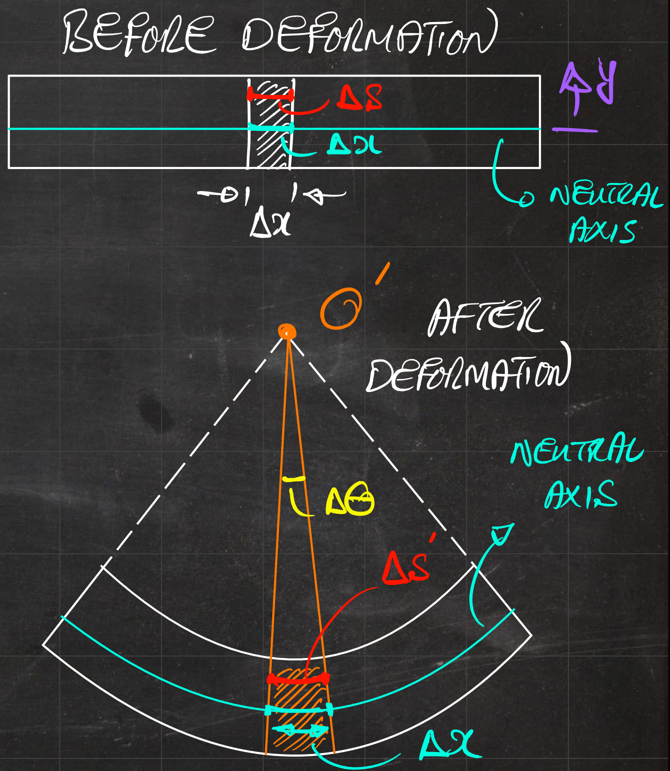


$-\epsilon_{max}$
c
y
$\epsilon = -\left(\frac{y}{c}\right)\epsilon_{max}$
Δx
$\varepsilon = -\frac{y}{\rho}$
O'
$\Delta\theta$
ρ
ρ
$\Delta s = \Delta x$
Δx
y
$\Delta s'$
Δx
M
M
y
(+)
CONVENTION
Before
deformation
After
deformation

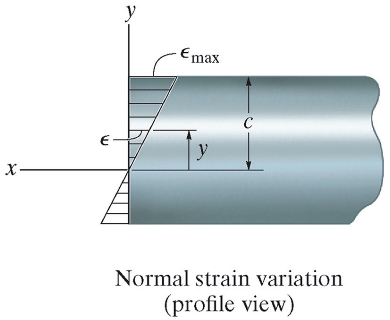


BEFORE DEFORMATION
Δs
Δx
Δy
Δx
NEUTRAL AXIS
O'
AFTER DEFORMATION
$\Delta\theta$
$\Delta s'$
NEUTRAL AXIS
Δx

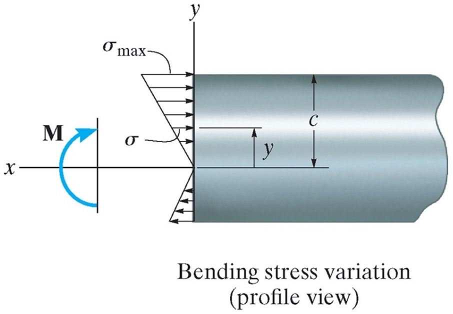
THE FLEXURE FORMULA



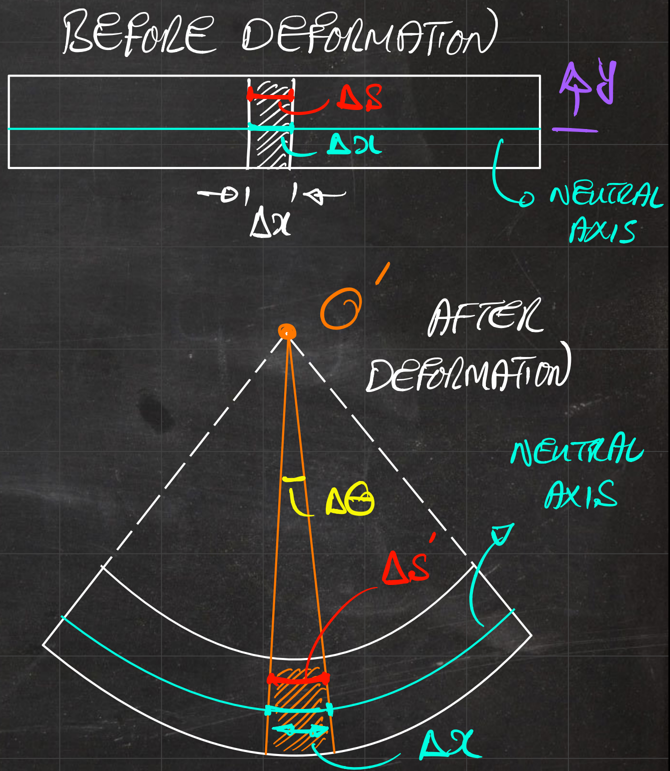
THE FLEXURE FORMULA



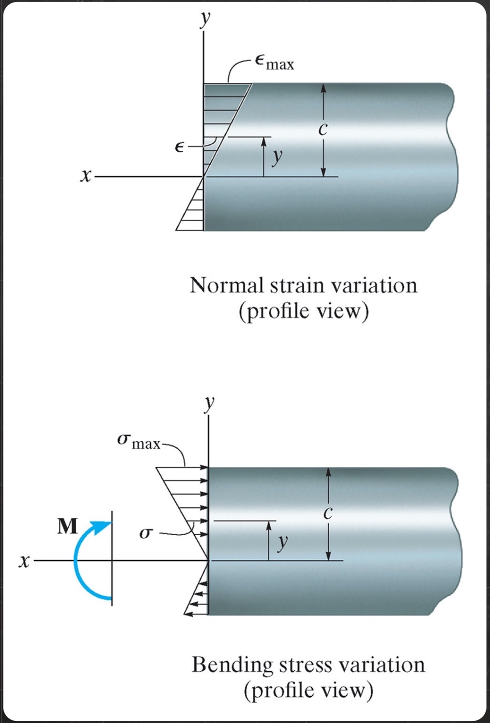
Normal strain variation
(profile view)



Bending stress variation
(profile view)



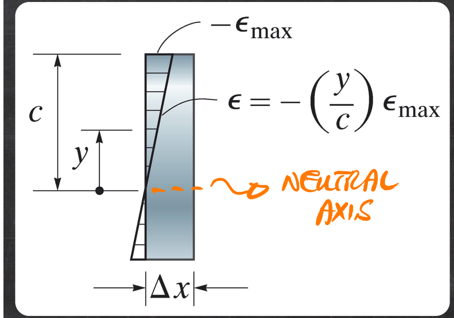
THE FLEXURE FORMULA



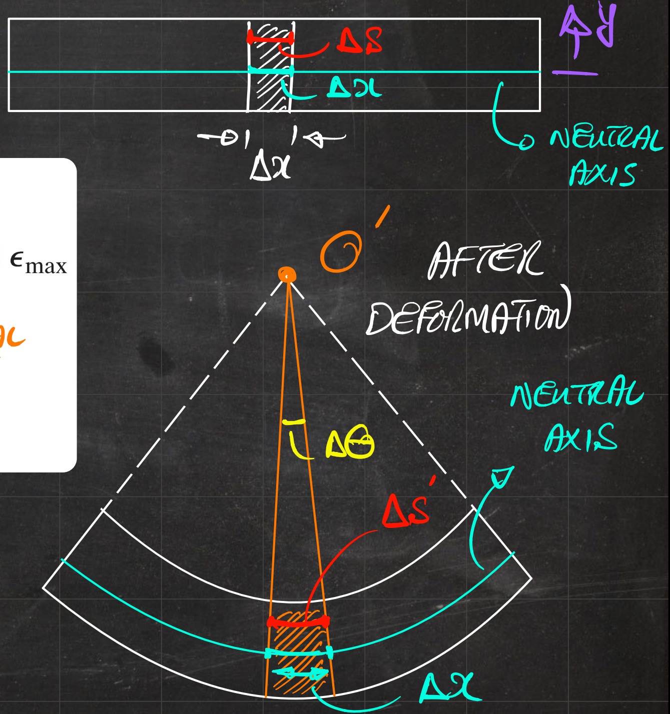
Normal strain variation
(profile view)

Bending stress variation
(profile view)

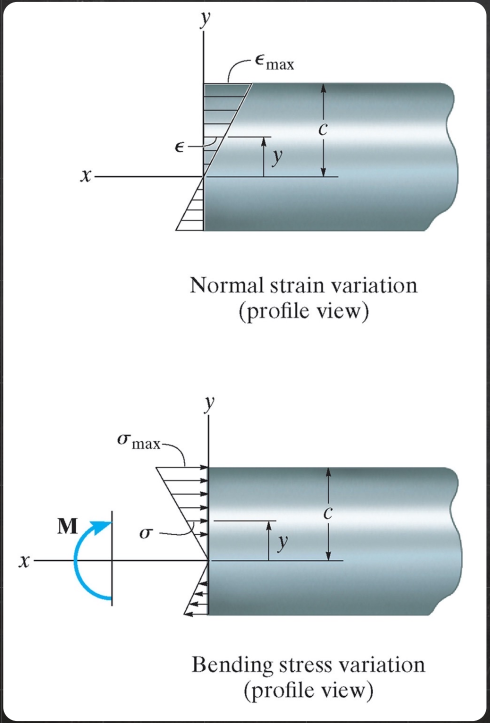
NORMAL STRAIN DISTRIBUTION



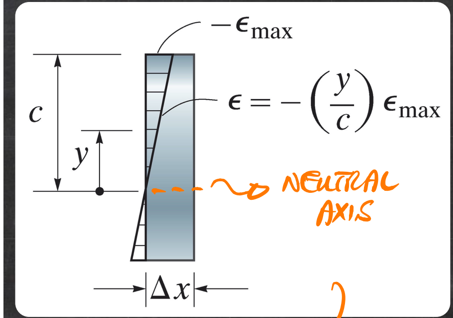
BEFORE DEFORMATION



The Flexure Formula



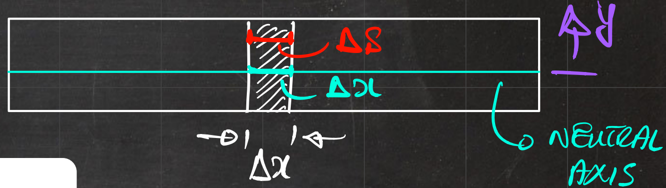
Normal strain distribution

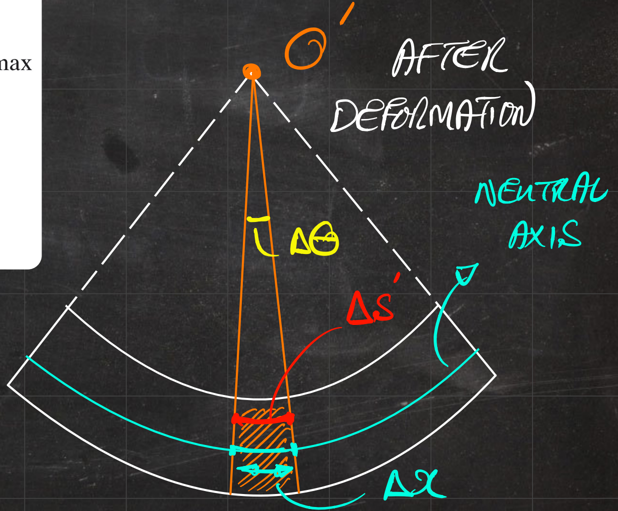


$$\sigma = E\epsilon$$

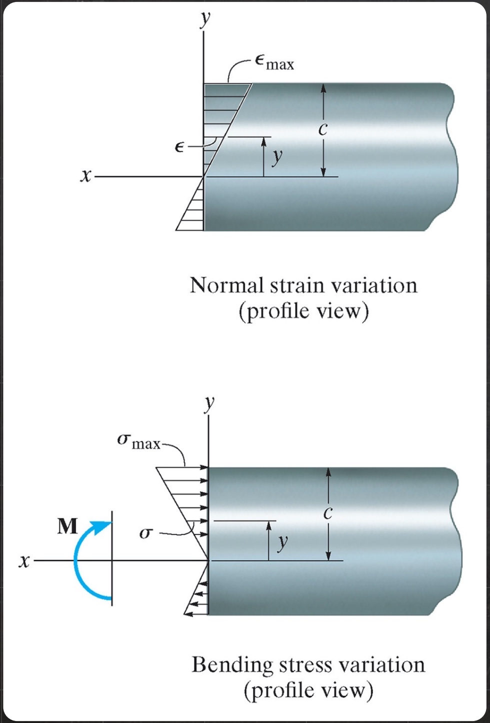
$$\Rightarrow \sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

Before deformation





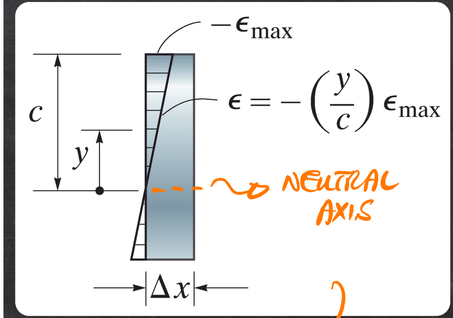
The Flexure Formula



Normal strain variation
(profile view)

Bending stress variation
(profile view)

NORMAL STRAIN DISTRIBUTION



$$\sigma = E\epsilon$$

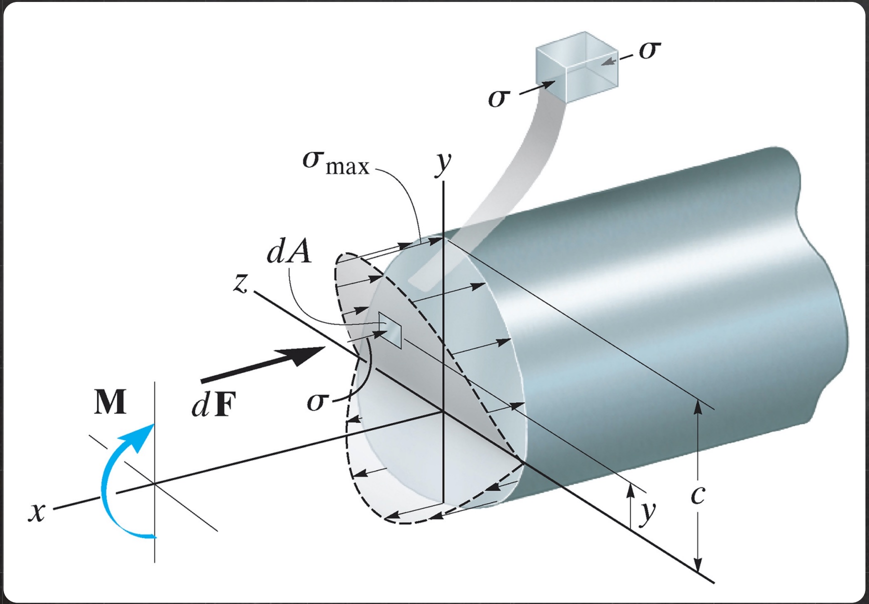
$$\Rightarrow \sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

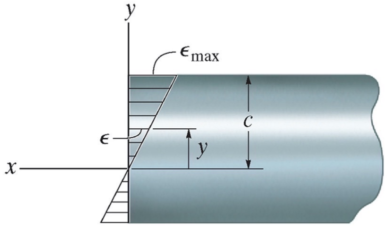
TO PROCEED, WE NEED TO CALCULATE THE LOCATION OF THE NEUTRAL AXIS

↳ EQUILIBRIUM

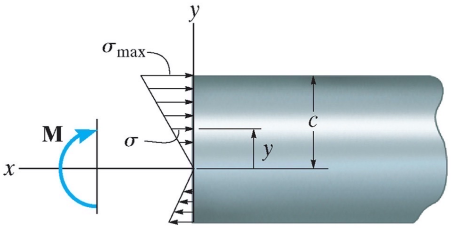
↳ Resultant Force $\overset{!}{=} 0$

LOCATION OF NEUTRAL AXIS





Normal strain variation (profile view)

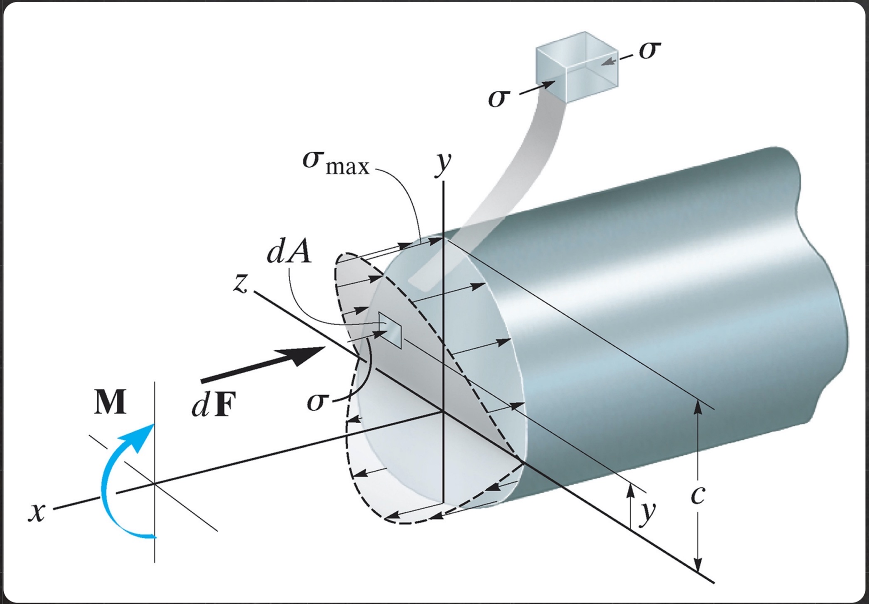


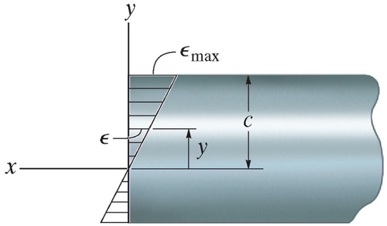
Bending stress variation (profile view)

$$\int_A dF \overset{!}{=} 0$$

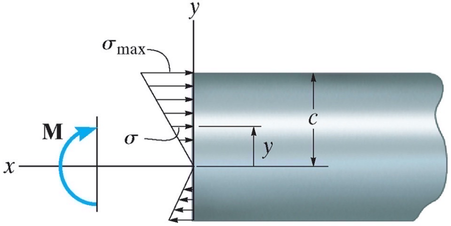
$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

Location of Neutral Axis





Normal strain variation
(profile view)

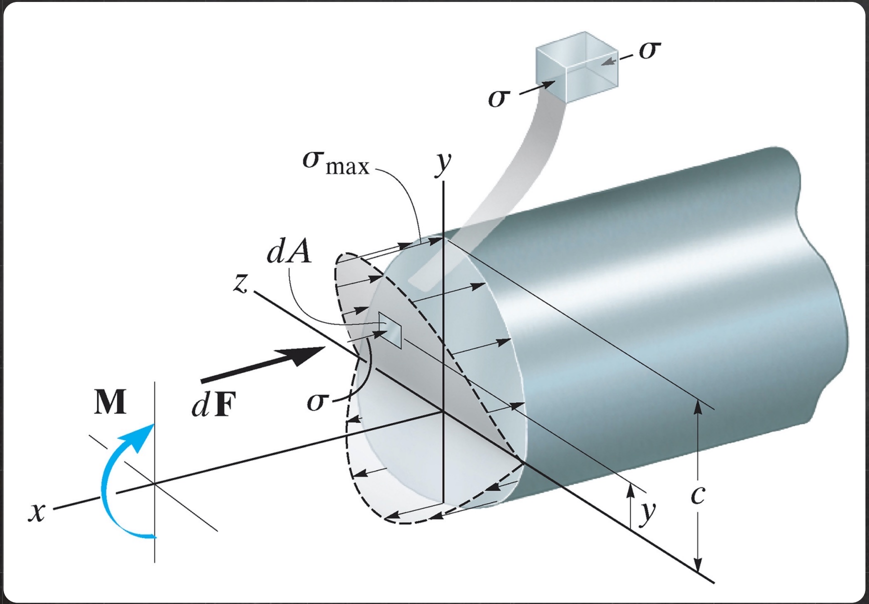


Bending stress variation
(profile view)

$$\int_A dF \overset{!}{=} 0 \quad , \quad dF = \sigma \, dA$$

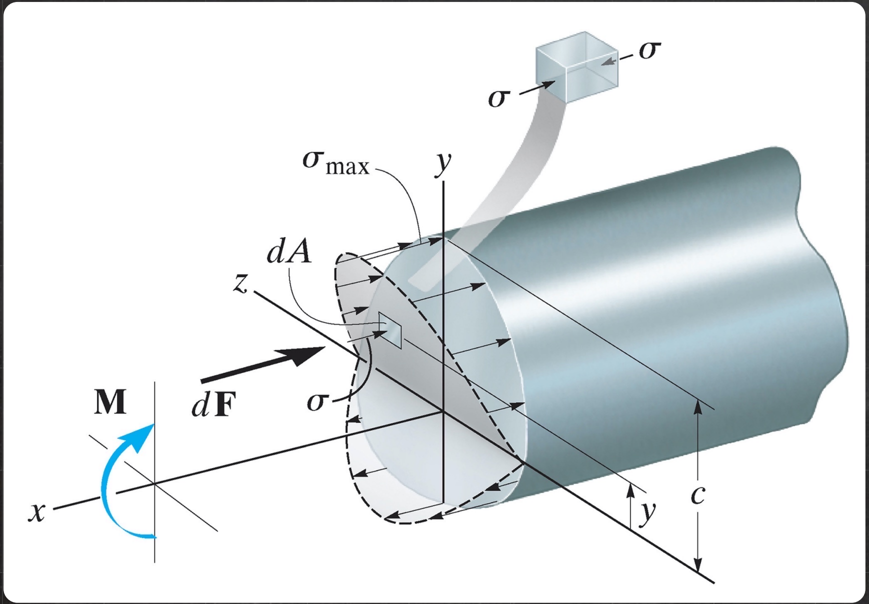
$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

LOCATION OF NEUTRAL AXIS



$$\int_A dF = \int_A \sigma \, dA$$

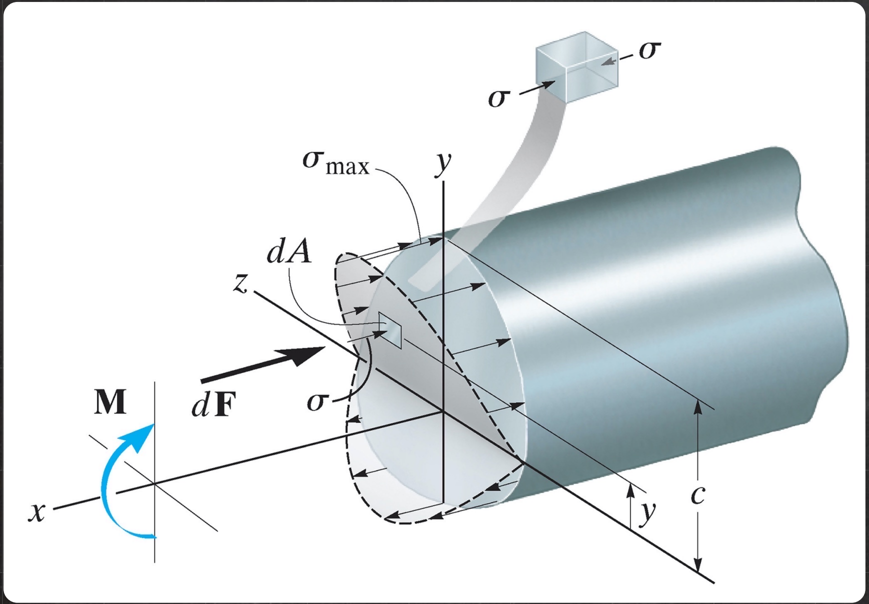
LOCATION OF NEUTRAL AXIS



$$\int_A dF = \int_A \sigma \, dA$$

$$= \int_A -\frac{y}{c}\sigma_{max} \, dA$$

LOCATION OF NEUTRAL AXIS

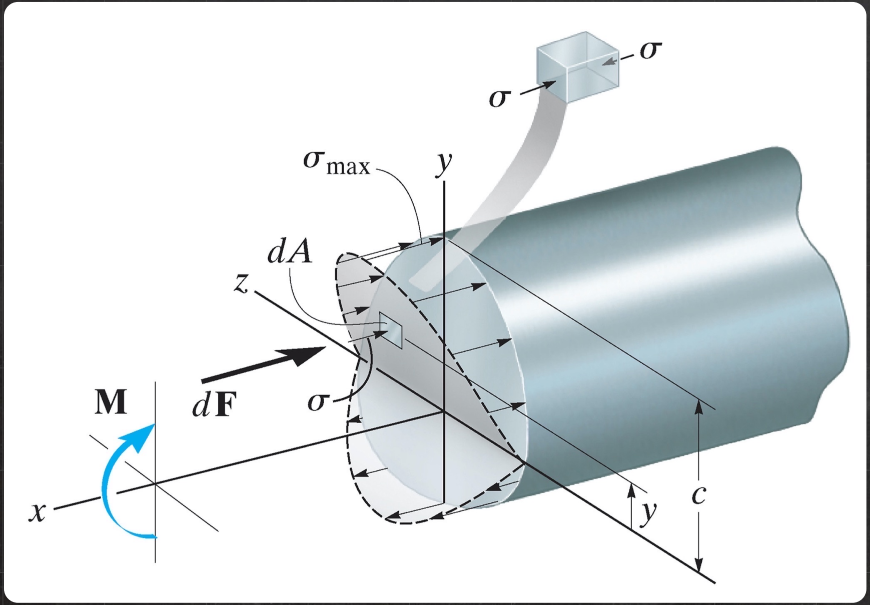


$$\int_A dF = \int_A \sigma \, dA$$

$$= \int_A -\frac{y}{c}\sigma_{max} \, dA$$

$$= -\frac{\sigma_{max}}{c}\int_A y \, dA$$

LOCATION OF NEUTRAL AXIS



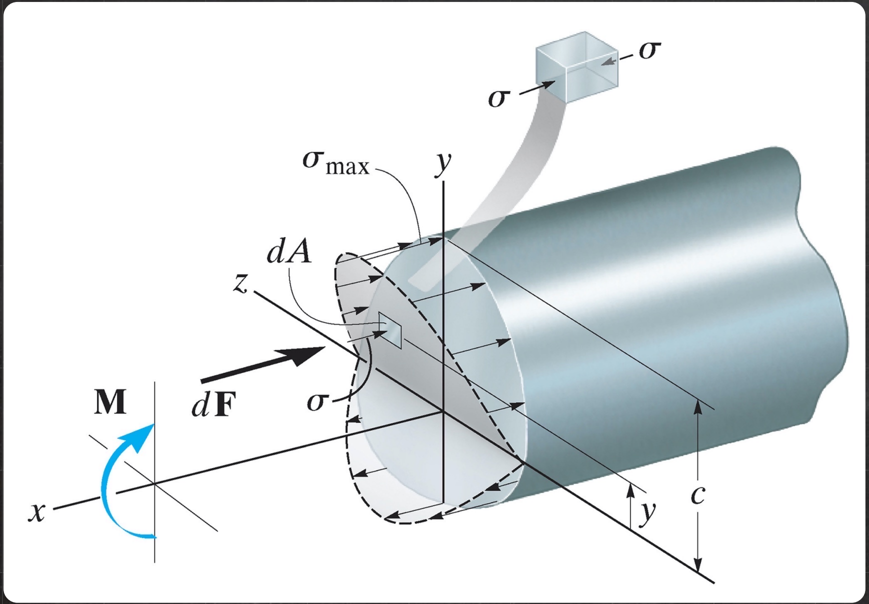
$$\int_A dF = \int_A \sigma \, dA$$

$$= \int_A -\frac{y}{c}\sigma_{max} \, dA$$

$$= -\frac{\sigma_{max}}{c}\int_A y \, dA \overset{!}{=} 0$$

$\neq 0$

LOCATION OF NEUTRAL AXIS



$$\int_A dF = \int_A \sigma \, dA$$

$$= \int_A -\frac{y}{c} \sigma_{max} \, dA$$

$$= -\frac{\sigma_{max}}{c} \int_A y \, dA \overset{!}{=} 0$$

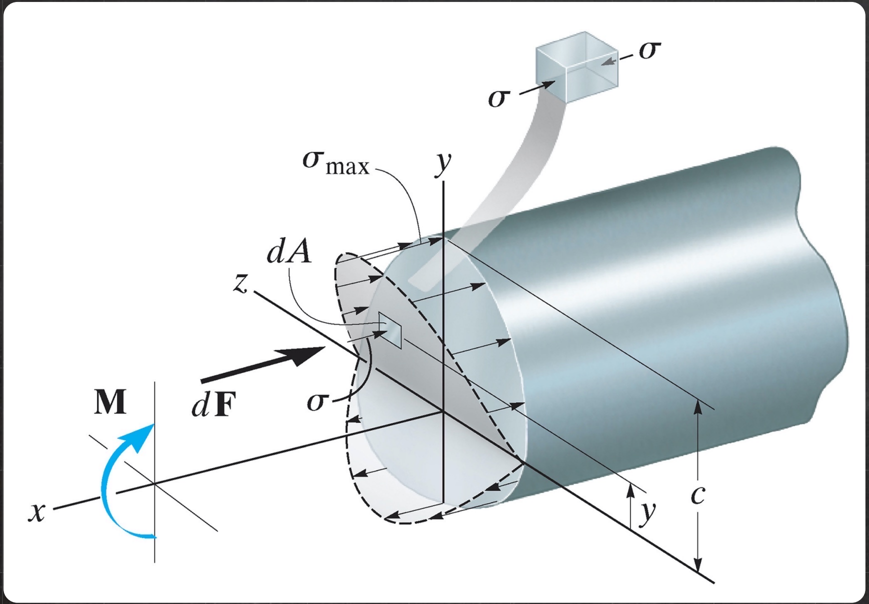
$\neq 0$

$$\Rightarrow \int_A y \, dA = 0$$

The first moment of cross-sectional area about neutral axis is ZERO

LOCATION OF NEUTRAL AXIS

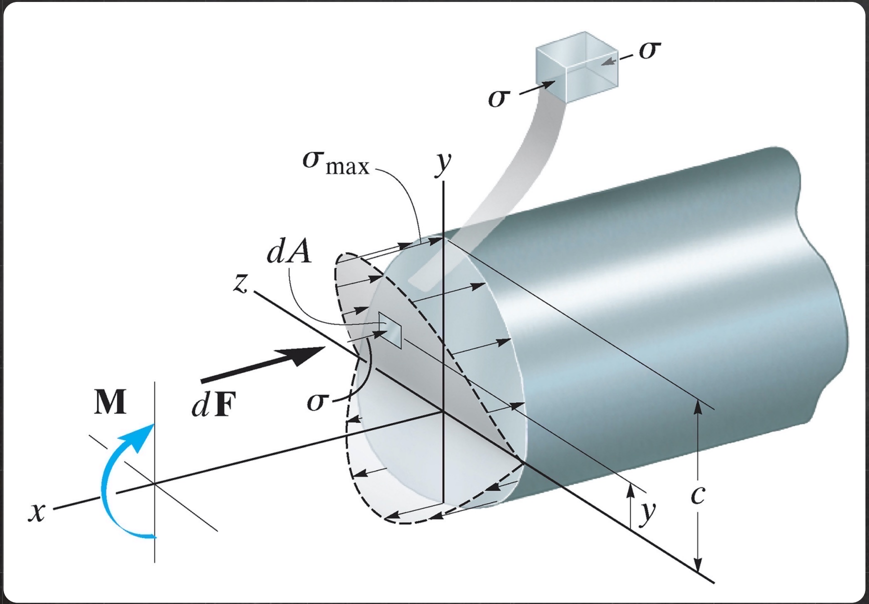
RECALL : $\bar{y} = \dfrac{\int_A y\,dA}{A}$



$$\Rightarrow \int_A y\,dA = 0$$

The first moment of cross-sectional area about neutral axis is ZERO

LOCATION OF NEUTRAL AXIS



RECALL : $\bar{y} = \dfrac{\int_A y\, dA}{A} = 0$

⇓

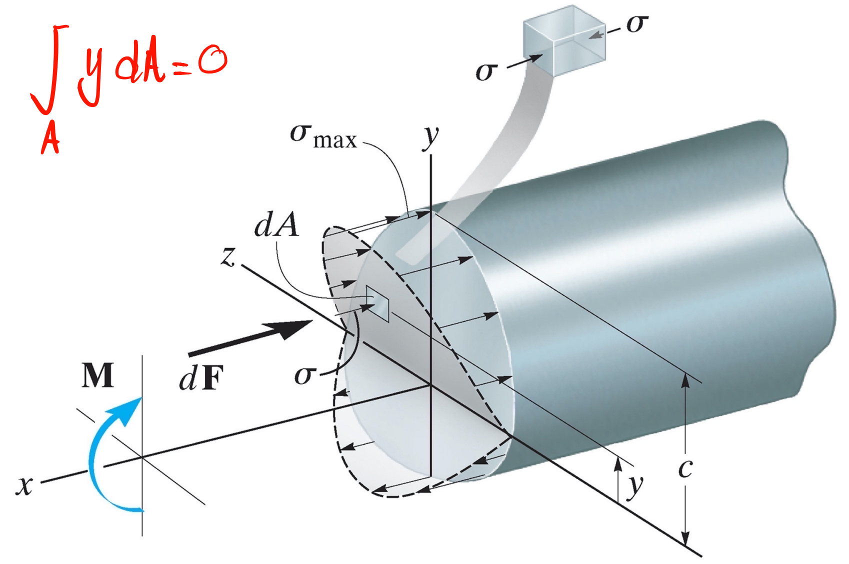
THE NEUTRAL AXIS IS ALSO THE HORIZONTAL CENTROIDAL AXIS FOR THE CROSS SECTION.

$\Rightarrow \int_A y\, dA = 0$

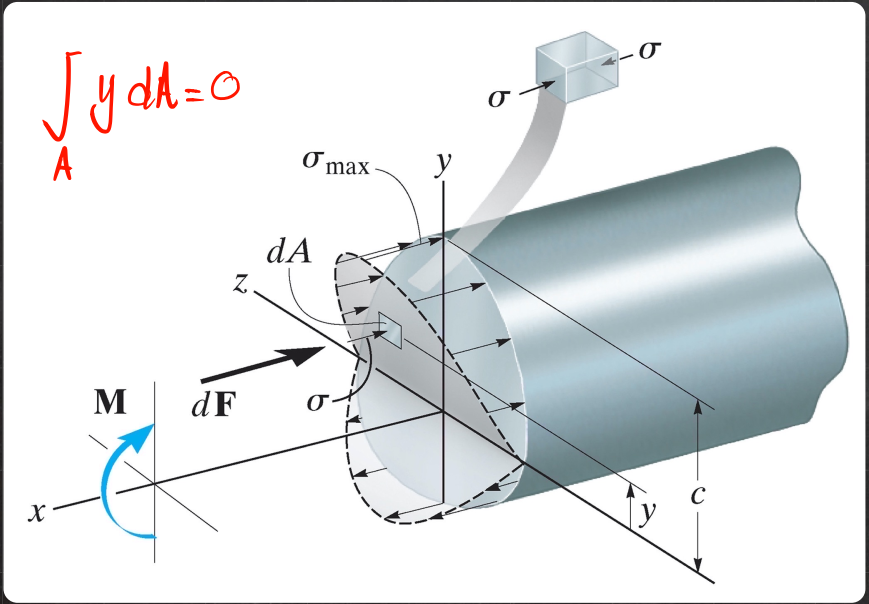
The first moment of cross-sectional area about neutral axis is ZERO

BENDING MOMENT

$$\int_A y\, dA = 0$$

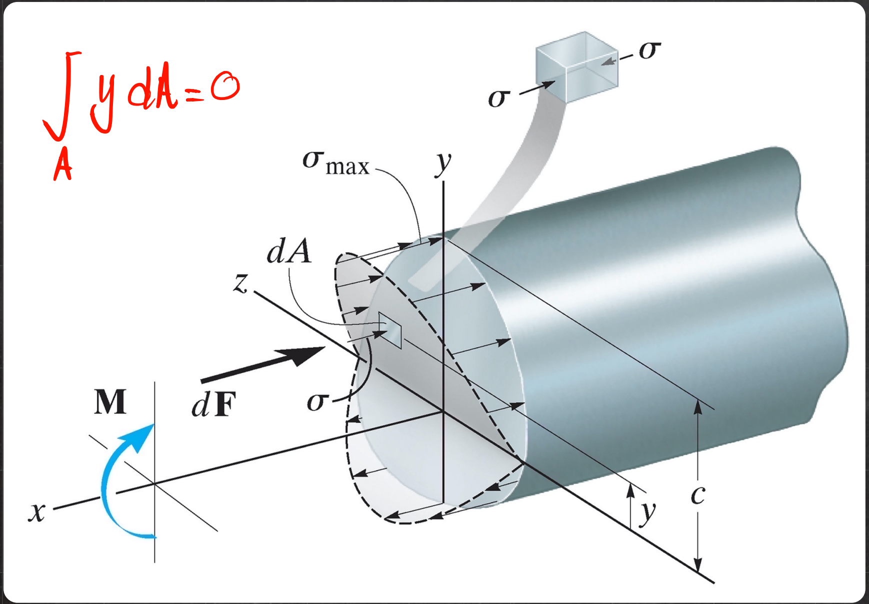


BENDING MOMENT



$$M = \int_A y\, dF$$

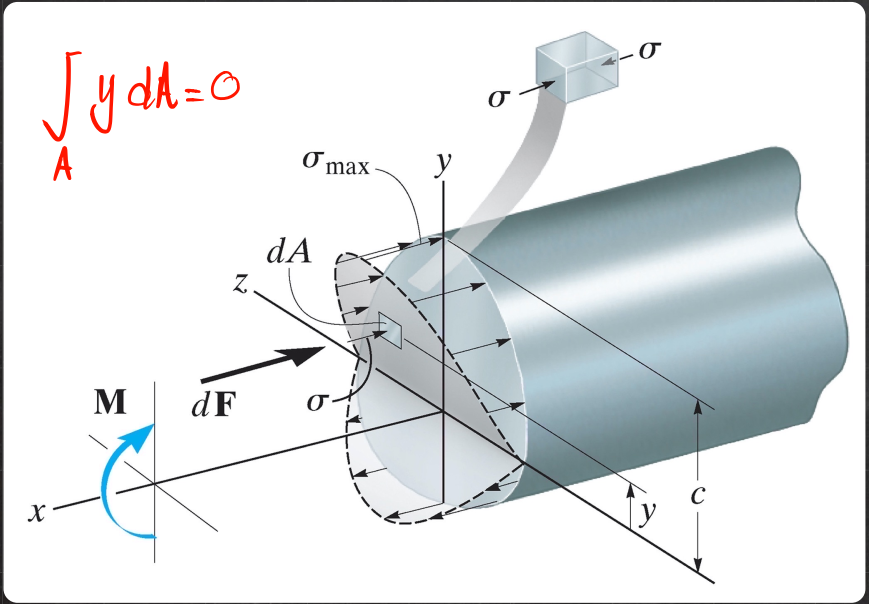
BENDING MOMENT



$$M = \int_A y\, dF$$

$$= \int_A y\, \sigma\, dA$$

Bending moment

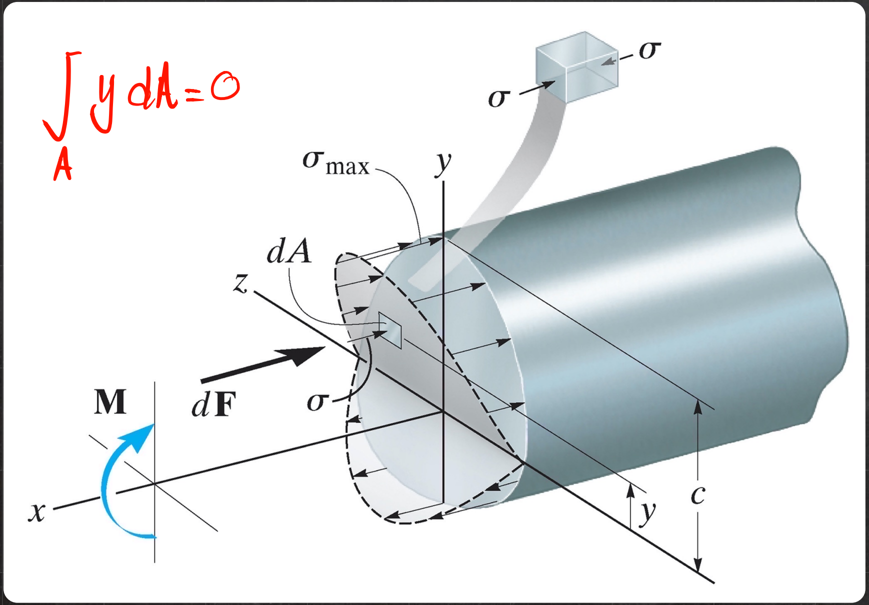


$$M = \int_A y\, dF$$

$$= \int_A y\, \sigma\, dA$$

$$= \int_A y\, \frac{y}{c}\, \sigma_{max}\, dA$$

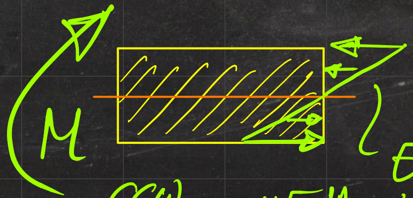
Bending moment



$$M = \int_A y\,dF$$

$$= \int_A y\,\sigma\,dA$$

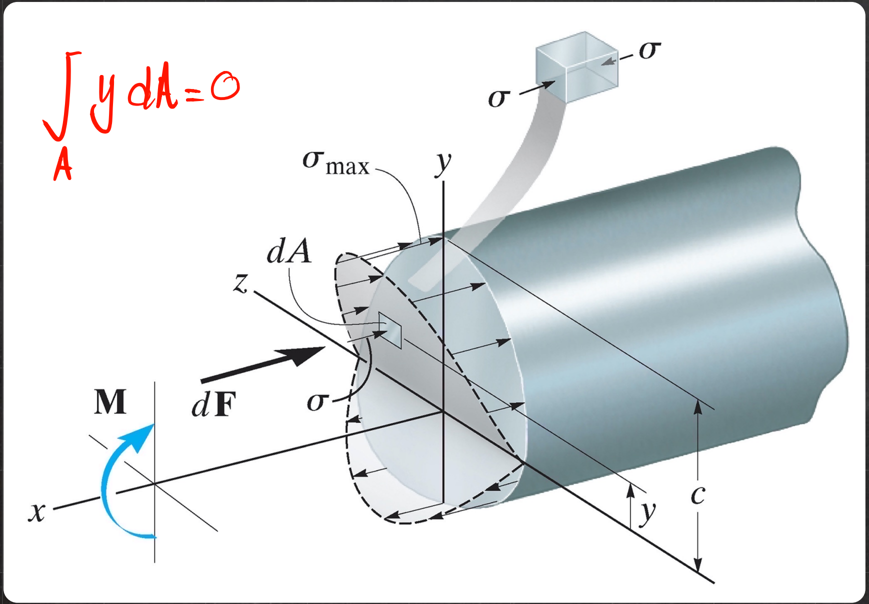
$$= \int_A y\,\frac{y}{c}\,\sigma_{max}\,dA$$



$$\sigma = \frac{y}{c}\sigma_{max}$$

CCW (+) $y\sigma dA$

BENDING MOMENT



$$\int_A y\,dA = 0$$

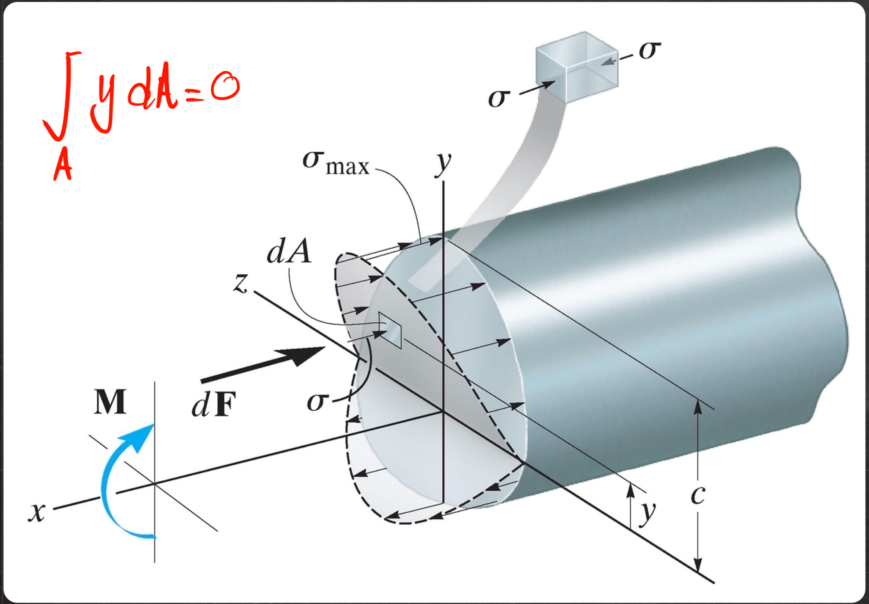
$$M = \int_A y\,dF$$

$$= \int_A y\,\sigma\,dA$$

$$= \int_A y\,\frac{y}{c}\,\sigma_{max}\,dA$$

$$= \frac{\sigma_{max}}{c}\int_A y^2\,dA$$

BENDING MOMENT



$$\int_A y\,dA = 0$$

$$M = \int_A y\,dF$$

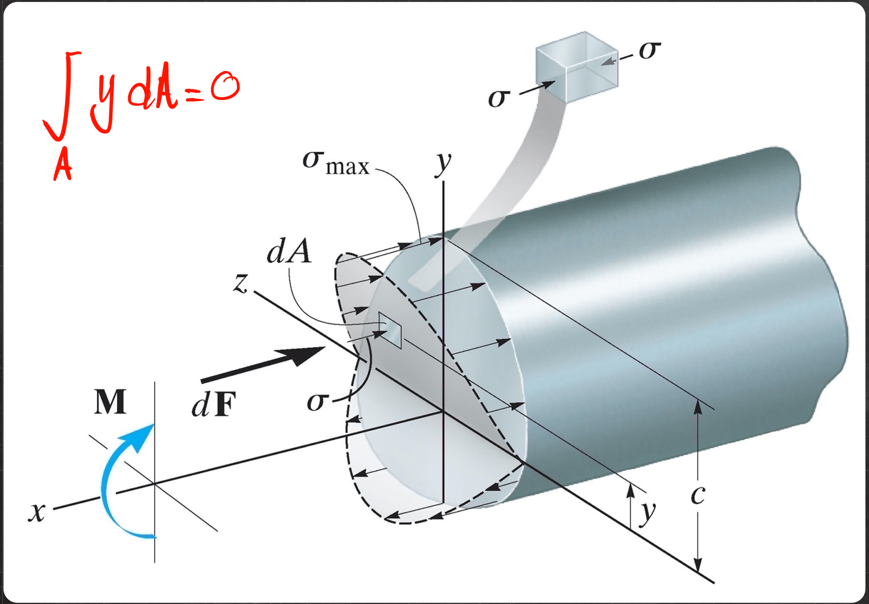
$$= \int_A y\,\sigma\,dA$$

$$= \int_A y\,\frac{y}{c}\,\sigma_{max}\,dA$$

$$= \frac{\sigma_{max}}{c}\int_A y^2\,dA$$

I

BENDING MOMENT



$$M = \int_A y\,dF$$

$$= \int_A y\,\sigma\,dA$$

$$= \int_A y\,\frac{y}{c}\,\sigma_{max}\,dA$$

$$= \frac{\sigma_{max}}{c}\int_A y^2\,dA \quad (I)$$

$$M = \frac{\sigma_{max} I}{c} \Rightarrow \sigma_{max} = \frac{Mc}{I}$$

BENDING MOMENT

$$\sigma_{max} = \frac{Mc}{I}$$

BENDING MOMENT

$$\sigma_{max} = \frac{Mc}{I} \iff \sigma = -\frac{My}{I}$$

BENDING MOMENT

$$\sigma_{max} = \frac{Mc}{I} \iff \sigma = -\frac{My}{I}$$

FLEXURE FORMULA
(either one)

BENDING MOMENT

$$\sigma_{max} = \frac{Mc}{I} \iff \sigma = -\frac{My}{I}$$

FLEXURE FORMULA (either one)

σ_{max} : maximum normal stress that occurs at a point on cross-sectional area farthest away from the neutral axis → distance "C"

M : resultan internal moment determined from the method of sections

I : moment of inertia of the cross-sectional area about the neutral axis

HOW TO COMPUTE I ?

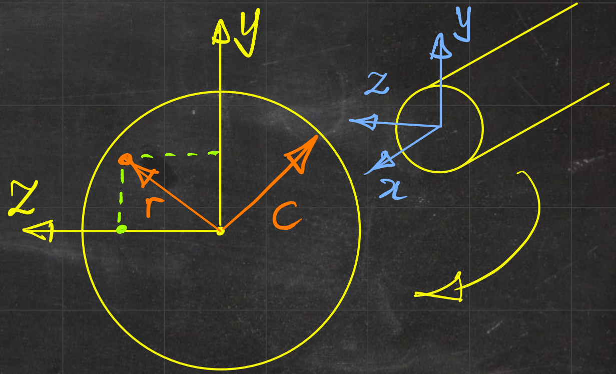
EXAMPLE:

I: moment of inertia of the cross-sectional area about the neutral axis

HOW TO COMPUTE I ?

EXAMPLE:

I: moment of inertia of the cross-sectional area about the neutral axis



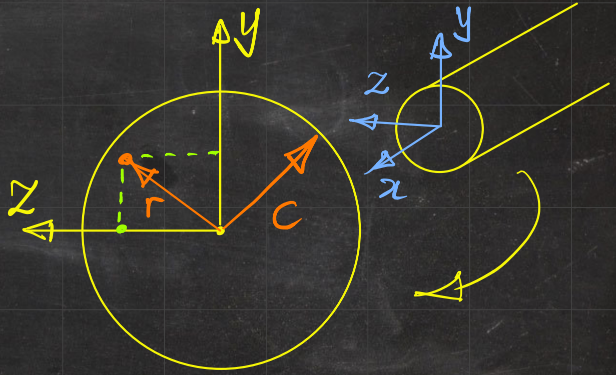
HOW TO COMPUTE I ?

EXAMPLE:

$$I_z = \int y^2 dA$$

w.r.t. z-axis

I: moment of inertia of the cross-sectional area about the neutral axis



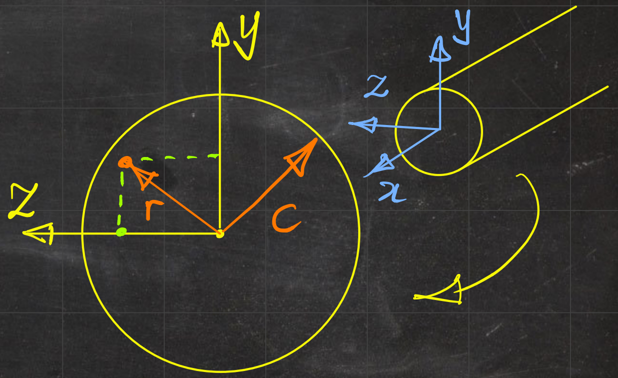
HOW TO COMPUTE I ?

I: moment of inertia of the cross-sectional area about the neutral axis

EXAMPLE:

$$I_z = \int y^2 \, dA \quad y^2 + z^2 = r^2$$

w.r.t. z-axis



HOW TO COMPUTE I ?

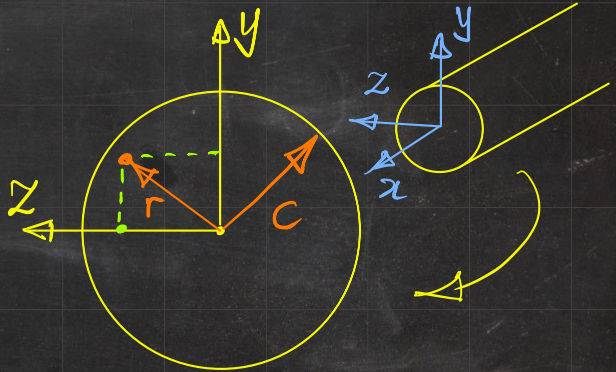
I : moment of inertia of the cross-sectional area about the neutral axis

EXAMPLE :

$$I_z = \int y^2 \, dA \qquad y^2 + z^2 = r^2$$

w.r.t. z-axis

$$= \int_A (r^2 - z^2) \, dA$$



HOW TO COMPUTE I ?

I : moment of inertia of the cross-sectional area about the neutral axis

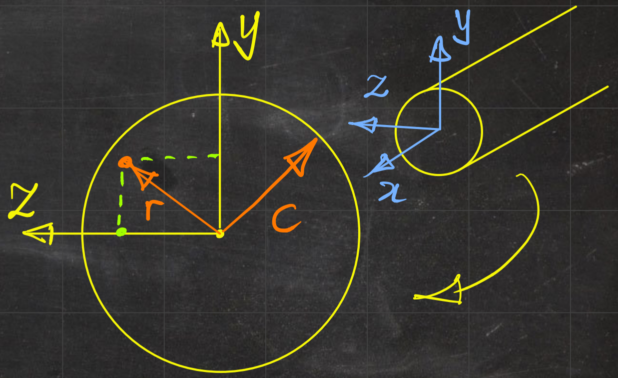
EXAMPLE :

$$I_z = \int y^2 \, dA \qquad y^2 + z^2 = r^2$$

w.r.t. z-axis

$$= \int_A (r^2 - z^2) \, dA$$

$$= \int_A r^2 \, dA - \int_A z^2 \, dA$$



HOW TO COMPUTE I ?

I: moment of inertia of the cross-sectional area about the neutral axis

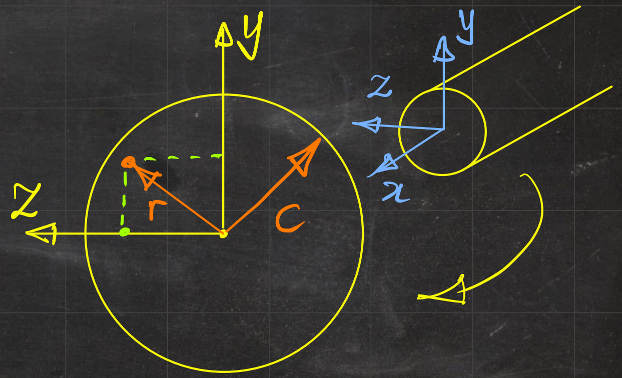
EXAMPLE:

$$I_z = \int y^2 \, dA \qquad y^2 + z^2 = r^2$$

w.r.t. z-axis

$$= \int_A (r^2 - z^2) \, dA$$

$$= \underbrace{\int_A r^2 \, dA}_{J = \frac{\pi}{2} c^4} - \overbrace{\int_A z^2 \, dA}^{I_y}$$



HOW TO COMPUTE I ?

I: moment of inertia of the cross-sectional area about the neutral axis

EXAMPLE:

$$I_z = \int y^2 \, dA \quad \leftarrow \quad y^2 + z^2 = r^2$$

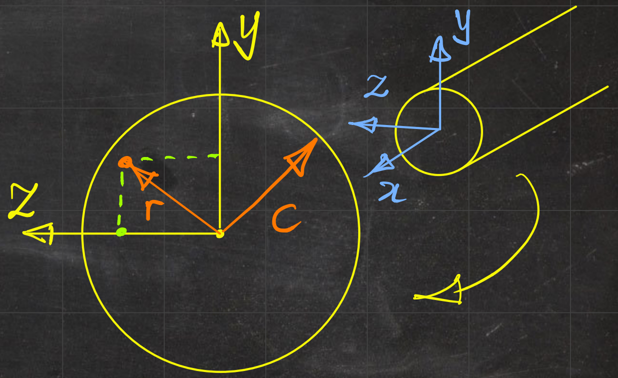
w.r.t. z-axis

$$= \int_A (r^2 - z^2) \, dA$$

$$= \underbrace{\int_A r^2 \, dA}_{J = \frac{\pi}{2} c^4} - \underbrace{\int_A z^2 \, dA}_{I_y}$$

SYMMETRY

$$I_y = I_z$$



HOW TO COMPUTE I ?

I : moment of inertia of the cross-sectional area about the neutral axis

EXAMPLE:

$$I_z = \int y^2 \, dA \qquad y^2 + z^2 = r^2$$

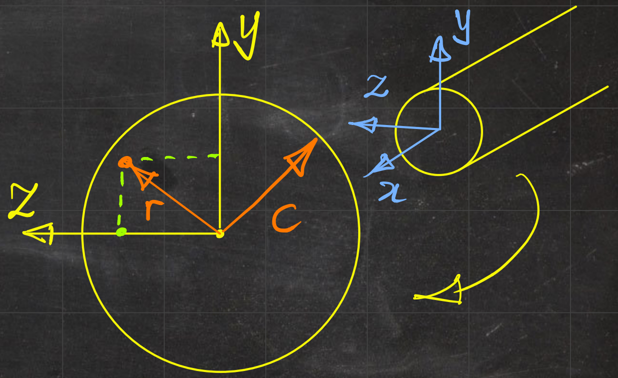
w.r.t. z-axis

$$= \int_A (r^2 - z^2) \, dA$$

$$= \underbrace{\int_A r^2 \, dA}_{J = \frac{\pi}{2} c^4} - \overbrace{\int_A z^2 \, dA}^{I_y}$$

SYMMETRY

$$I_y = I_z$$

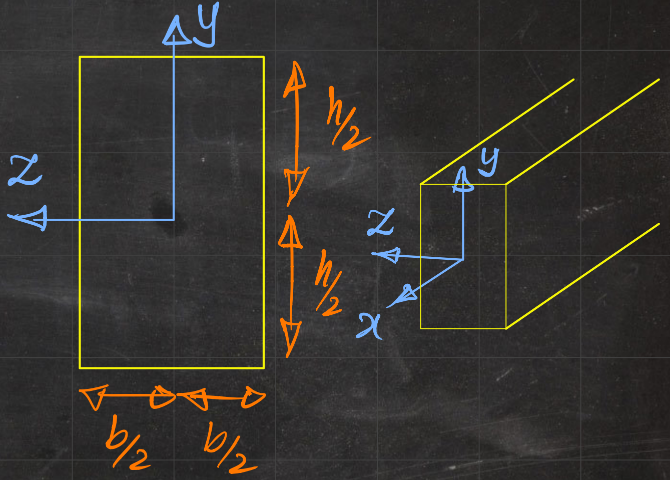


$$\Rightarrow 2 I_z = J \Rightarrow I_z = \frac{\pi}{4} c^4$$

HOW TO COMPUTE I ?

I : moment of inertia of the cross-sectional area about the neutral axis

EXAMPLE :



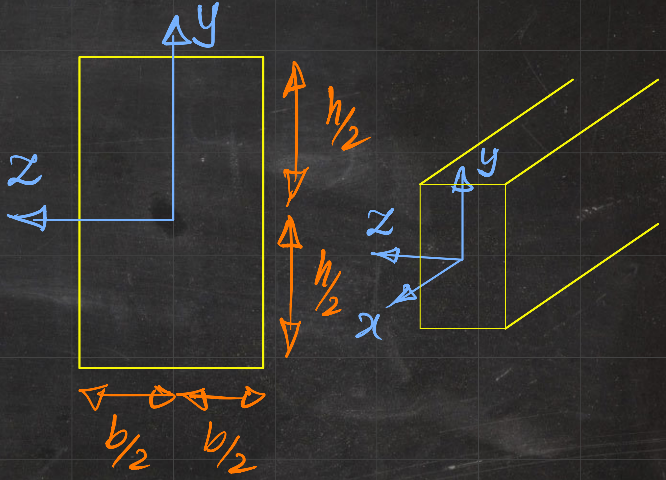
How to compute I ?

EXAMPLE:

$$I_z = \int y^2 \, dA$$

w.r.t. z-axis

I: moment of inertia of the cross-sectional area about the neutral axis



HOW TO COMPUTE I ?

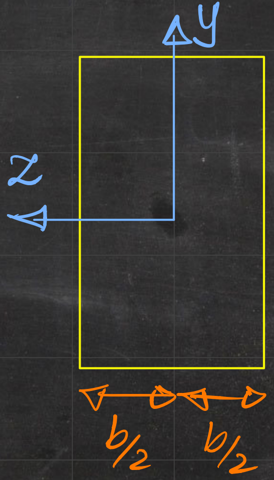
I : moment of inertia of the cross-sectional area about the neutral axis

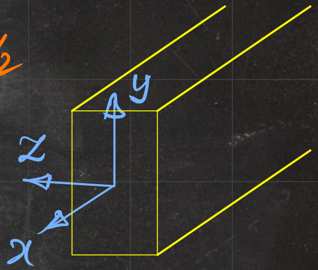
EXAMPLE :

$$I_z = \int y^2 \, dA$$

w.r.t. z-axis

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \, dA \qquad (dA = b\,dy)$$





HOW TO COMPUTE I ?

I : moment of inertia of the cross-sectional area about the neutral axis

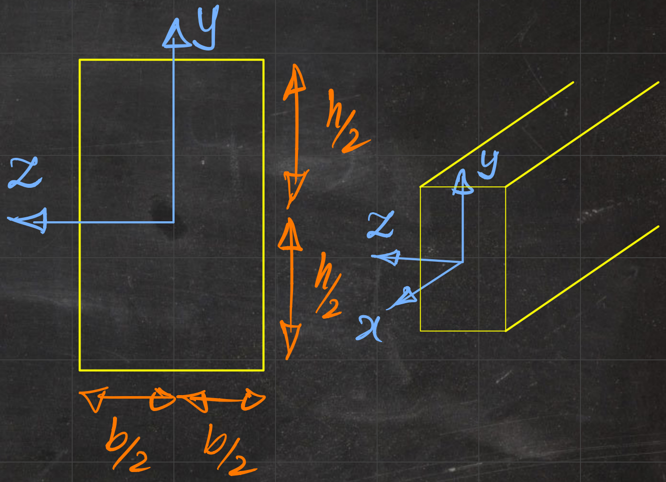
EXAMPLE :

$$I_z = \int y^2 \, dA$$

w.r.t. z-axis

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \, dA \qquad (dA = b\,dy)$$

$$= 2b \int_0^{h/2} y^2 \, dA$$



HOW TO COMPUTE I ?

I: moment of inertia of the cross-sectional area about the neutral axis

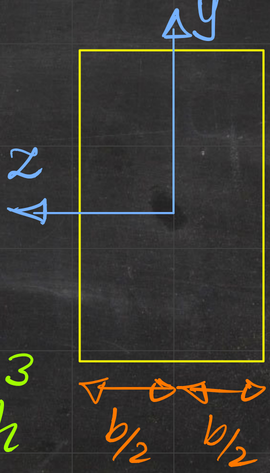
EXAMPLE:

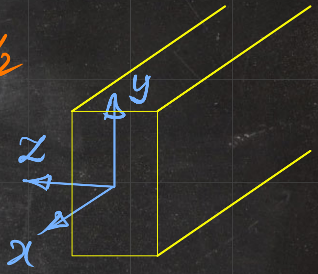
$$I_z = \int y^2 dA$$

w.r.t. z-axis

$$= \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \, dA \qquad (dA = b\,dy)$$

$$= 2b \int_0^{h/2} y^2 dA \Rightarrow I_z = \frac{1}{12} b h^3$$

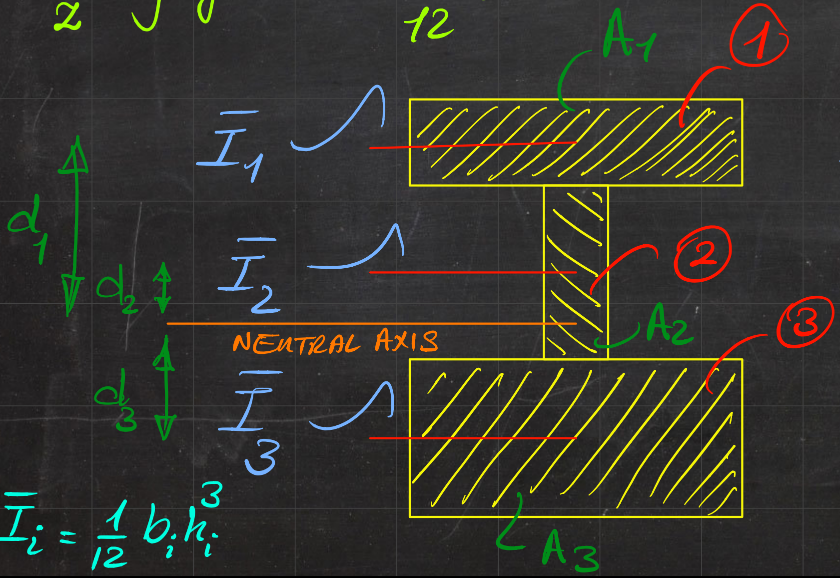




How to compute I ?

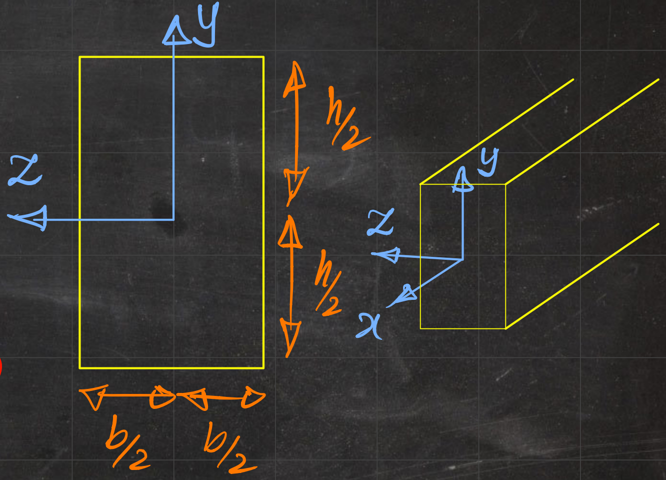
Example:

$$I_z = \int y^2 dA = \frac{1}{12} b h^3$$



$$\bar{I}_i = \frac{1}{12} b_i h_i^3$$

I: moment of inertia of the cross-sectional area about the neutral axis

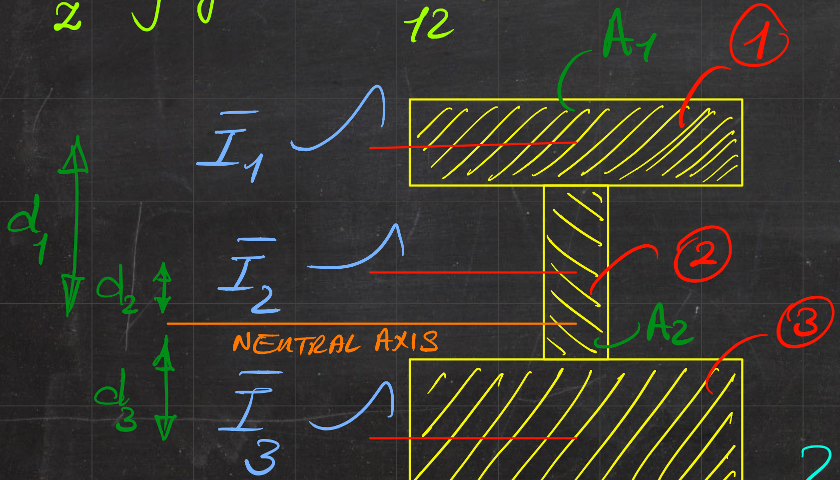


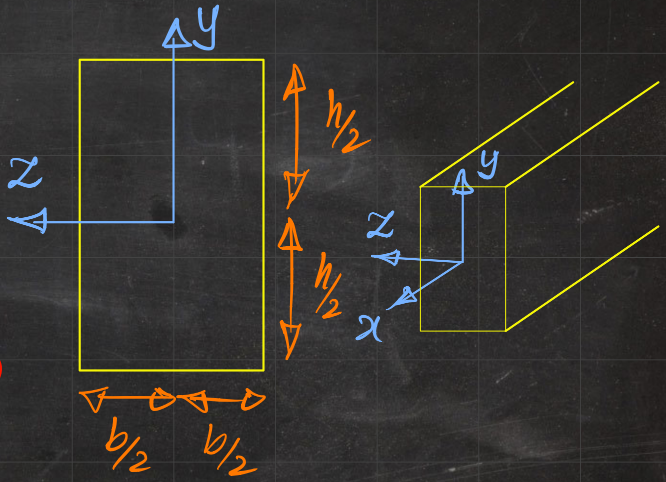
How to compute I ?

EXAMPLE:

$$I_z = \int y^2 \, dA = \frac{1}{12} b h^3$$

I: moment of inertia of the cross-sectional area about the neutral axis





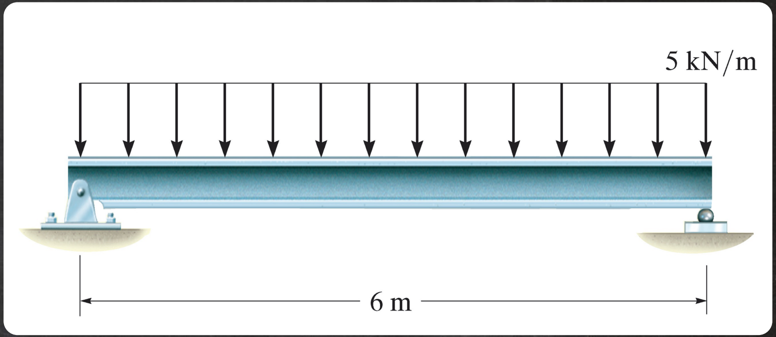
$$\bar{I}_i = \frac{1}{12} b_i h_i^3$$

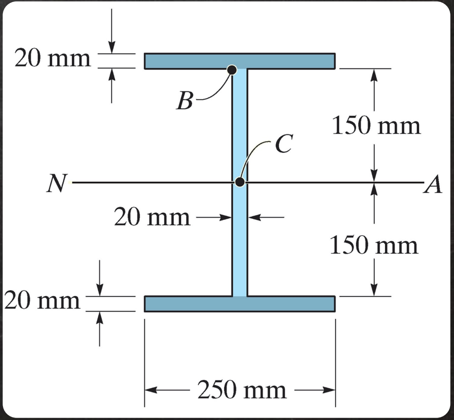
2▷ $I = \sum \bar{I}_i + A_i d_i^2$

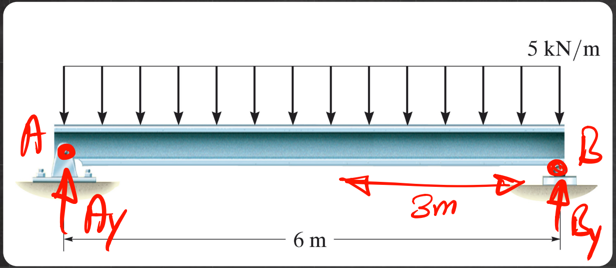
Exercise 1 . [similar to ... P. 294 ... 6.12]

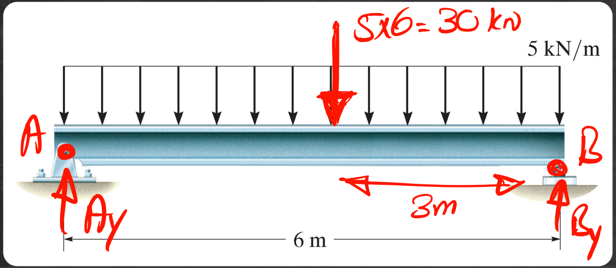
THE SIMPLY SUPPORTED BEAM SHOWN IN THE FIGURE HAS THE CROSS-SECTIONAL AREA GIVEN.

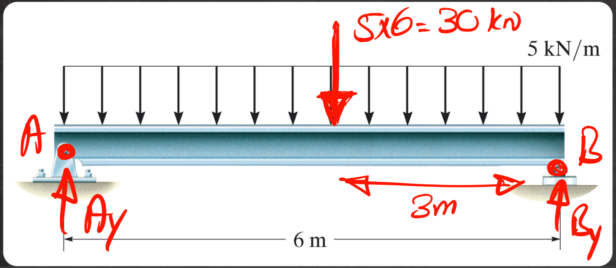
DETERMINE THE ABSOLUTE MAXIMUM BENDING STRESS IN THE BEAM AND DRAW THE STRESS DISTRIBUTION OVER THE CROSS SECTION AT THIS LOCATION.






5 kN/m
A
B
Ay
By
3m
6 m

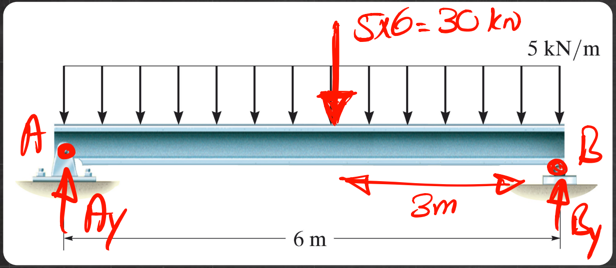

5x6=30 kN
5 kN/m
A
B
3m
Ay
By
6 m



$$+\uparrow \sum F_y = 0 \Rightarrow +A_y + B_y - 5\times 6 = 0$$

$$\circlearrowleft + \sum M_A = 0 \Rightarrow +B_y \times 6 - 30\times 30 = 0$$

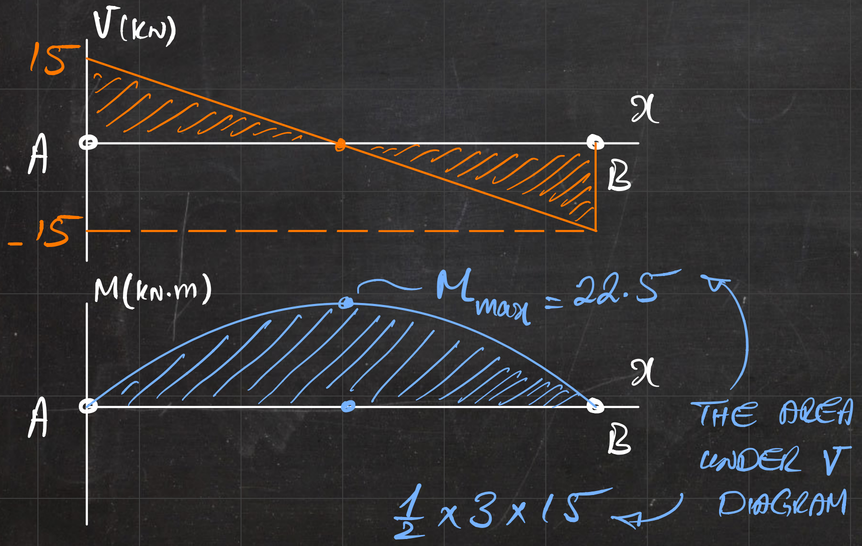
$$\Rightarrow A_y = 15 \quad , \quad B_y = 15$$

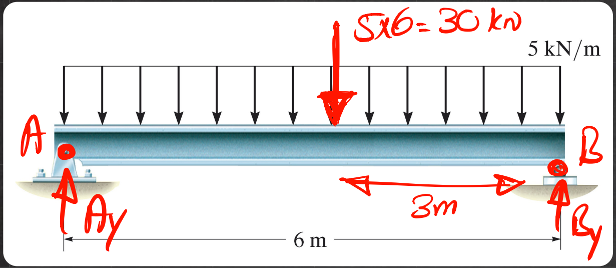


$+\uparrow \sum F_y = 0 \quad \Rightarrow +A_y + B_y - 5\times 6 = 0$

$\circlearrowleft + \sum M_A = 0 \quad \Rightarrow +B_y \times 6 - 30\times 30 = 0$

$\Rightarrow A_y = 15 \quad , \quad B_y = 15$

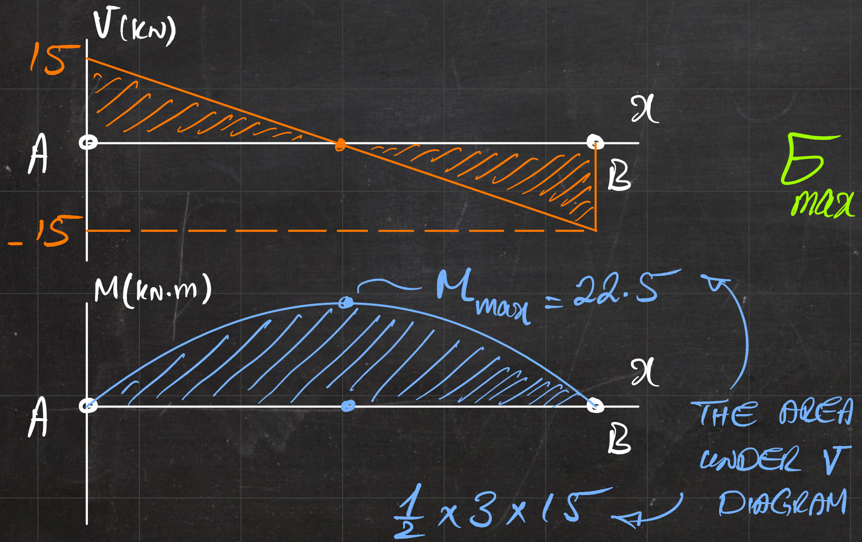




$$+\uparrow \sum F_y = 0 \;\Rightarrow\; +A_y + B_y - 5\times 6 = 0$$

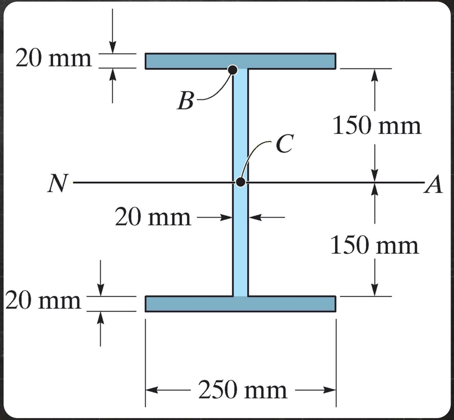
$$\circlearrowleft + \sum M_A = 0 \;\Rightarrow\; +B_y \times 6 - 30\times 30 = 0$$

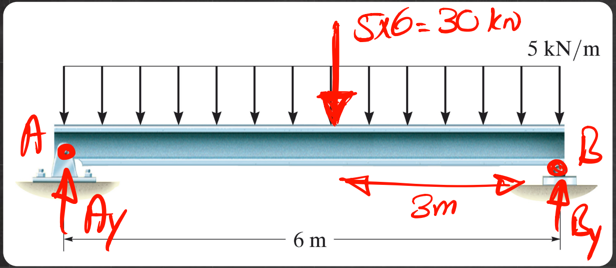
$$\Rightarrow A_y = 15 \;,\; B_y = 15$$



$$\sigma_{max} = \frac{Mc}{I}$$

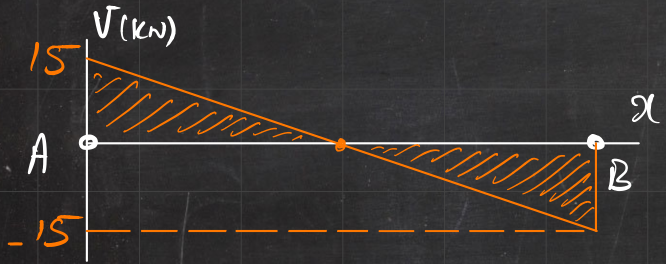
$\hookrightarrow M \rightarrow$ INTERNAL MOMENT $\rightarrow M_{max} = 22.5$

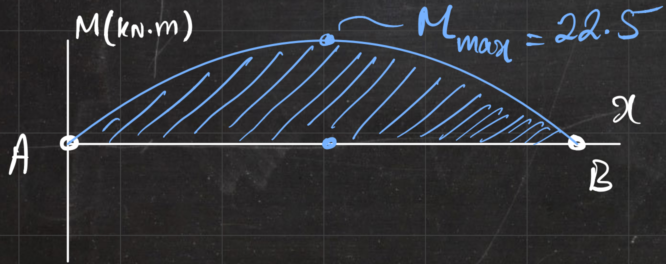


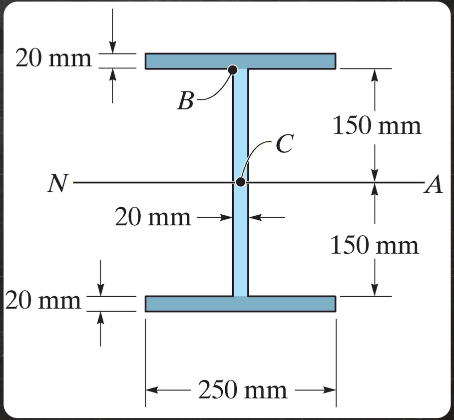


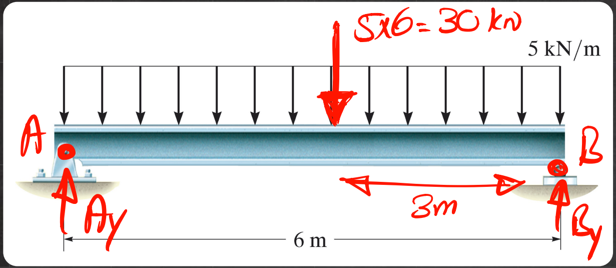
$$I = \sum_{i=1}^{3} \bar{I}_i + A_i d_i^2$$

NEUTRAL AXIS PASSES THROUGH C DUE TO SYMMETRY





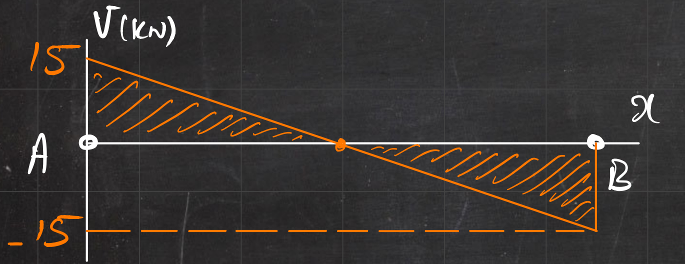


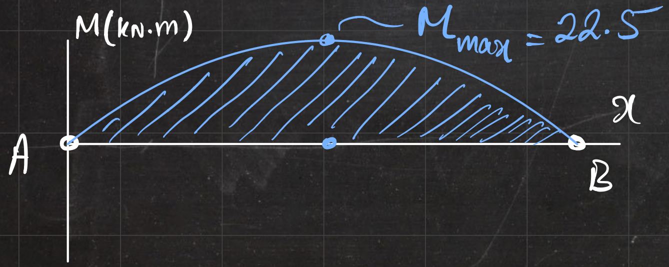


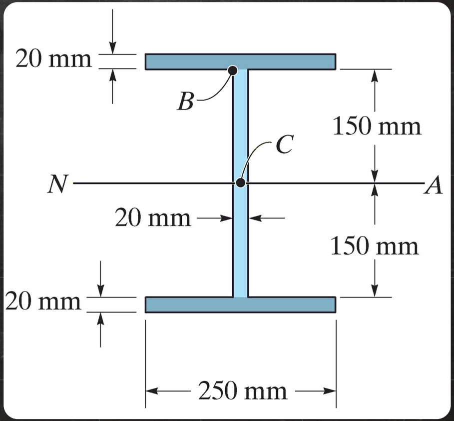
$$I = \sum_{i=1}^{3} \bar{I}_i + A_i d_i^2$$

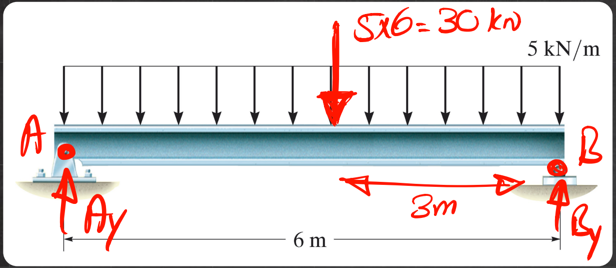
NEUTRAL AXIS PASSES THROUGH C DUE TO SYMMETRY

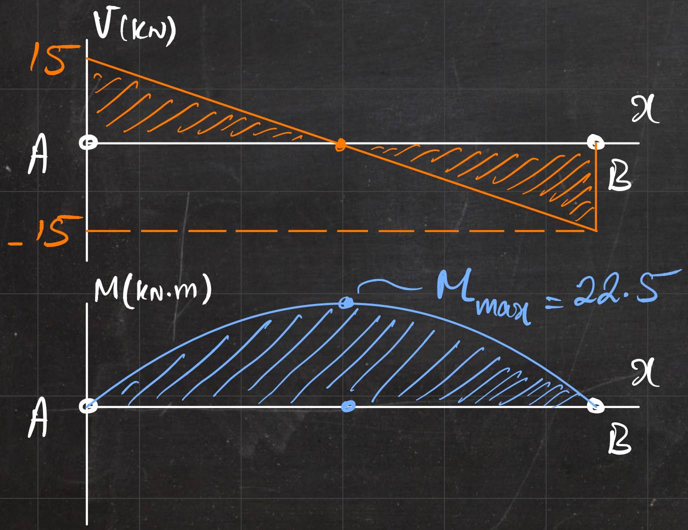
$$= \frac{1}{12} \times 0.02 \times 0.3^3 + 2\left(\frac{1}{12} \times 0.25 \times 0.02^3 + 0.25 \times 0.02 \times 0.16^2\right)$$











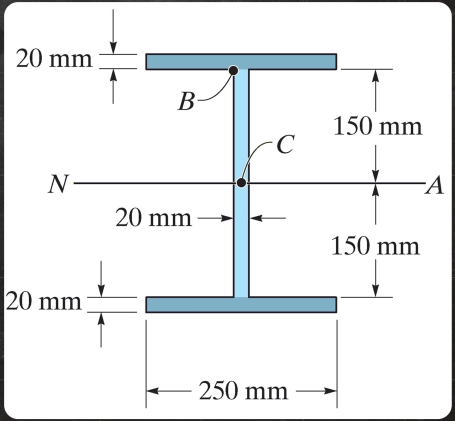
$$I = \sum_{i=1}^{3} \bar{I}_i + A_i d_i^2$$

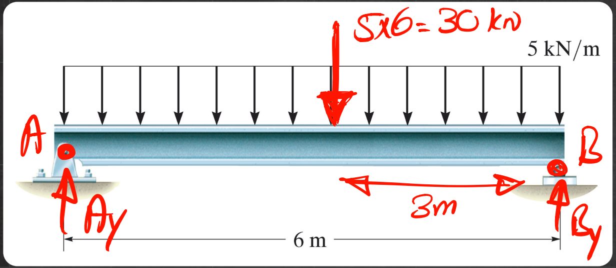
NEUTRAL AXIS PASSES THROUGH C DUE TO SYMMETRY

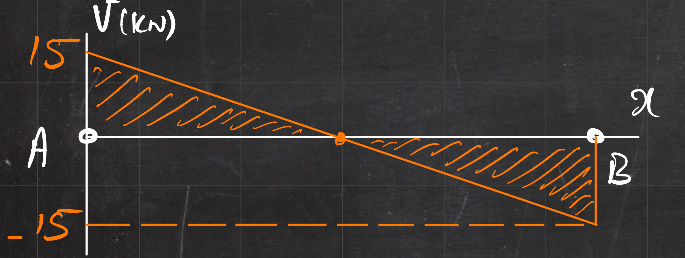
$$= \frac{1}{12} \times 0.02 \times 0.3^3 + 2\left(\frac{1}{12} \times 0.25 \times 0.02^3 + 0.25 \times 0.02 \times 0.16^2\right)$$

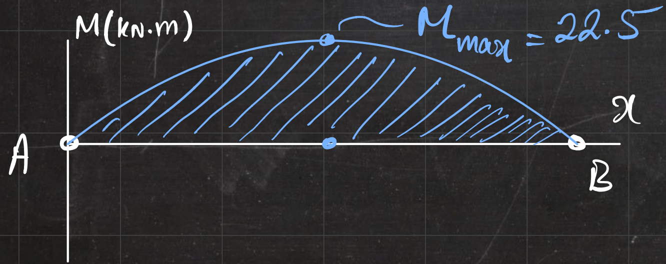
$$\Downarrow$$

$$I = 301.3 \times 10^{-6}\ m^4$$









$$I = \sum_{i=1}^{3} \bar{I}_i + A_i d_i^2$$

← NEUTRAL AXIS PASSES THROUGH C DUE TO SYMMETRY

$$= \frac{1}{12} \times 0.02 \times 0.3^3 + 2\left(\frac{1}{12} \times 0.25 \times 0.02^3 + 0.25 \times 0.02 \times 0.16^2\right)$$

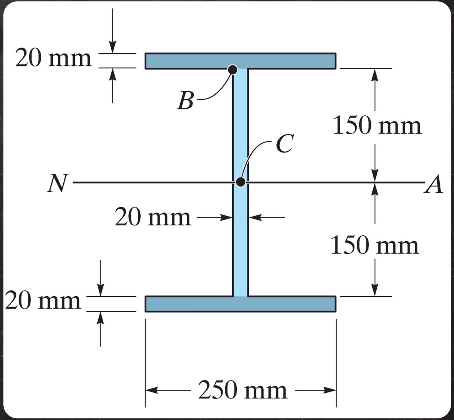
⇓

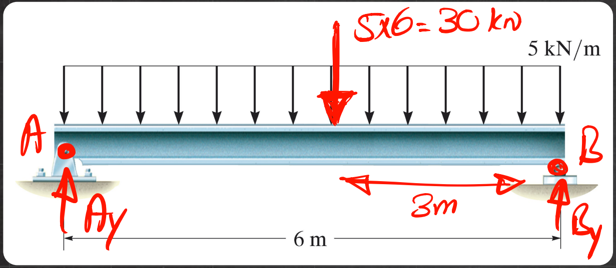
$$I = 301.3 \times 10^{-6}\ m^4$$

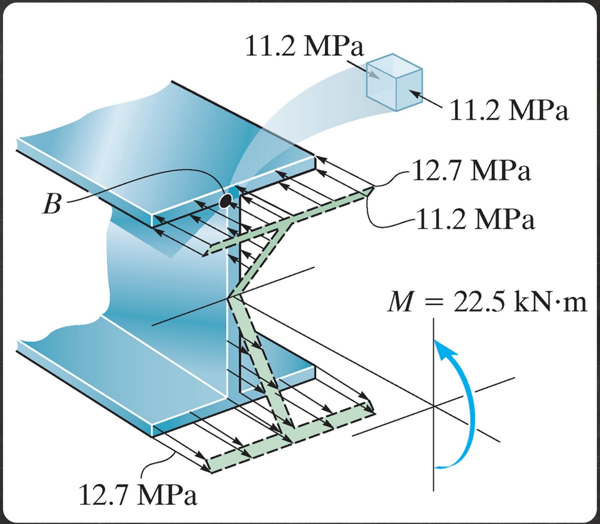
⇓

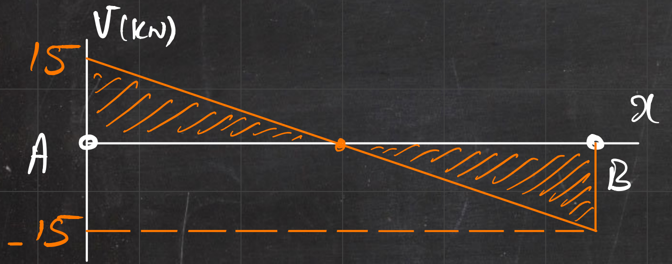
$$\sigma_{max} = \frac{M_{max}\, c}{I} = 12.7\ \text{MPa}$$

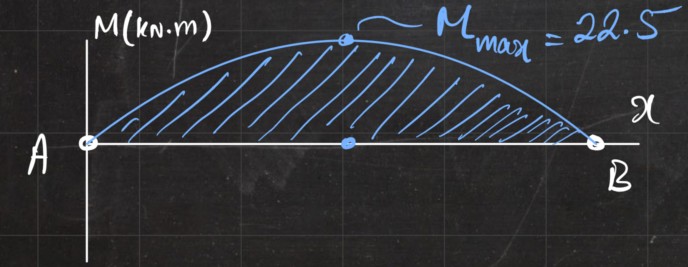
(c ↙ 0.17)



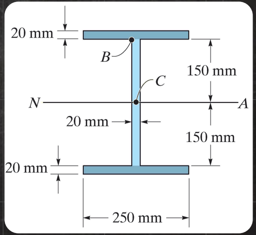








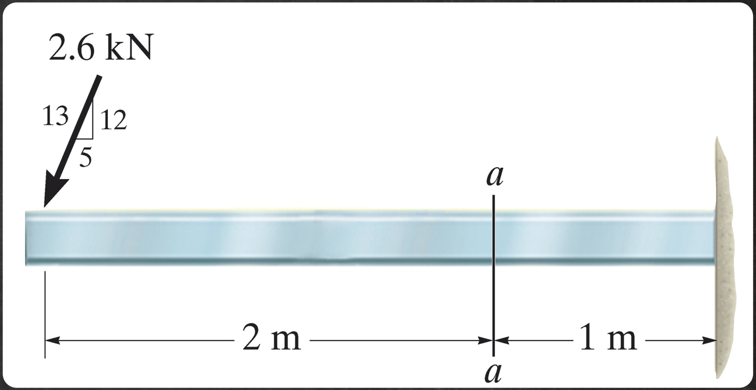
$$\sigma_{max} = \frac{M_{max}\, c}{I} = 12.7 \text{ MPa}$$

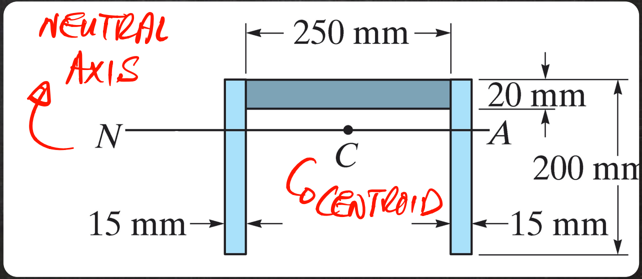


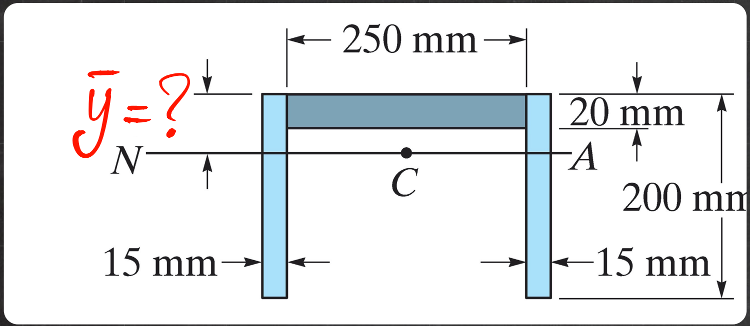
Exercise 2 . [similar to … P. 295 … 6.13]

THE BEAM SHOWN IN THE FIGURE HAS THE CROSS-SECTIONAL AREA GIVEN.

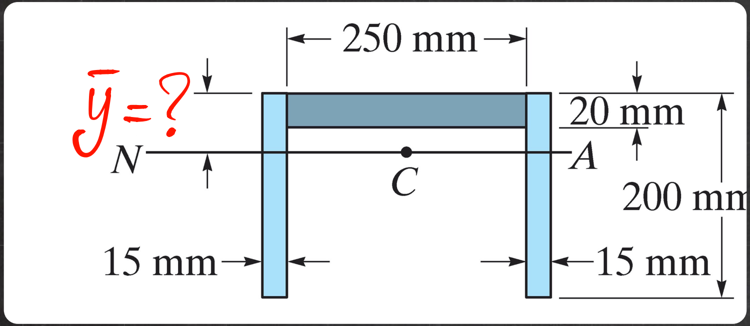
DETERMINE THE MAXIMUM BENDING STRESS THAT OCCURS AT SECTION a-a.



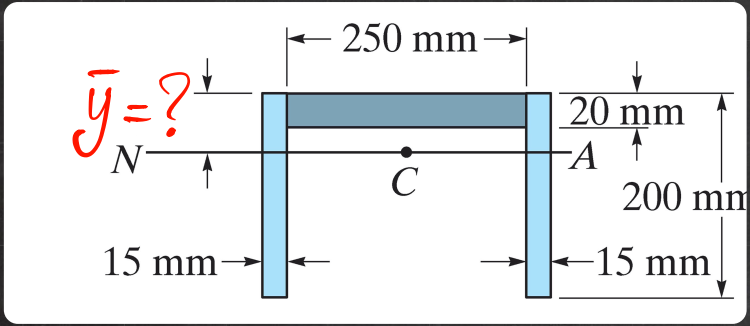




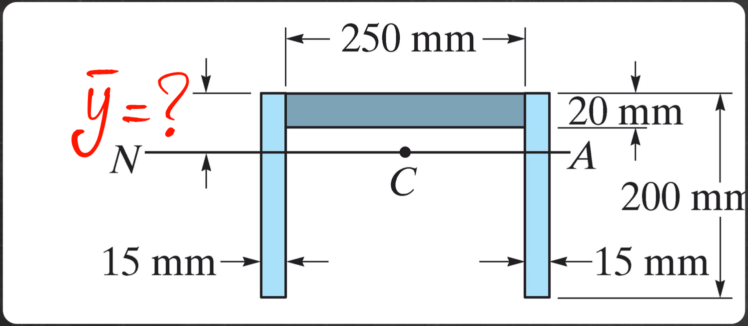
250 mm
$\bar{y}$ = ?
20 mm
N
A
C
200 mm
15 mm
15 mm

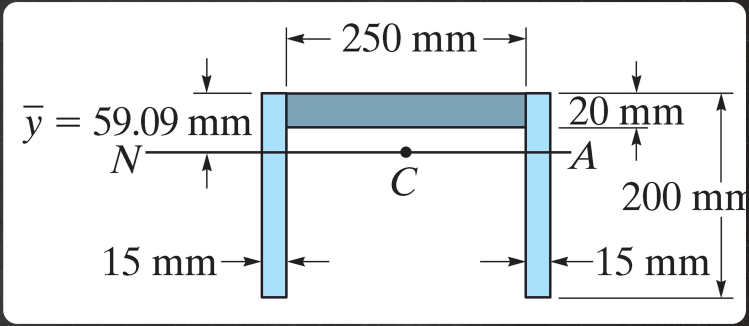


$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$



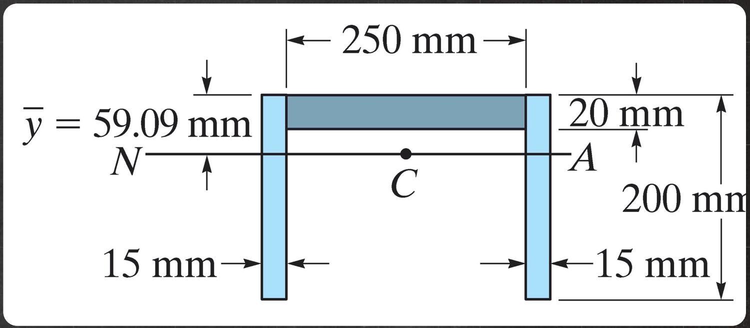
$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{2 \times 0.1 \times 0.2 \times 0.015 + 0.01 \times 0.02 \times 0.25}{2 \times 0.2 \times 0.015 + 0.02 \times 0.25}$$

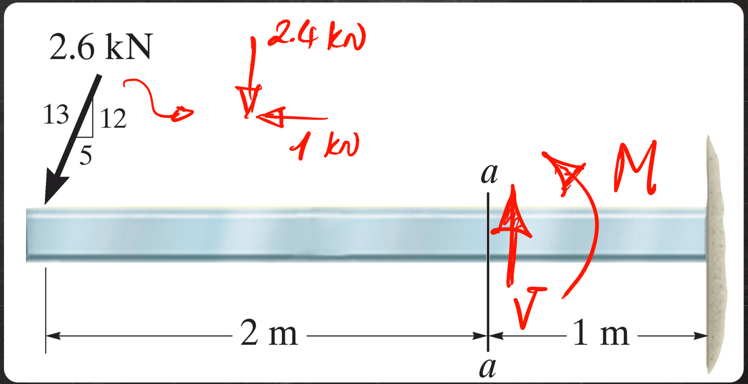




$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{2 \times 0.1 \times 0.2 \times 0.015 + 0.01 \times 0.02 \times 0.25}{2 \times 0.2 \times 0.015 + 0.02 \times 0.25}$$

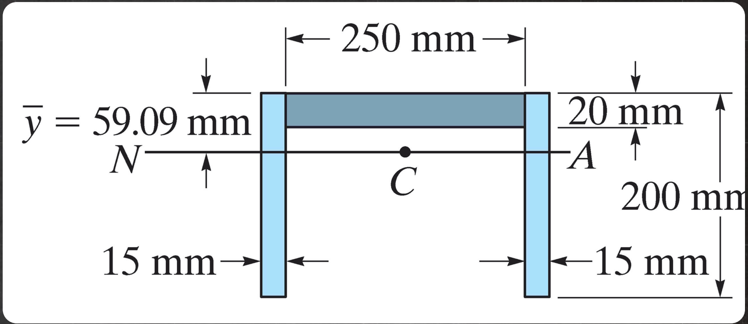
$$\Rightarrow \bar{y} = 0.05909 \text{ m} = 59.09 \text{ mm}$$

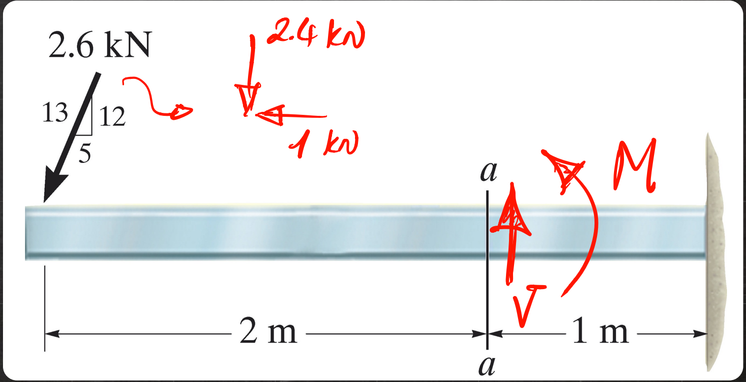

250 mm
$\bar{y}$ = 59.09 mm
20 mm
N
A
C
200 mm
15 mm
15 mm


2.6 kN
13
12
5
2.4 kN
1 kN
a
M
V
2 m
1 m
a

$\circlearrowleft + \sum M = 0$

$+M + 2.4 \times 2 + 1 \times 0.05909 = 0$

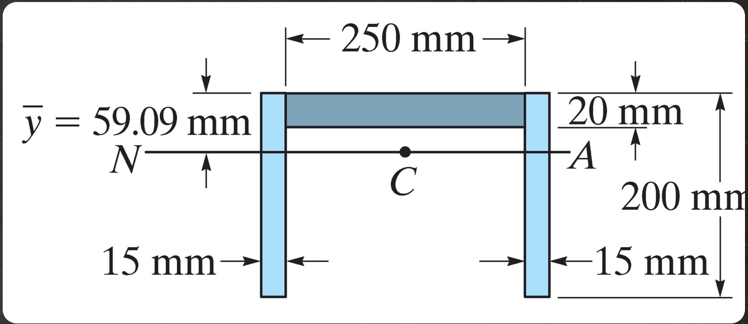


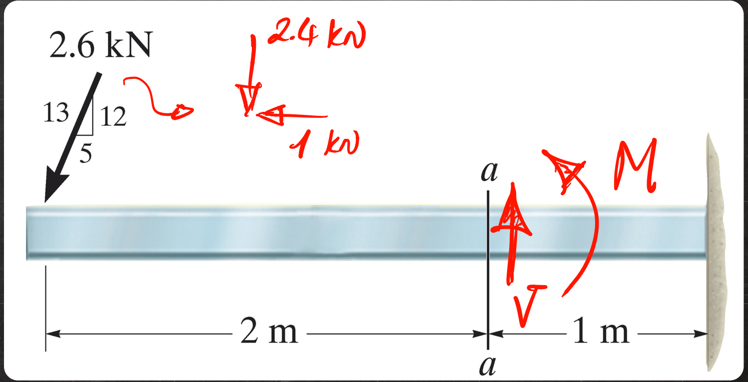


$\circlearrowleft + \sum M = 0$

$+M + 2.4 \times 2 + 1 \times 0.05909 = 0$

$\Rightarrow M = -4.859 \text{ kN.m}$



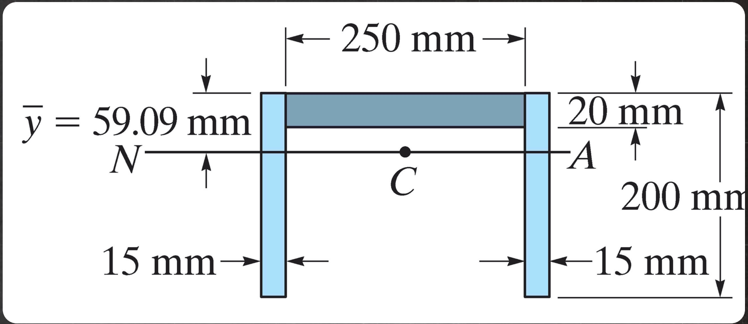


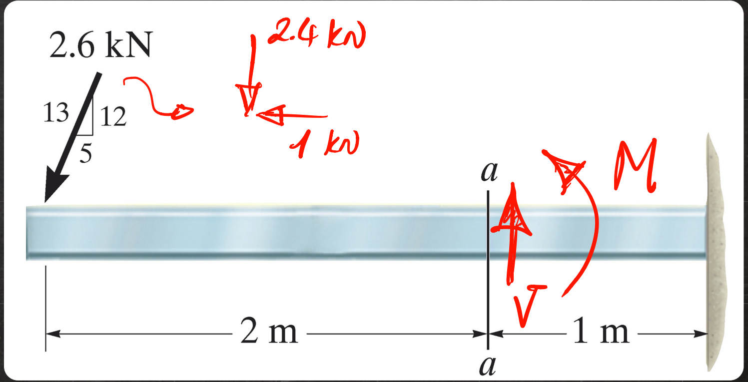
$\circlearrowleft + \sum M = 0$

$+M + 2.4 \times 2 + 1 \times 0.05909 = 0$

$\Rightarrow M = -4.859 \text{ kN.m}$

$I = \sum \bar{I}_i + A_i d_i^2$





$\circlearrowleft + \sum M = 0$

$+M + 2.4 \times 2 + 1 \times 0.05909 = 0$

$\Rightarrow M = -4.859 \text{ kN.m}$

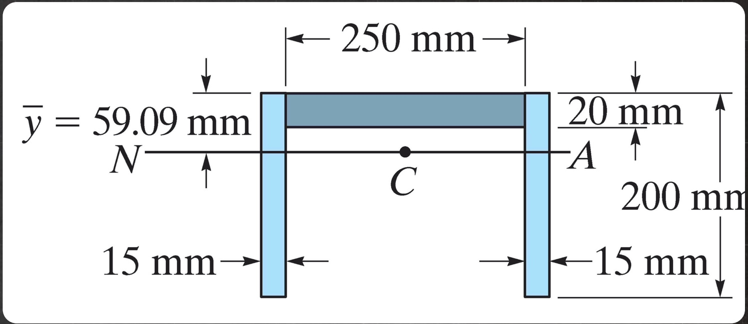
$I = \sum \bar{I}_i + A_i d_i^2$

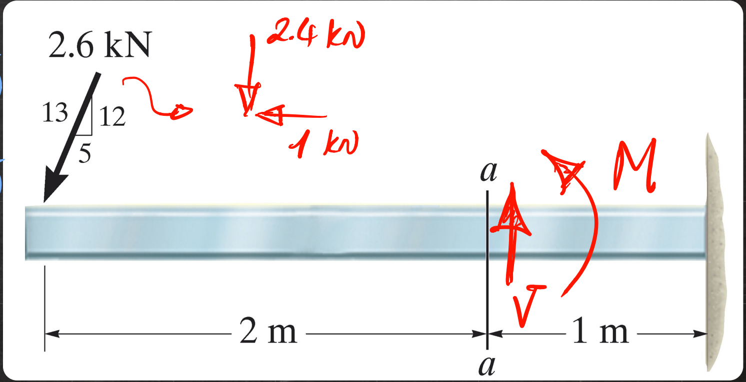
$= \frac{1}{12} \times 0.25 \times 0.02^3 + 0.25 \times 0.02 \times (0.059 - 0.01)^2$

$+ \frac{1}{12} \times 0.015 \times 0.2^3 + 0.015 \times 0.2 \times (0.1 - 0.059)^2$

$+ \frac{1}{12} \times 0.015 \times 0.2^3 + 0.015 \times 0.2 \times (0.1 - 0.059)^2$

identical





$$\circlearrowleft^{+} \sum M = 0$$

$$+M + 2.4 \times 2 + 1 \times 0.05909 = 0$$

$$\Rightarrow M = -4.859 \text{ kN.m}$$

$$I = \sum \bar{I}_i + A_i d_i^2$$

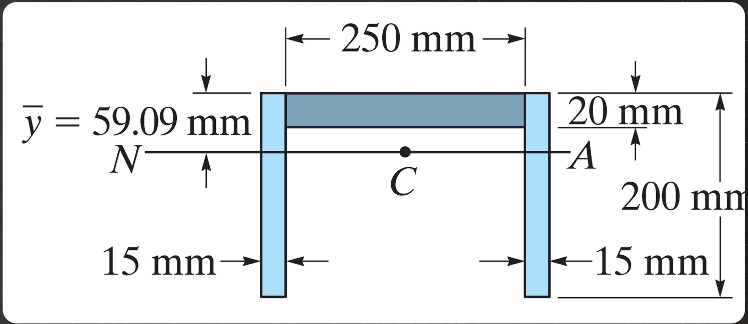
$$= \frac{1}{12} \times 0.25 \times 0.02^3 + 0.25 \times 0.02 \times (0.059 - 0.01)^2$$

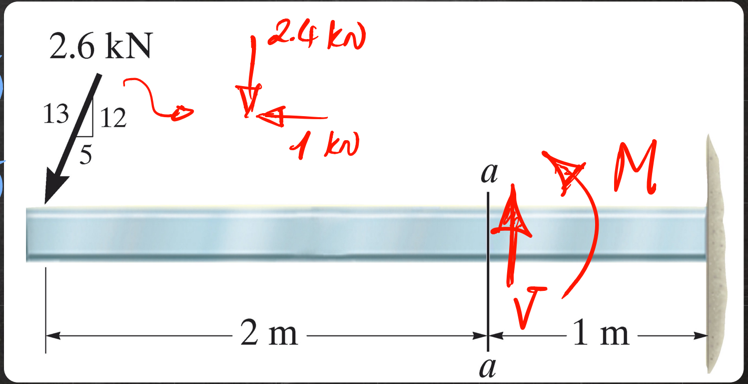
$$+ \frac{1}{12} \times 0.015 \times 0.2^3 + 0.015 \times 0.2 \times (0.1 - 0.059)^2$$

identical

$$+ \frac{1}{12} \times 0.015 \times 0.2^3 + 0.015 \times 0.2 \times (0.1 - 0.059)^2$$

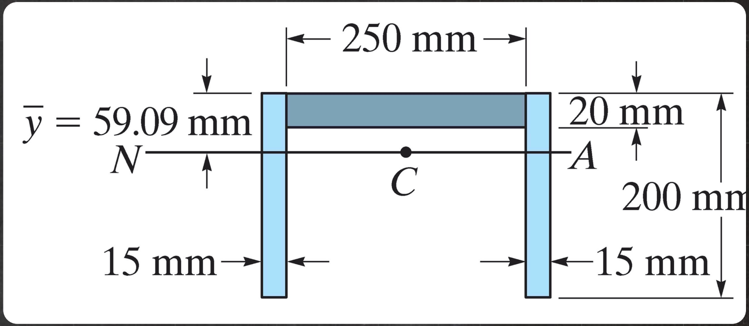
$$\Rightarrow I = 42.26 \times 10^{-6} \text{ m}^4$$

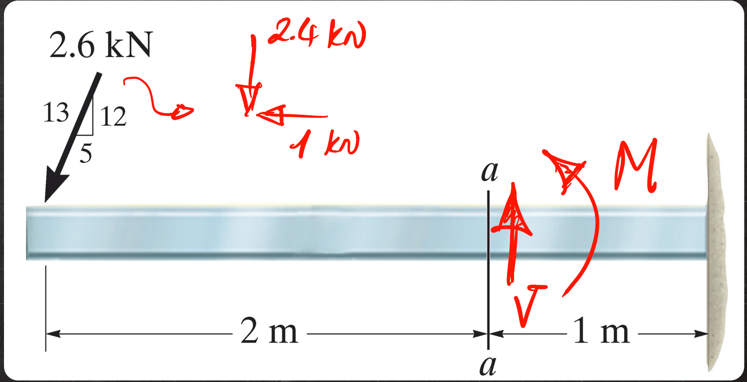




$\Rightarrow M = -4.859 \text{ kN.m}$

$\Rightarrow I = 42.26 \times 10^{-6} \text{ m}^4$



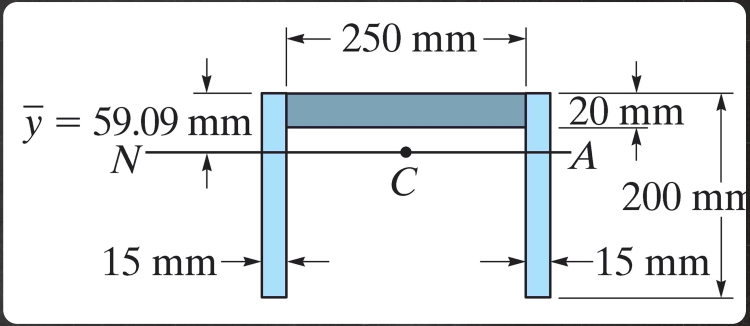


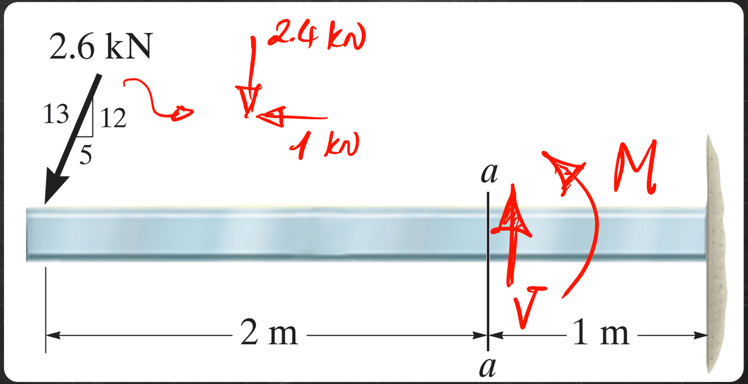
$\Rightarrow M = -4.859 \text{ kN.m}$

$\Rightarrow I = 42.26 \times 10^{-6} \text{ m}^4$

$\sigma_{max} = \frac{Mc}{I}$

$c = 0.2 - 0.059 = 0.141$



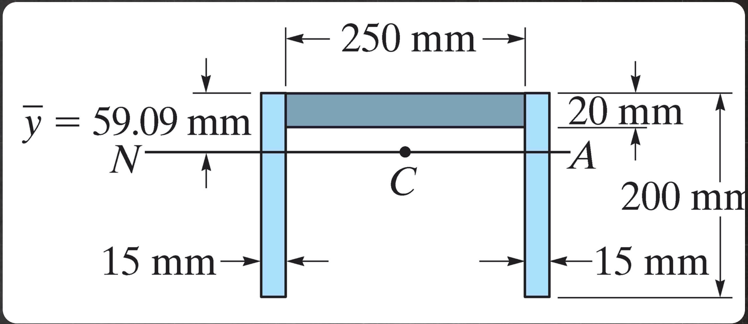


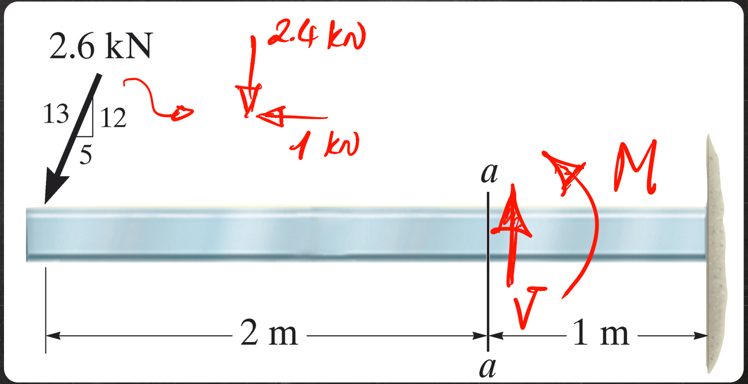
$\Rightarrow M = -4.859 \text{ kN.m}$

$\Rightarrow I = 42.26 \times 10^{-6} \text{ m}^4$

$\sigma_{max} = \dfrac{Mc}{I}$ $\quad c = 0.2 - 0.059 = 0.141$

$\Rightarrow \sigma_{max} = 16.2 \text{ MPa}$





$\Rightarrow M = -4.859$ kN.m

$\Rightarrow I = 42.26 \times 10^{-6}\ m^4$

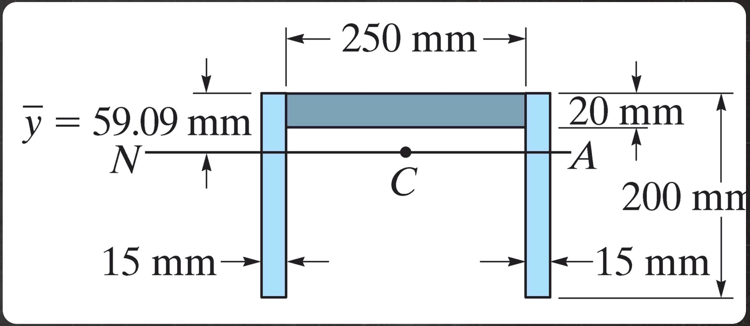
$\sigma_{max} = \frac{Mc}{I}$

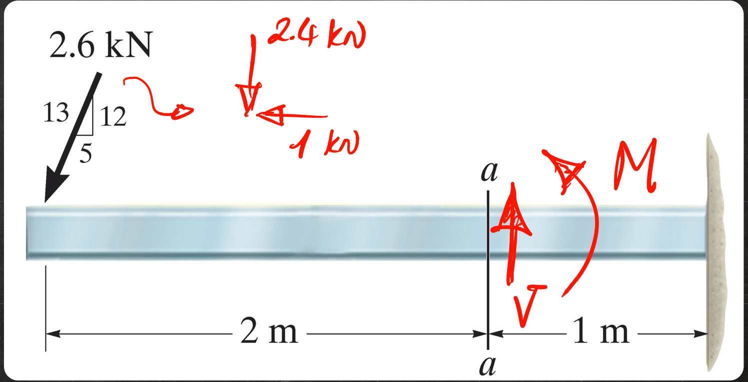
$C = 0.2 - 0.059 = 0.141$

BOTTOM HALF

COMPRESSION

$\Rightarrow \sigma_{max} = 16.2$ MPa





MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

Bending iii

Sections … 6.3 – 6.4

Chap. 6

[Hibbeler 9th edition]

MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

18