MECHANICS AND MATERIALS I

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11

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Bilkent University

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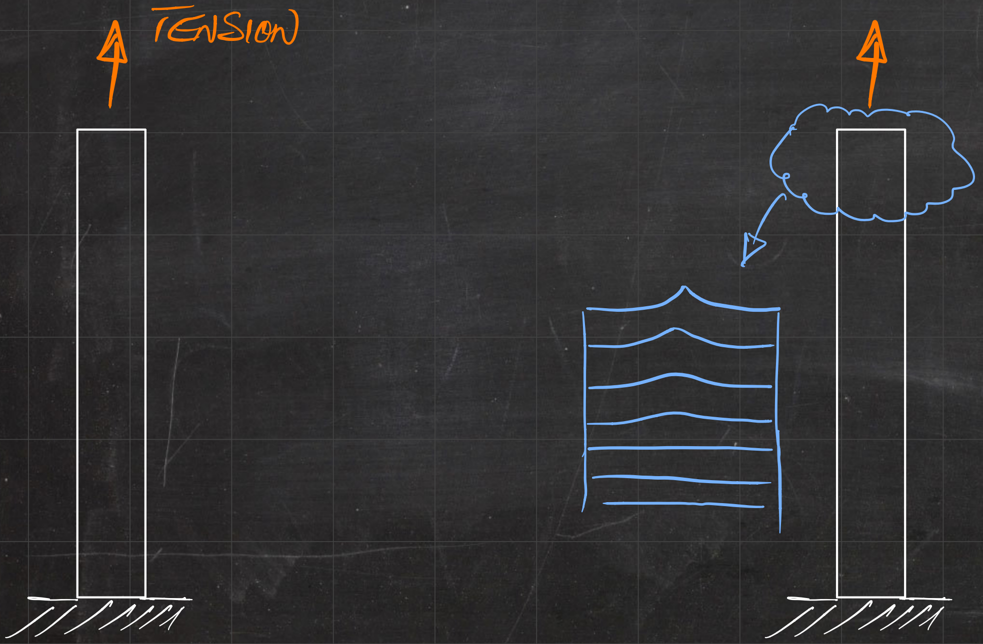
Axial loading

Sections ... 4.1 – 4.5 ... 4.7 – 4.8

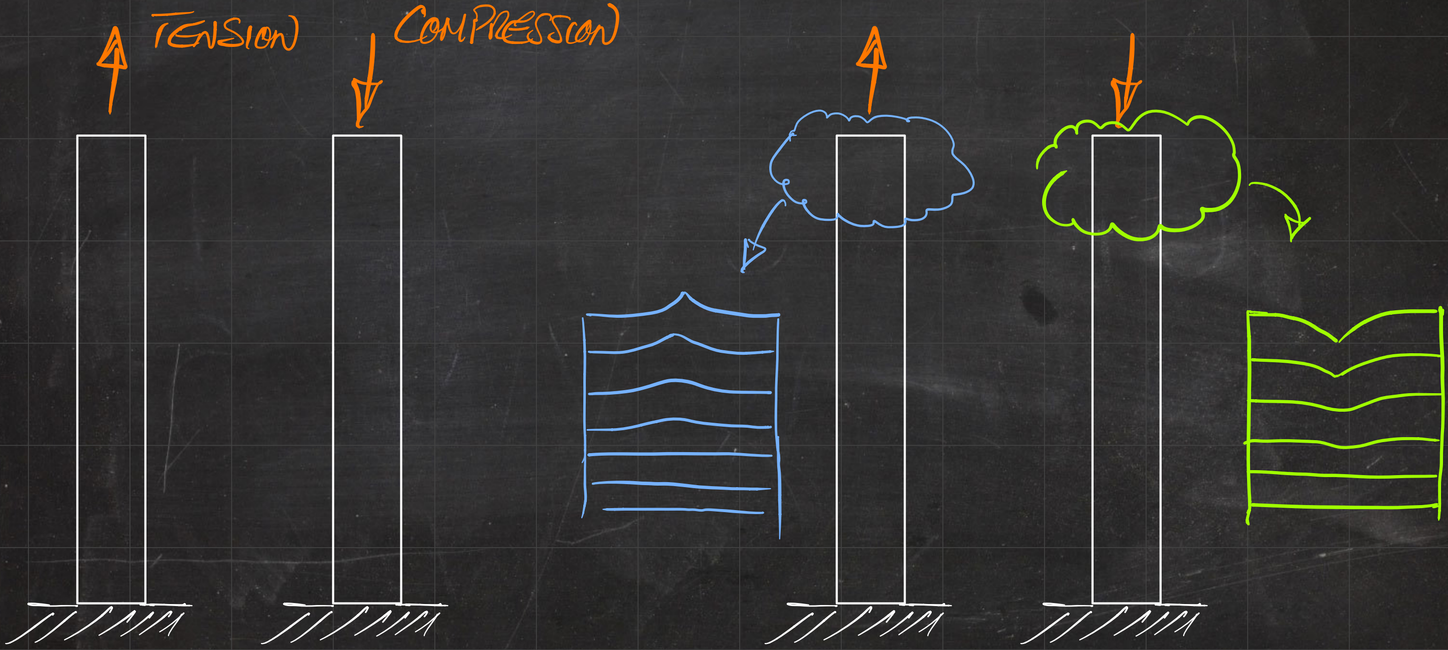
Chap. 4

[Hibbeler 9th edition]

AXIAL LOAD

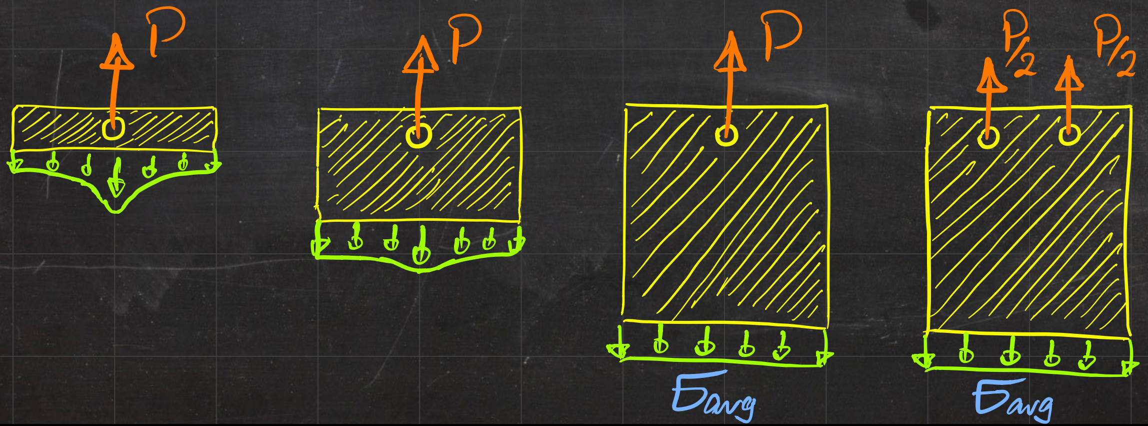


AXIAL LOAD



SAINT-VENANT'S PRINCIPLE (1855)

↳ Stress and strain produced at points in a body sufficiently away (far) from the load, is the same for any statically equivalent resultant applied in the same region.



$$\sigma_{avg} = \frac{P}{A}$$

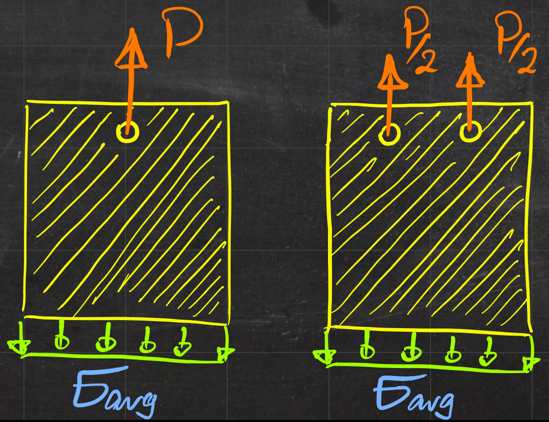
SAINT-VENANT'S PRINCIPLE (1855)

↳ Stress and strain produced at points in a body sufficiently away (far) from the load, is the same for any statically equivalent resultant applied in the same region.

FROM NOW ON ASSUME,

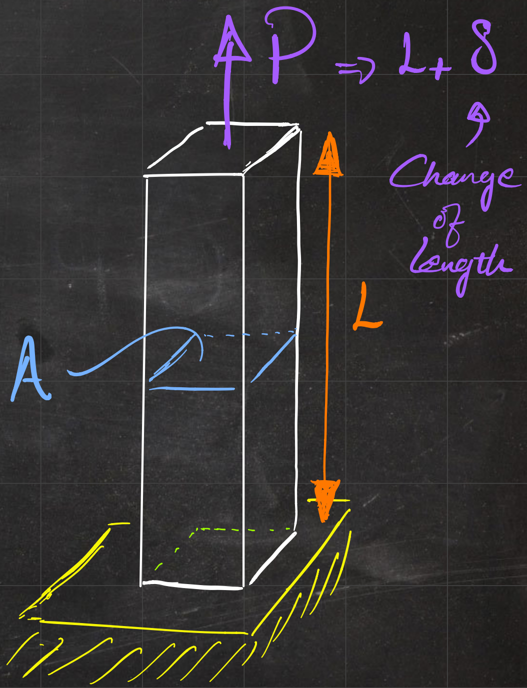
WE DEAL WITH SECTIONS SUFFICIENTLY FAR!

(UNLESS SPECIFIED OTHERWISE) $\Rightarrow \sigma = \sigma_{avg} = \dfrac{P}{A}$



$$\sigma_{avg} = \frac{P}{A}$$

ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER



ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

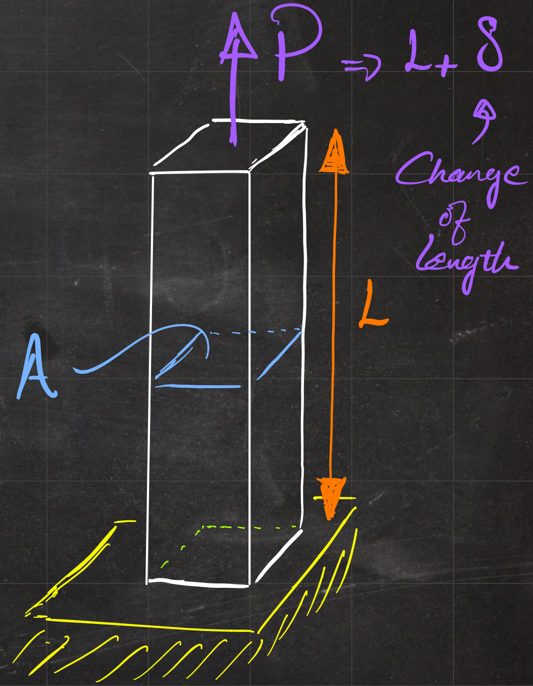
Deformation ⟷ ? ⟷ Loading

Chap. 2 ↕ Strain

Chap. 1 ↕ Stress

Strain ⟷ Stress

Chap. 3



ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

Deformation ⟷ (?) ⟷ Loading

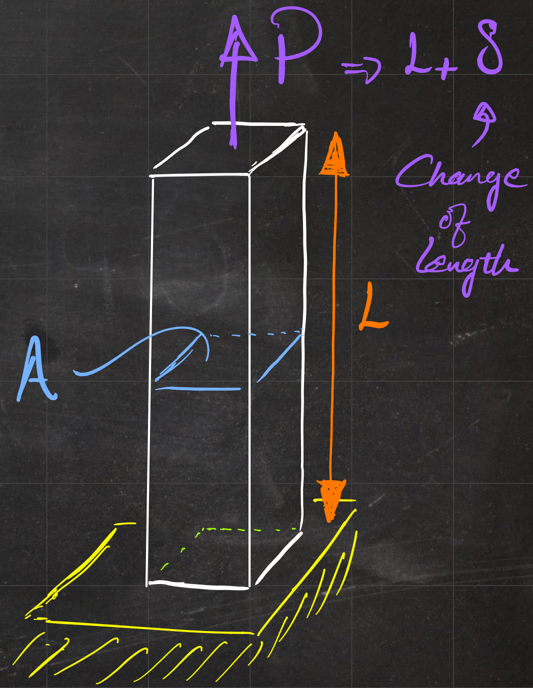
Chap. 2 ↕ (Deformation – Strain)

Chap. 1 ↕ (Loading – Stress)

Strain ⟷ Stress

Chap. 3

$$\varepsilon = \frac{\delta}{L} \qquad \sigma = E\varepsilon \qquad \sigma = \frac{P}{A}$$



ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

Deformation ⟷ ? ⟷ Loading

Chap. 2 ↕

$$\delta = \frac{PL}{EA}$$

↕ Chap. 1

Strain ⟷ Stress

Chap. 3

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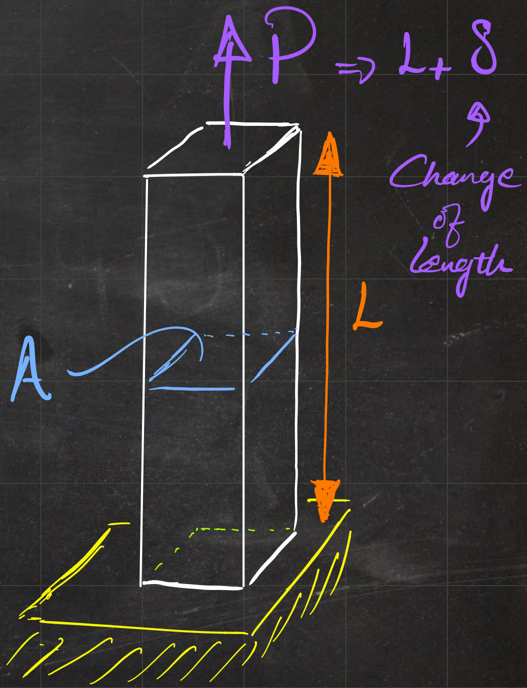
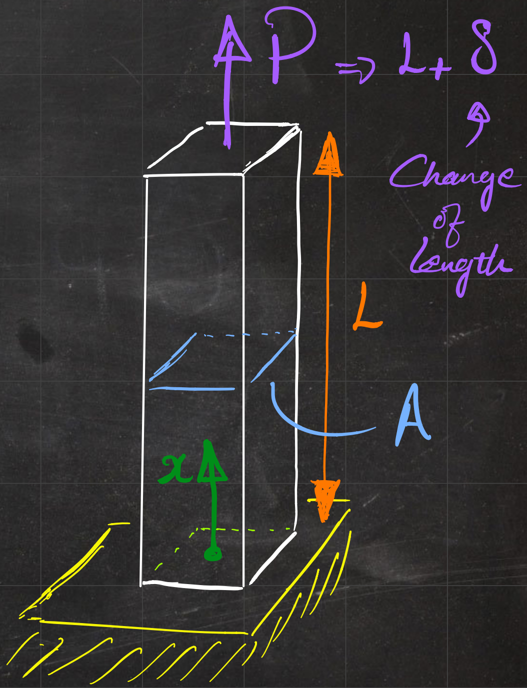


ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$$\delta(L) \leftarrow \delta = \frac{PL}{EA}$$

What is displacement at x ?



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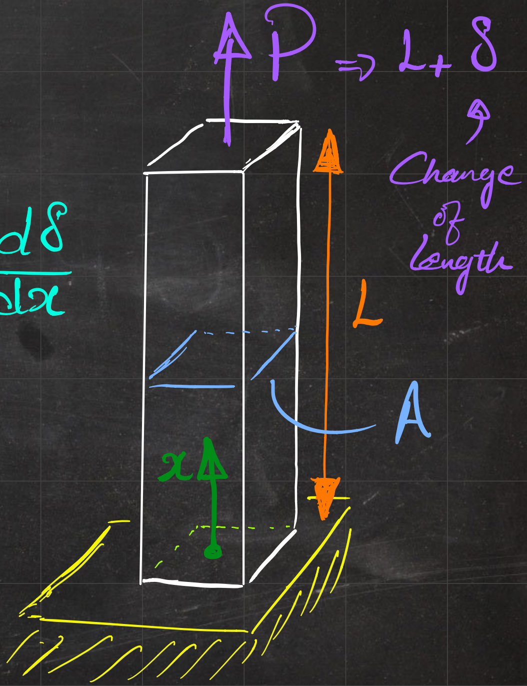
What is displacement at x ? $\leftarrow \varepsilon = \frac{d\delta}{dx}$



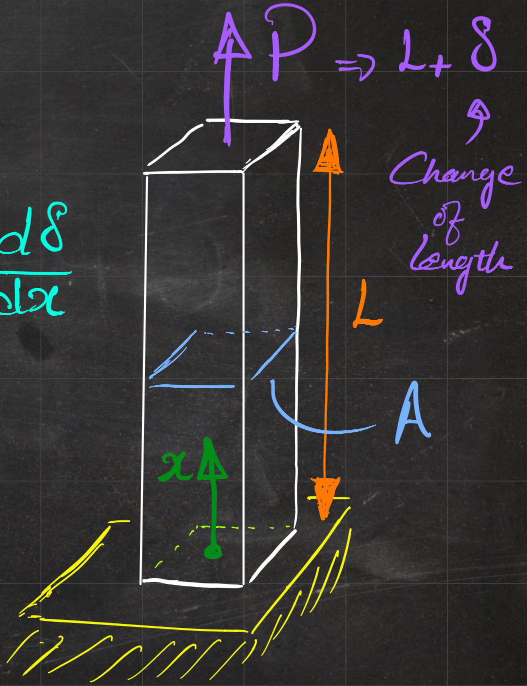
ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$\delta(L) \leftarrow \delta = \dfrac{PL}{EA}$

What is displacement at x ? $\leftarrow \varepsilon = \dfrac{d\delta}{dx}$

$$\delta(x) = \int_0^x \varepsilon \, dx \quad , \quad \varepsilon = \frac{\sigma}{E} = \frac{P}{EA}$$

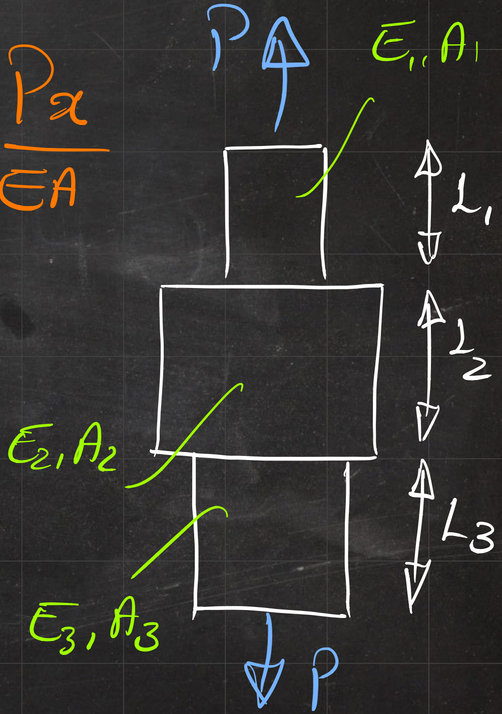
$$\Rightarrow \quad \delta(x) = \frac{Px}{EA}$$



ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$$\delta(L) = \frac{PL}{EA} \quad , \quad \delta(x) = \frac{Px}{EA}$$

How about multiple cross sections?

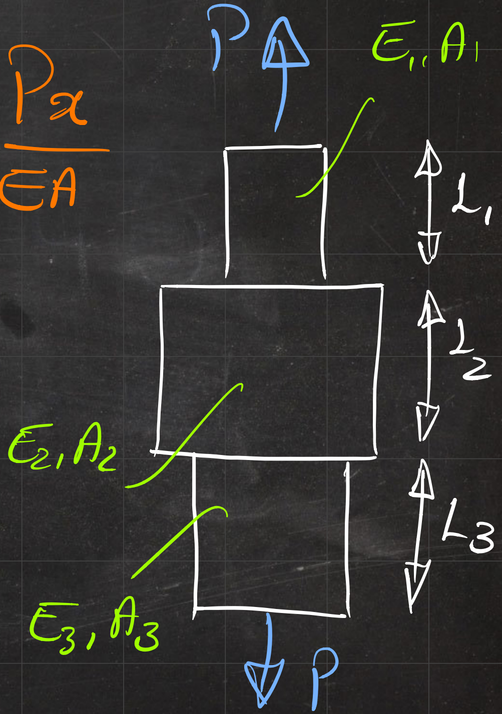


ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

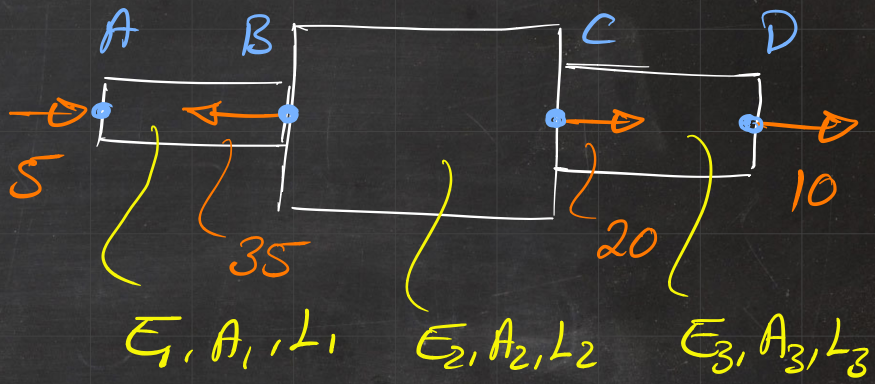
$$\delta(L) = \frac{PL}{EA} \quad , \quad \delta(x) = \frac{Px}{EA}$$

How about multiple cross sections?

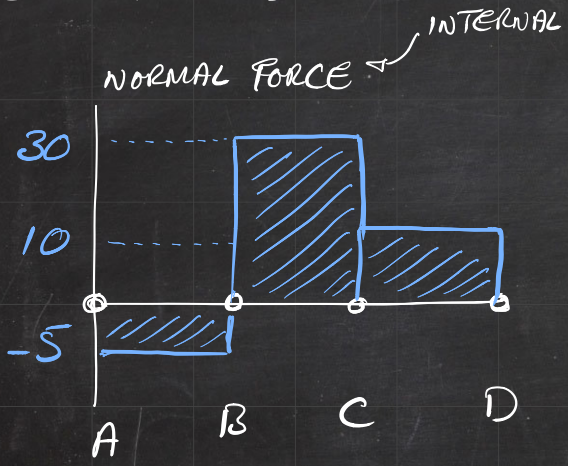
$$\delta_{tot} = \sum_{i=1}^{3} \delta_i = \sum \frac{P L_i}{E_i A_i}$$



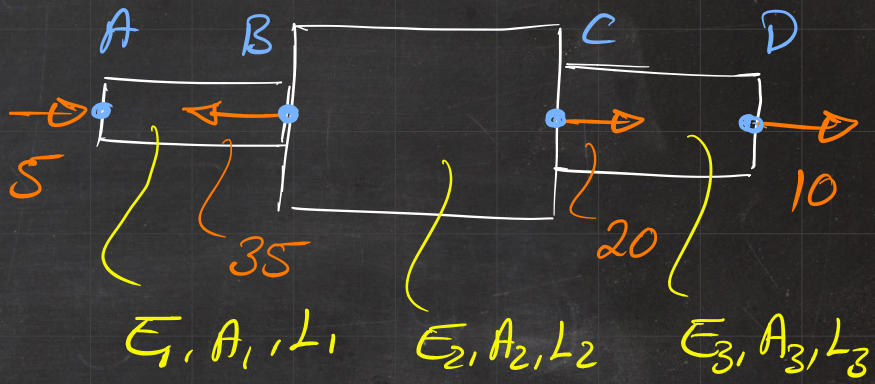
EXAMPLE :



EXAMPLE :

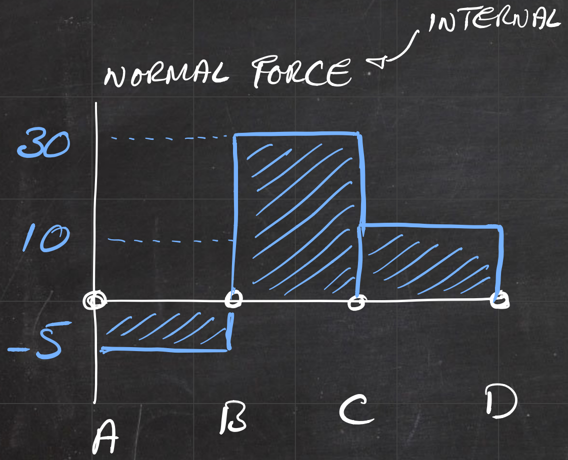


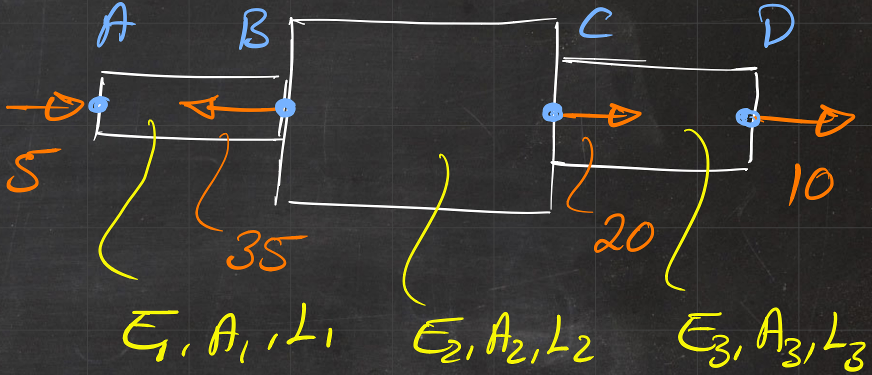
INTERNAL
NORMAL FORCE
30
10
-5
A
B
C
D



A
B
C
D
5
35
20
10
E_1, A_1, L_1
E_2, A_2, L_2
E_3, A_3, L_3

EXAMPLE :

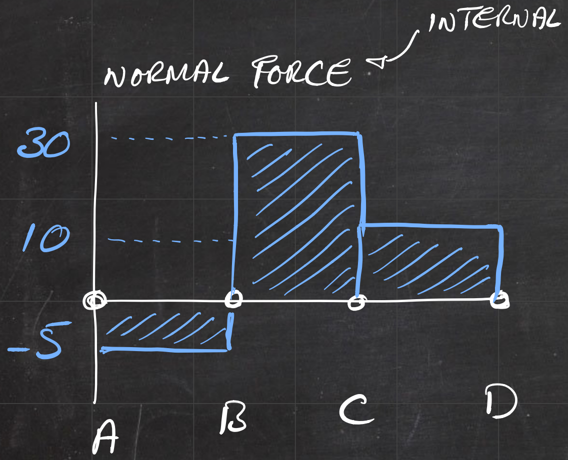


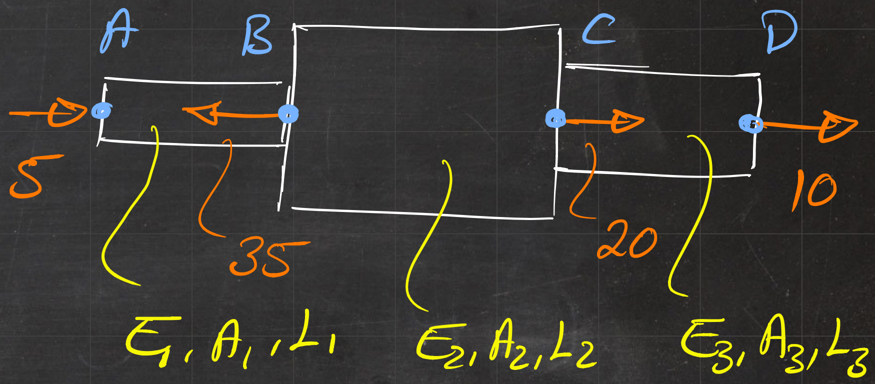


$$\delta_{tot} = \delta_{D/A} = \underbrace{\delta_{D/C}}_{\delta_3} + \underbrace{\delta_{C/B}}_{\delta_2} + \underbrace{\delta_{B/A}}_{\delta_1}$$

$\delta_{D/A}$ ↳ from A to D (D w.r.t. A)

EXAMPLE :





$$\delta_{tot} = \delta_{D/A} = \underbrace{\delta_{D/C}}_{\delta_3} + \underbrace{\delta_{C/B}}_{\delta_2} + \underbrace{\delta_{B/A}}_{\delta_1} = \frac{P_{CD} L_3}{E_3 A_3} + \frac{P_{BC} L_2}{E_2 A_2} + \frac{P_{AB} L_1}{E_1 A_1}$$

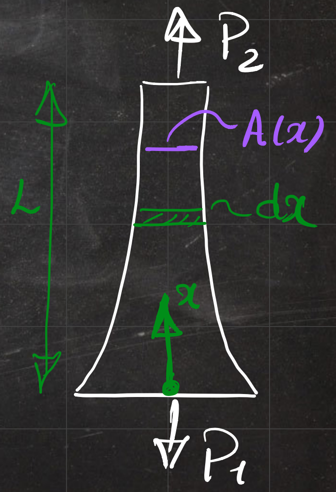
$P_{CD} = 10$, $P_{BC} = 30$, $P_{AB} = -5$

$\delta_{D/A}$: from A to D (D w.r.t. A)

ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$$\delta(L) = \frac{PL}{EA}, \quad \delta(x) = \frac{Px}{EA}, \quad \delta_{tot} = \sum_{i=1}^{3} \delta_i = \sum \frac{PL_i}{E_i A_i}$$

GENERIC CASE:

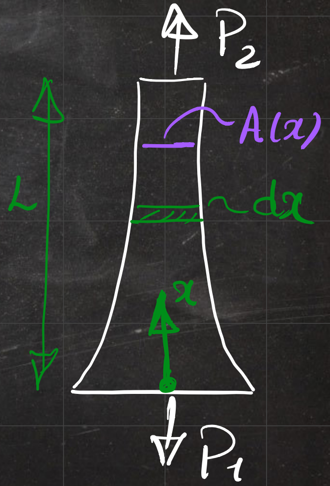


ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$$\delta(L) = \frac{PL}{EA}, \quad \delta(x) = \frac{Px}{EA}, \quad \delta_{tot} = \sum_{i=1}^{3} \delta_i = \sum \frac{PL_i}{E_i A_i}$$

GENERIC CASE:

$$\delta = \int_0^x \varepsilon(x)\, dx = \int_0^x \frac{\sigma}{E}\, dx = \int_0^x \frac{P(x)}{E(x)\, A(x)}\, dx$$

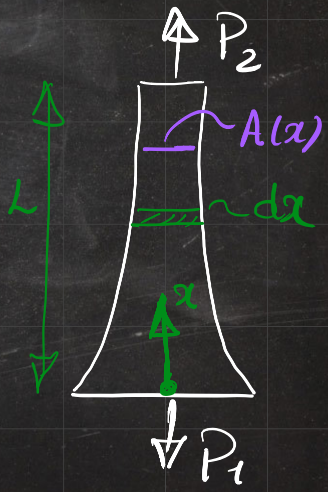


ELASTIC DEFORMATION OF AN AXIALLY LOADED MEMBER

$$\delta(L) = \frac{PL}{EA}\ , \quad \delta(x) = \frac{Px}{EA}\ , \quad \delta_{tot} = \sum_{i=1}^{3} \delta_i = \sum \frac{PL_i}{E_i A_i}$$

GENERIC CASE:

$$\Rightarrow \delta(L) = \int_0^L \frac{P(x)}{E(x)\,A(x)}\,dx$$



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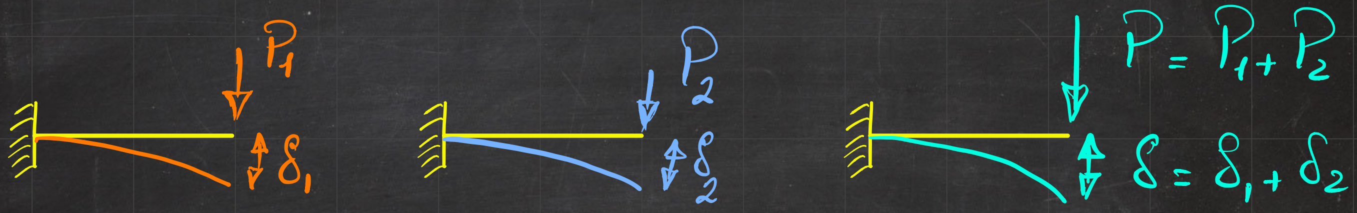
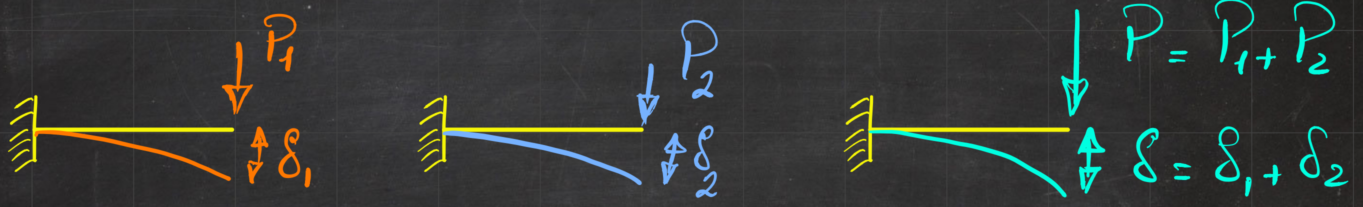
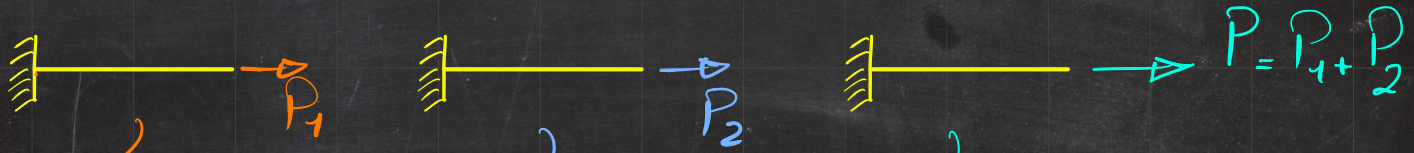
PRINCIPLE OF SUPERPOSITION

Principle of Superposition



Principle of Superposition





$$\delta_1 = \frac{P_1 L}{EA}$$

$$\delta_2 = \frac{P_2 L}{EA}$$

$$\delta = \delta_1 + \delta_2 = \frac{(P_1 + P_2) L}{EA}$$

PRINCIPLE OF SUPERPOSITION

↳ CAN BE APPLIED IF LOADING IS LINEARLY RELATED TO DISPLACEMENT

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↳ disp = linear function (loading)

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$$\delta = f(P)$$

$$f: \text{linear} \Rightarrow \begin{cases} f(ax) = a f(x) \\ f(x+y) = f(x) + f(y) \end{cases}$$

PRINCIPLE OF SUPERPOSITION

↳ CAN BE APPLIED IF LOADING IS LINEARLY RELATED TO DISPLACEMENT

↳ disp = linear function (loading)

$\delta = f(P)$

$\delta = \frac{PL}{EA} = \frac{L}{EA} P \Rightarrow \delta \propto P$

f: linear $\Rightarrow \begin{cases} f(ax) = a f(x) \\ f(x+y) = f(x) + f(y) \end{cases}$

STATICALLY INDETERMINATE AXIALLY LOADED MEMBERS

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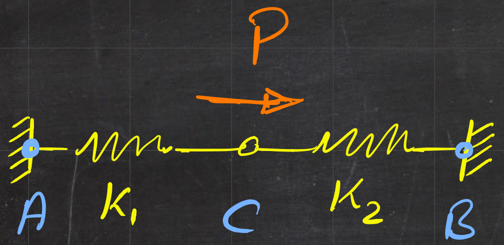
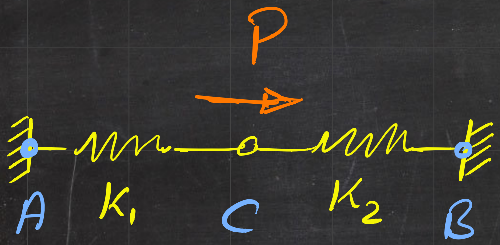
Example:



↳ What are the reactions at A and B?

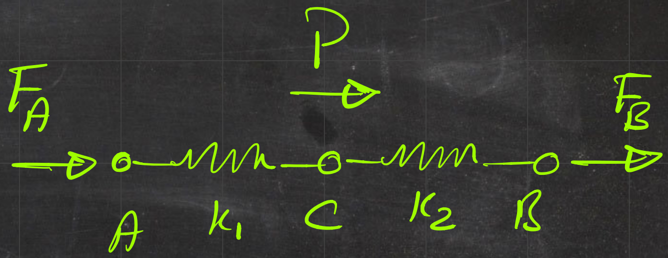
Statically Indeterminate Axially Loaded Members

Example:



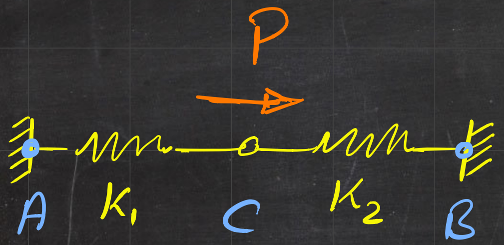
↳ What are the reactions at A and B?

$$\sum F_x = 0 \Rightarrow F_A + F_B + P = 0$$



STATICALLY INDETERMINATE AXIALLY LOADED MEMBERS

Example:



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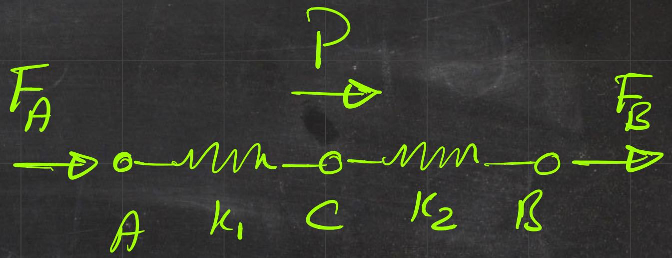
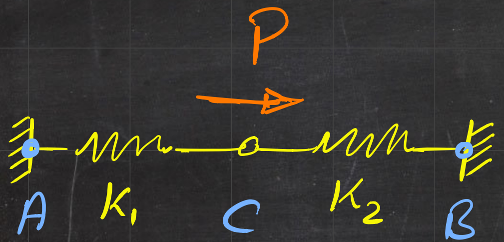


$K_1 = K_2$

↳ $F_A = F_B$

Statically Indeterminate Axially Loaded Members

Example:



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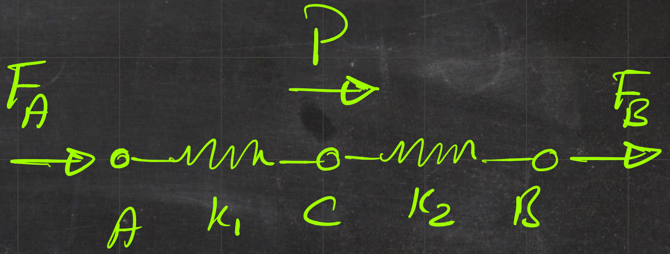
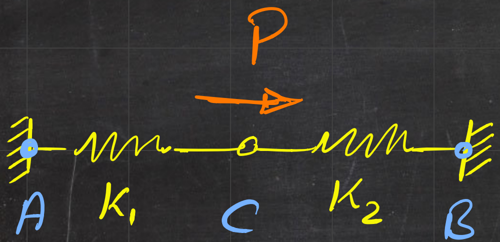


$K_1 = K_2$ ↳ $F_A = F_B$

$K_1 < K_2$ ↳ $F_A < F_B$

Statically Indeterminate Axially Loaded Members

Example:



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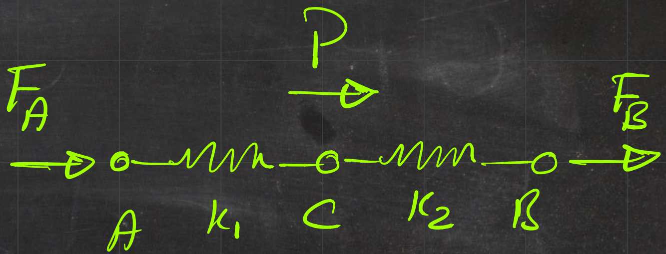


$K_1 = K_2$ ↳ $F_A = F_B$

$K_1 < K_2$ ↳ $F_A < F_B$

$K_1 > K_2$ ↳ $F_A > F_B$

Statically Indeterminate Axially Loaded Members

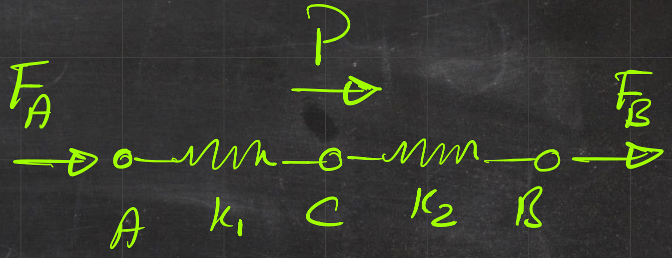
$$\delta = \frac{PL}{EA} \Rightarrow P = \frac{EA}{L}\delta$$

$$F = K \cdot \Delta x$$

Compare

$$\leadsto K \equiv \frac{EA}{L}$$

$$\Sigma F_x = 0 \Rightarrow F_A + F_B + P = 0$$



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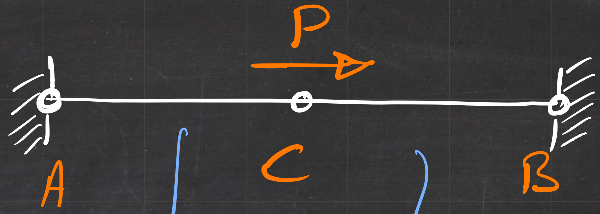


$$K_{AC} = \frac{EA}{L_{AC}}$$

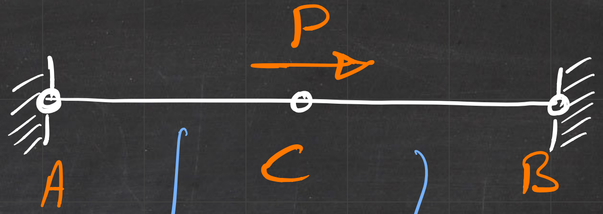
$$K_{CB} = \frac{EA}{L_{CB}}$$

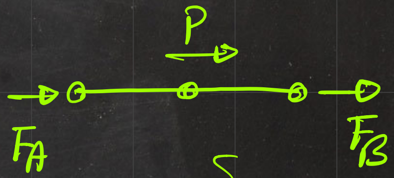
What are the reactions at A and B?

⇕

INTERNAL FORCES F_{AC}, F_{CB} ?

EXAMPLE:





What are the reactions at A and B?

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INTERNAL FORCES F_{AC}, F_{CB} ?

$$K_{AC} \equiv \frac{EA}{L_{AC}}$$

$$K_{CB} \equiv \frac{EA}{L_{CB}}$$

$$\sum F_x = 0$$

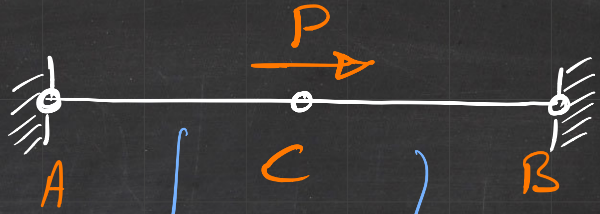
$$\Downarrow$$

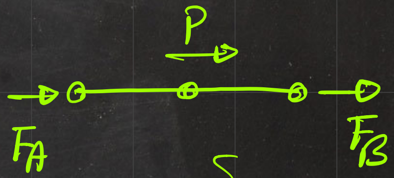
$$F_A + F_B + P = 0$$

$$\Downarrow$$

$$F_A + F_B = -P$$

EXAMPLE:





What are the reactions at A and B?

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INTERNAL FORCES F_{AC}, F_{CB}?

$$K_{AC} \equiv \frac{EA}{L_{AC}} \qquad K_{CB} \equiv \frac{EA}{L_{CB}}$$

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$$\Downarrow$$

$$F_A + F_B + P = 0$$

$$\Downarrow$$

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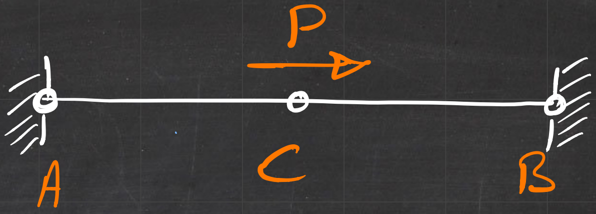
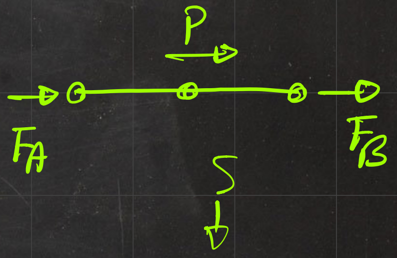
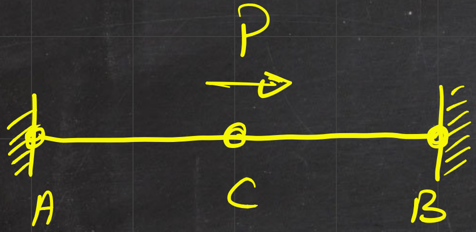
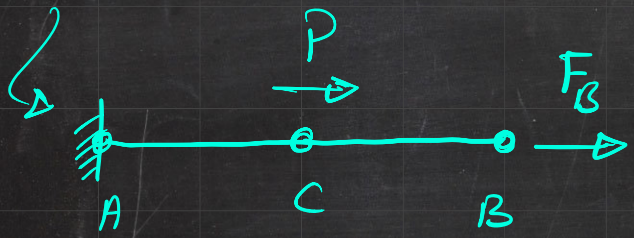






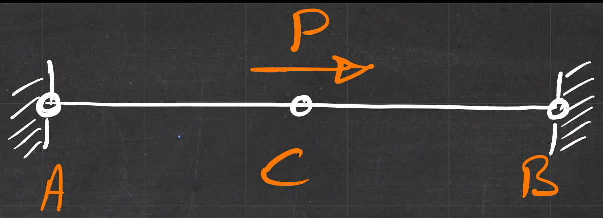


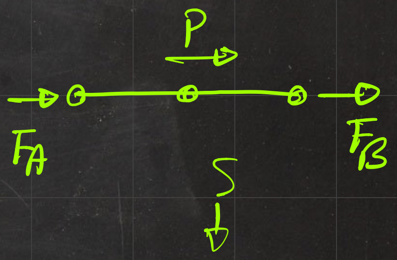
$$\sum F_x = 0$$

$$\Downarrow$$

$$F_A + F_B + P = 0$$

Example:





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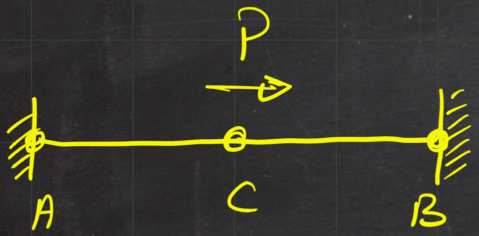
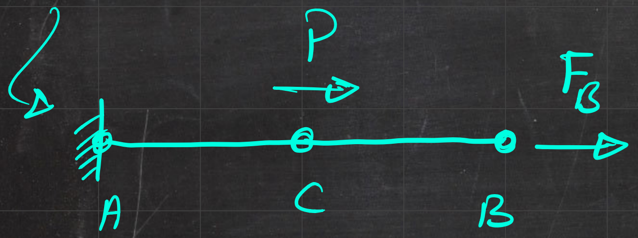
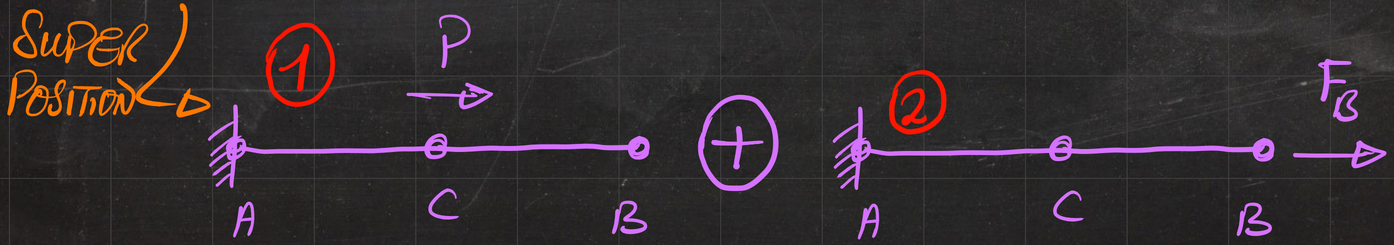
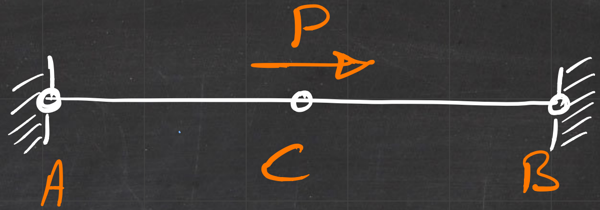
$$F_A + F_B + P = 0$$

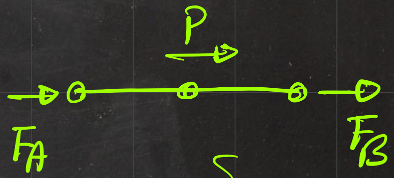






EXAMPLE:



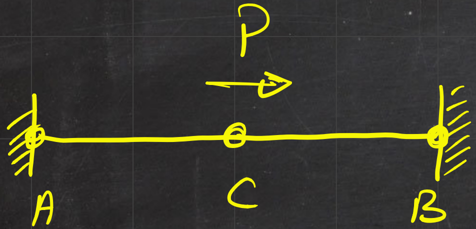


$\Downarrow$

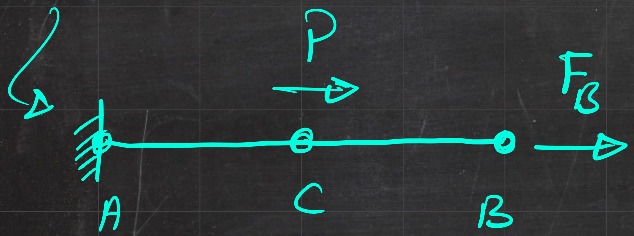
$$\sum F_x = 0$$

$\Downarrow$

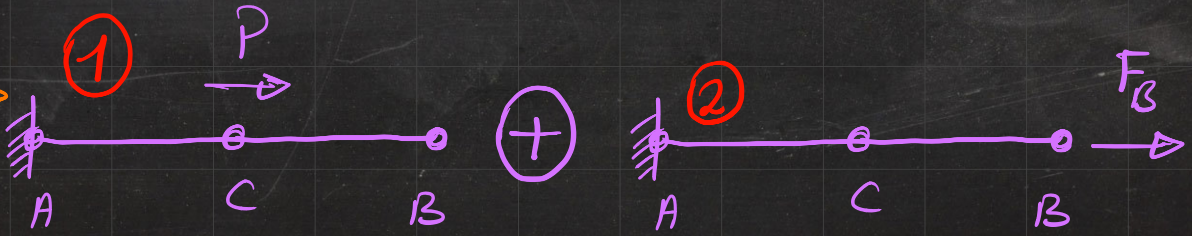
$$F_A + F_B + P = 0$$



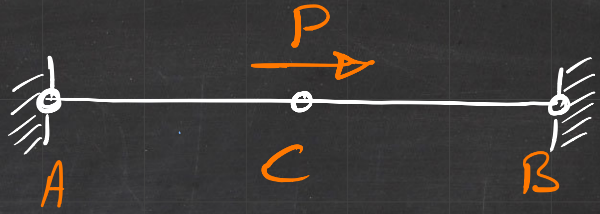
2. $\delta_B = 0 \Rightarrow \delta_B^1 + \delta_B^2 = 0$

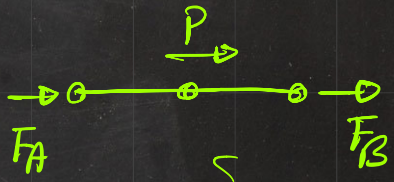


SUPER POSITION

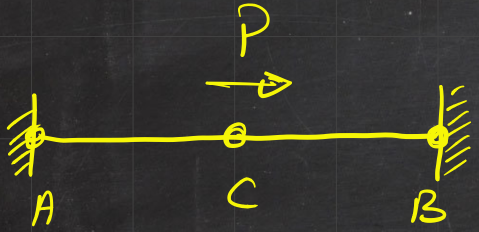


EXAMPLE:

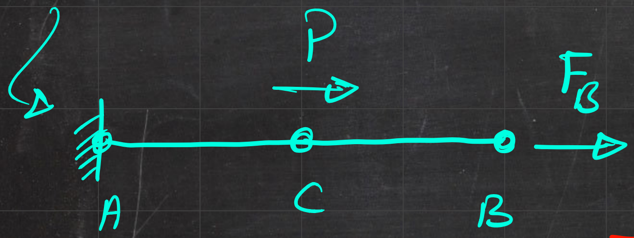




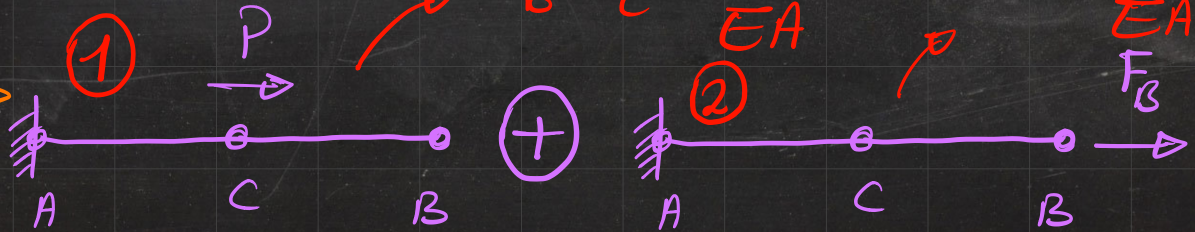
1. $\sum F_x = 0 \;\Rightarrow\; F_A + F_B + P = 0$



2. $\delta_B = 0 \Rightarrow \delta_B^1 + \delta_B^2 = 0$

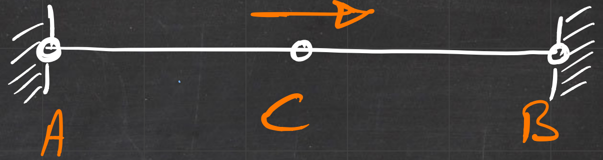


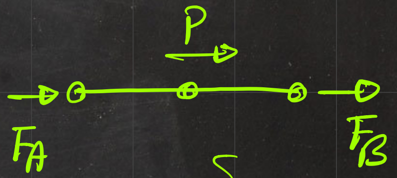
SUPERPOSITION



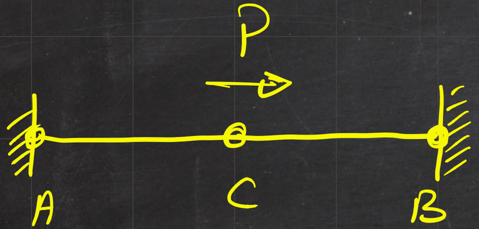
$$\delta_B^1 = \delta_C = \frac{P L_{AC}}{EA} \qquad \delta_B^2 = \frac{F_B L}{EA}$$

Example:



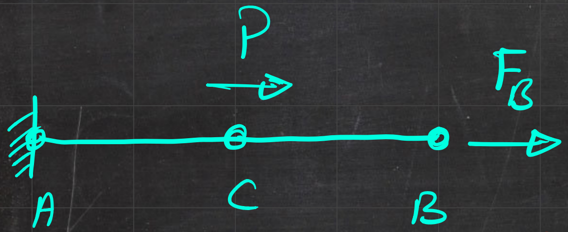


$$\sum F_x = 0 \Rightarrow F_A + F_B + P = 0$$



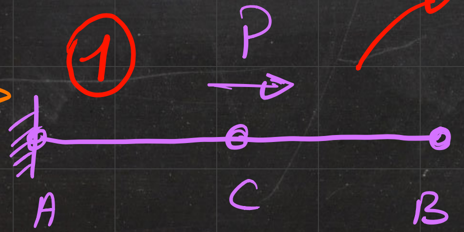
$$2. \quad \delta_B = 0 \Rightarrow \delta_B^1 + \delta_B^2 = 0$$

$$\Rightarrow \frac{P L_{AC}}{EA} + \frac{F_B L}{EA} = 0$$

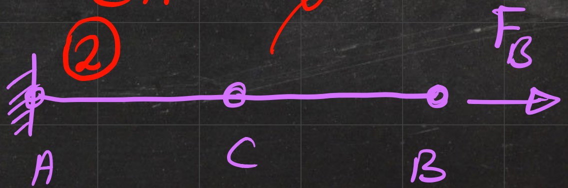


$$\delta_B^1 = \delta_C = \frac{P L_{AC}}{EA} \qquad \delta_B^2 = \frac{F_B L}{EA}$$

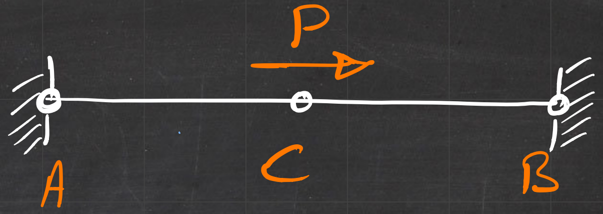
SUPERPOSITION

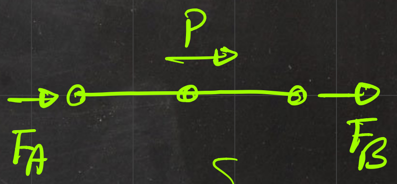


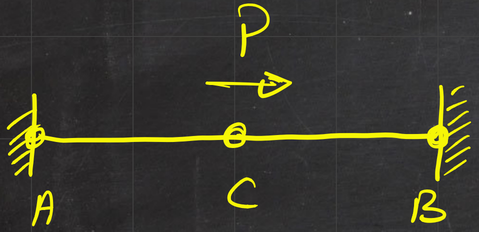
+



Example:



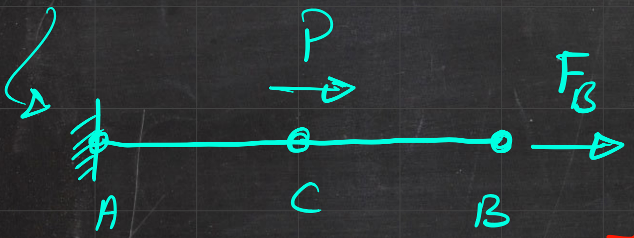




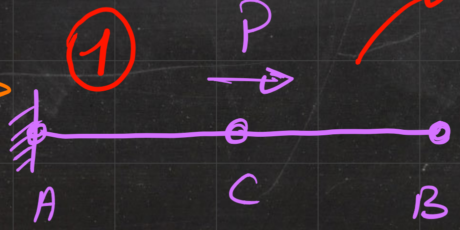
2) $\delta_B = 0 \Rightarrow \delta_B^1 + \delta_B^2 = 0$

$$\Rightarrow \frac{P L_{AC}}{EA} + \frac{F_B L}{EA} = 0$$

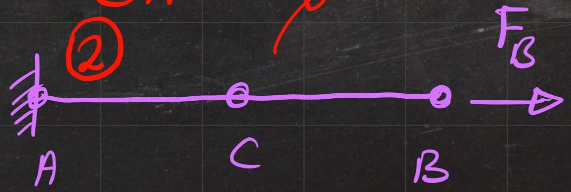
$$\Sigma F_x = 0 \Rightarrow F_A + F_B + P = 0$$



Superposition



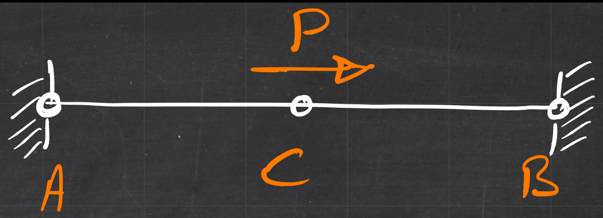
$\oplus$



$$\delta_B^1 = \delta_C = \frac{P L_{AC}}{EA}$$

$$\delta_B^2 = \frac{F_B L}{EA}$$

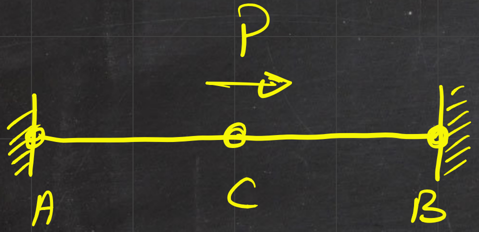
Example:



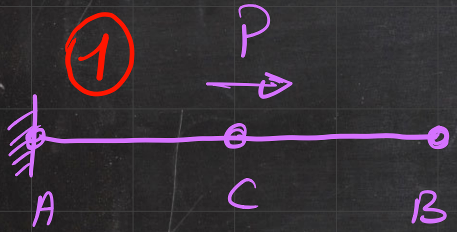
$$\Sigma F_x = 0$$

$$\Downarrow$$

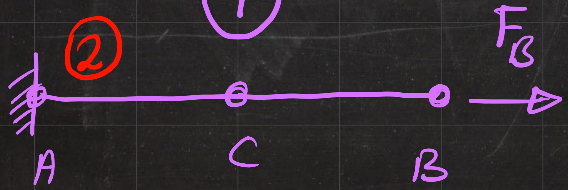
$$F_A + F_B + P = 0$$



zo $\delta_B = 0 \Rightarrow \delta_B^1 + \delta_B^2 = 0$



zo $\delta_B^1 = \delta_C = \dfrac{P L_{AC}}{EA}$



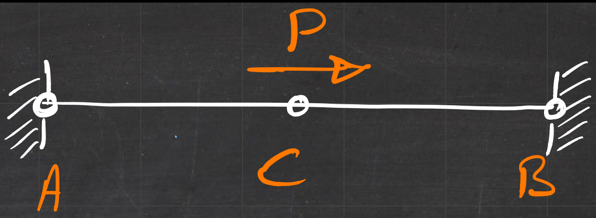
zo $\delta_B^2 = \dfrac{F_B L}{EA}$

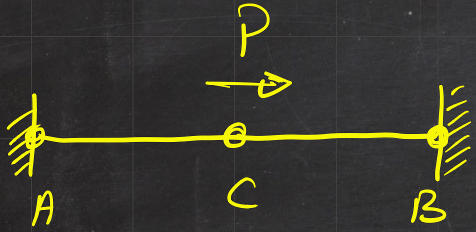
$$\frac{P L_{AC}}{EA} + \frac{F_B L}{EA} = 0$$

2 Equations

2 Unknowns

EXAMPLE:



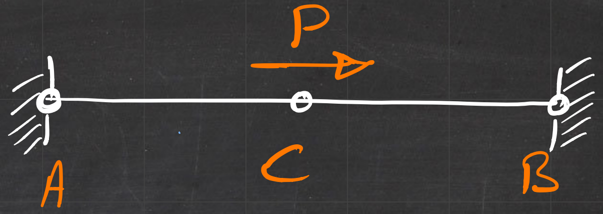


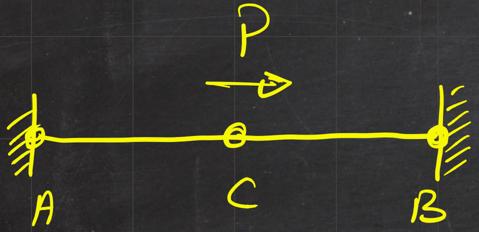
$$F_A + F_B + P = 0$$

$$\frac{P L_{AC}}{EA} + \frac{F_B L}{EA} = 0$$

$$\begin{cases} F_B = -\dfrac{L_{AC}}{L} P \\ F_A = -\dfrac{L_{BC}}{L} P \end{cases}$$

Example:





$$F_A + F_B + P = 0$$

$$\frac{P L_{AC}}{EA} + \frac{F_B L}{EA} = 0$$

$$\begin{cases} F_B = -\dfrac{L_{AC}}{L} P \\ F_A = -\dfrac{L_{BC}}{L} P \end{cases}$$

$$\Rightarrow \frac{F_A}{F_B} = \frac{L_{BC}}{L_{AC}}$$

$$\Downarrow$$

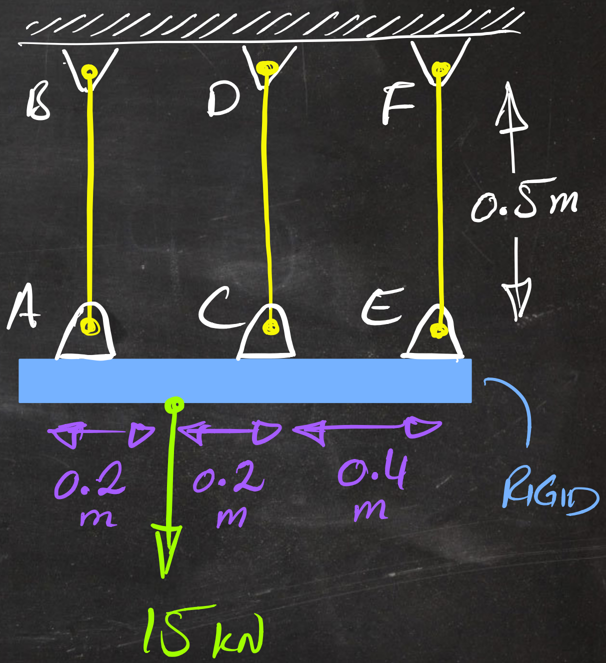
$$F_A L_{AC} = F_B L_{BC}$$

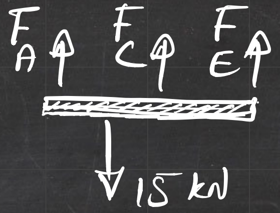
$$F_A = F_B \quad \text{if} \quad L_{AC} = L_{CB}$$

Exercise 1 . [similar to ... P. 143 ... 4.6]

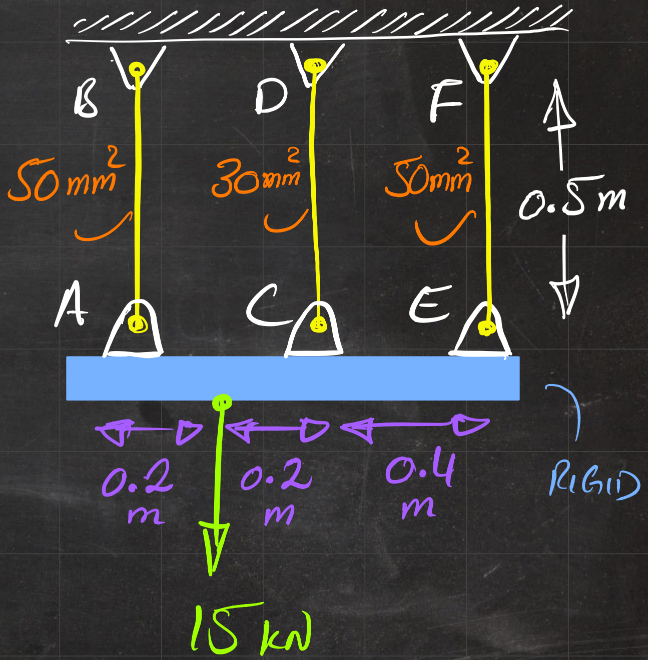
THE THREE STEEL BARS SHOWN IN THE FIGURE ARE PIN-CONNECTED TO A RIGID MEMBER. IF A LOAD OF 15 kN IS APPLIED AS SHOWN, DETERMINE THE FORCE DEVELOPED IN EACH BAR.

BARS AB AND EF EACH HAVE A CROSS-SECTIONAL AREA OF 50 mm^2, AND BAR CD HAS A CROSS-SECTIONAL AREA OF 30 mm^2.





F_A F_C F_E
15 kN



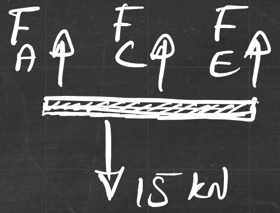
B
D
F
50 mm²
30 mm²
50 mm²
0.5 m
A
C
E
0.2 m
0.2 m
0.4 m
RIGID
15 kN

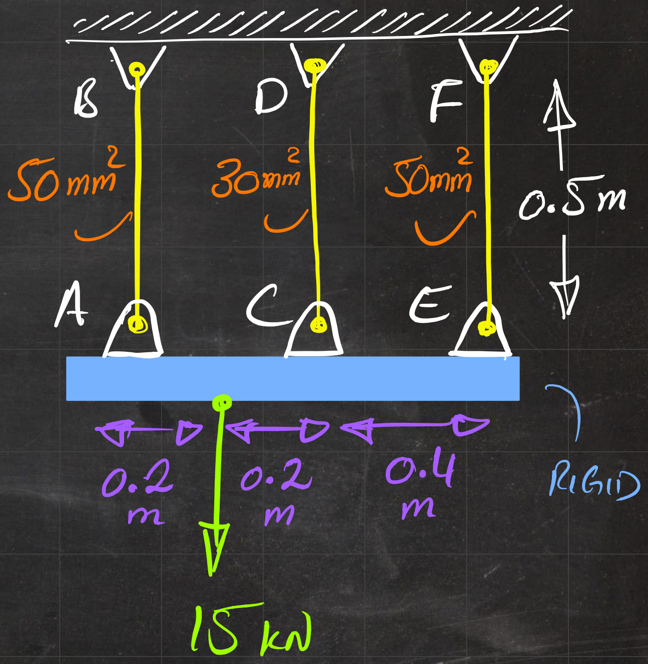
$+\uparrow \sum F_y = 0$

$F_A + F_C + F_E - 15 = 0$

$\circlearrowleft + \sum M_C = 0$

$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$





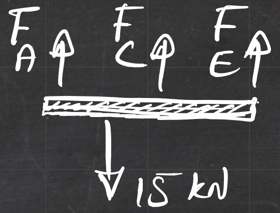
$+\uparrow \sum F_y = 0$

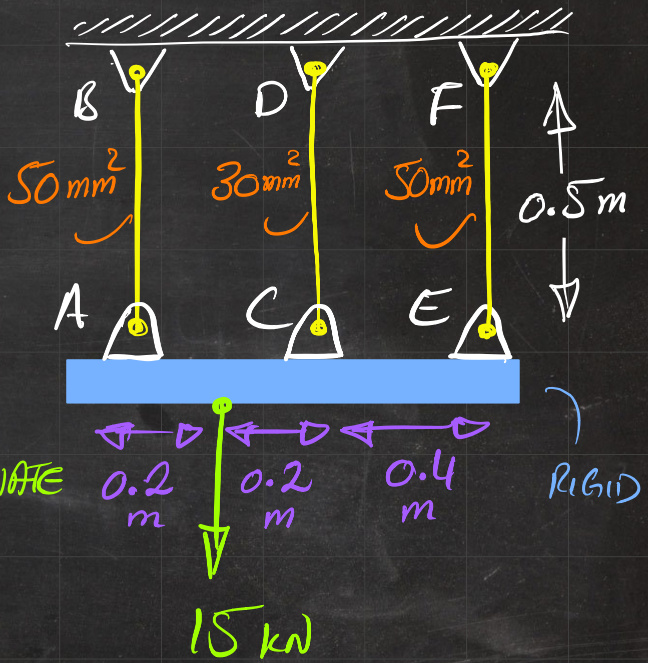
$F_A + F_C + F_E - 15 = 0$

$\circlearrowleft + \sum M_C = 0$

$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$

↳ 2 EQNs. & 3 UNKNOWNS ⇒ STATICALLY INDETERMINATE





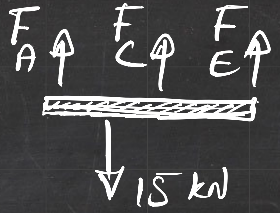
$+\uparrow \sum F_y = 0$

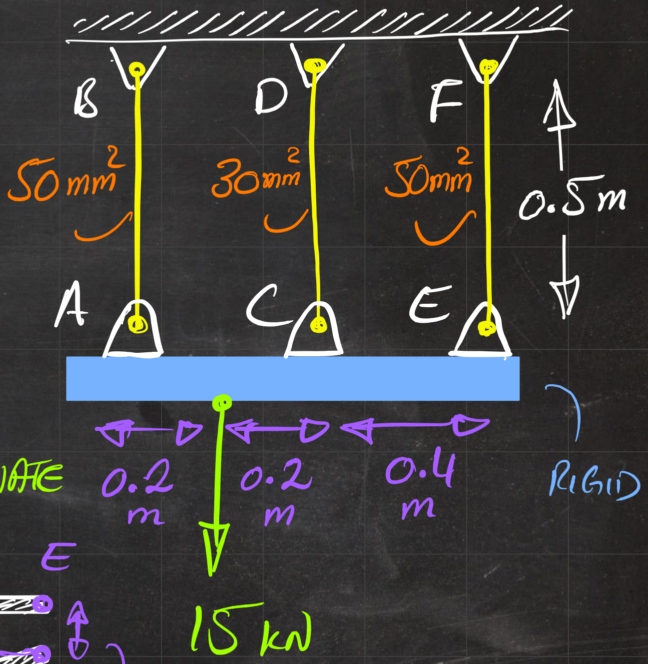
$F_A + F_C + F_E - 15 = 0$

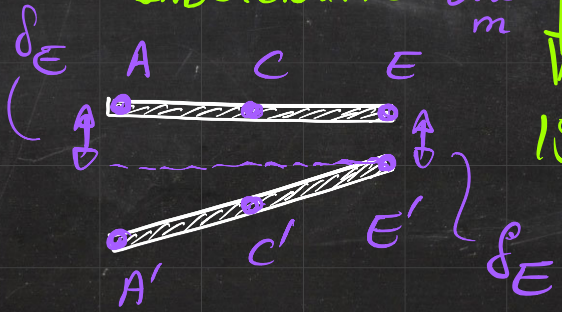
$\circlearrowleft + \sum M_C = 0$

$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$

↳ 2 EQNs. & 3 UNKNOWNS ⇒ STATICALLY INDETERMINATE

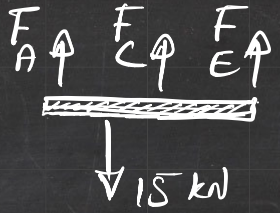






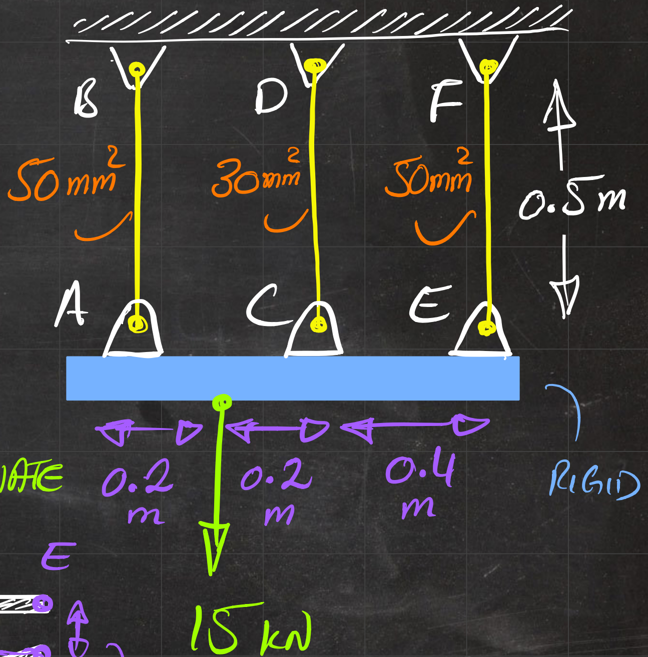
$+\uparrow \sum F_y = 0$

$$F_A + F_C + F_E - 15 = 0$$



$(+\circlearrowleft) \sum M_C = 0$

$$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$$

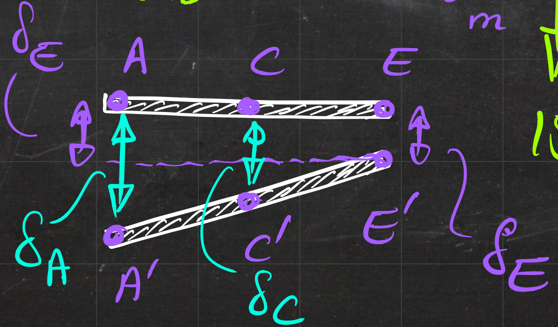


↳ 2 EQNs. & 3 UNKNOWNS ⇒ STATICALLY INDETERMINATE

3rd. Equation

$$\frac{\delta_A - \delta_E}{0.8} = \frac{\delta_C - \delta_E}{0.4}$$

Geometrical Constraint

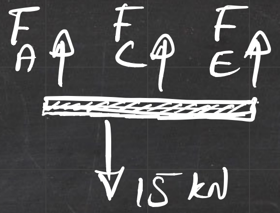


$+\uparrow \sum F_y = 0$

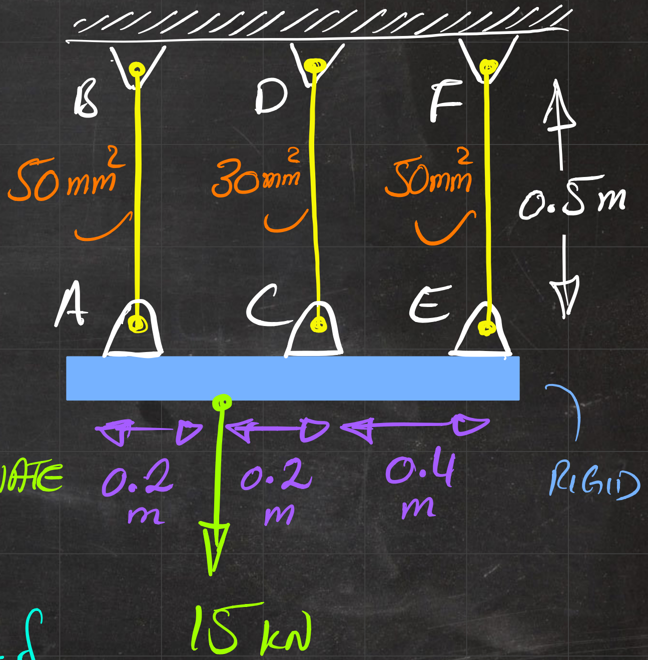
$$F_A + F_C + F_E - 15 = 0$$

$\circlearrowleft + \sum M_C = 0$

$$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$$



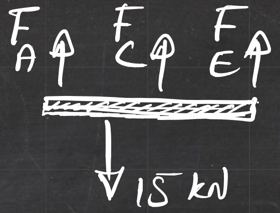
↳ 2 EQNs. & 3 UNKNOWNS ⇒ STATICALLY INDETERMINATE

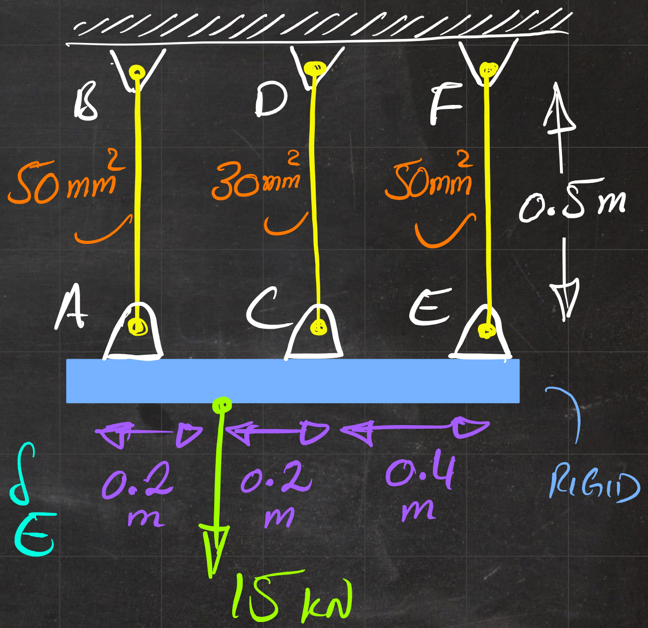


3rd. Equation

$$\frac{\delta_A - \delta_E}{0.8} = \frac{\delta_C - \delta_E}{0.4} \Rightarrow \delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E$$

Geometrical Constraint





$+\uparrow \sum F_y = 0$

$$F_A + F_C + F_E - 15 = 0$$

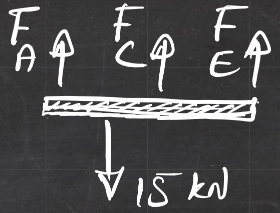
$\circlearrowleft + \sum M_C = 0$

$$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$$

$$\frac{\delta_A - \delta_E}{0.8} = \frac{\delta_C - \delta_E}{0.4} \Rightarrow \delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E$$

$$\delta_C = \frac{F_C L}{E A_{CD}} \qquad \delta_A = \frac{F_A L}{E A_{AB}} \qquad \delta_E = \frac{F_E L}{E A_{EF}}$$

$$+\uparrow \sum F_y = 0 \qquad \text{(I)}$$



$$F_A + F_C + F_E - 15 = 0$$

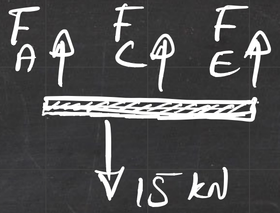
$$(+\circlearrowleft) \sum M_C = 0 \qquad \text{(II)}$$

$$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$$

$$\frac{\delta_A - \delta_E}{0.8} = \frac{\delta_C - \delta_E}{0.4} \Rightarrow \delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E \Rightarrow F_C = 0.3F_A + 0.3F_E \qquad \text{(III)}$$

$$\delta_C = \frac{F_C L}{E A_{CD}} \qquad \delta_A = \frac{F_A L}{E A_{AB}} \qquad \delta_E = \frac{F_E L}{E A_{EF}}$$

$+\uparrow \sum F_y = 0$ (I)



$$F_A + F_C + F_E - 15 = 0$$

$\circlearrowleft + \sum M_C = 0$ (II)

$$-F_A \times 0.4 + 15 \times 0.2 + F_E \times 0.4 = 0$$

$\Rightarrow$ $F_A = 9.52$ kN, $F_C = 3.46$ kN, $F_E = 2.02$ kN

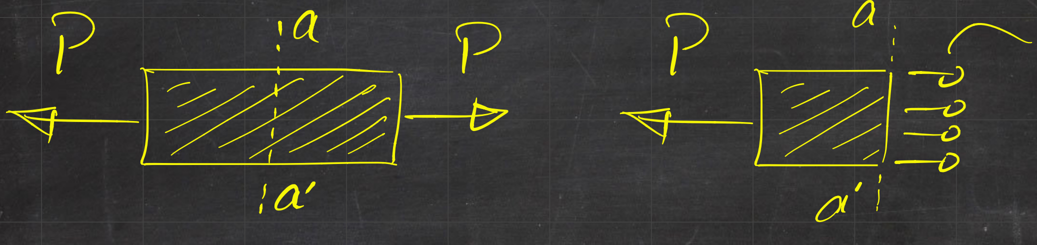
(III)

$$\frac{\delta_A - \delta_E}{0.8} = \frac{\delta_C - \delta_E}{0.4} \Rightarrow \delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E \Rightarrow F_C = 0.3 F_A + 0.3 F_E$$

$$\delta_C = \frac{F_C L}{E A_{CD}} \qquad \delta_A = \frac{F_A L}{E A_{AB}} \qquad \delta_E = \frac{F_E L}{E A_{EF}}$$

STRESS CONCENTRATION

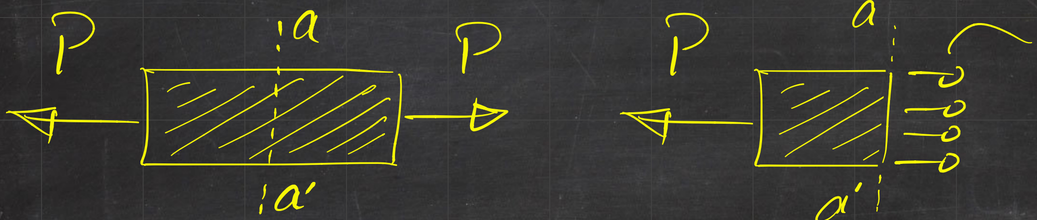
STRESS CONCENTRATION



$$\sigma = \frac{P}{A}$$

$$\sigma_{avg} = \frac{P}{A}$$

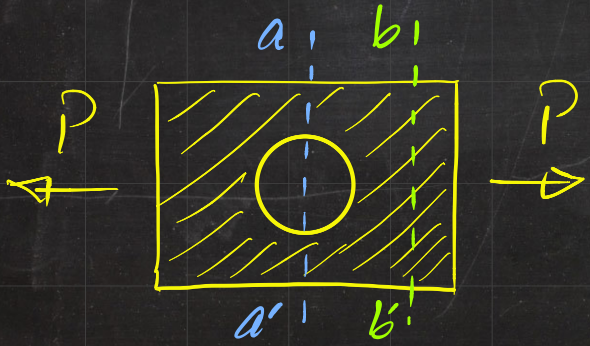
STRESS CONCENTRATION



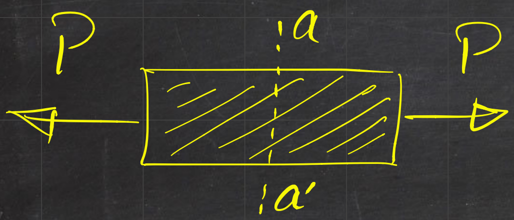
$\sigma = \frac{P}{A}$

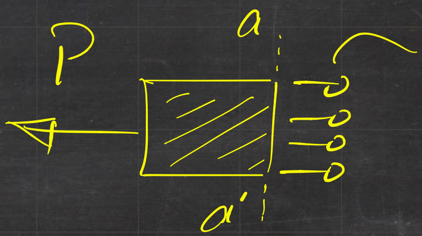
$\sigma_{avg} = \frac{P}{A}$

PLATE WITH HOLE:



STRESS CONCENTRATION

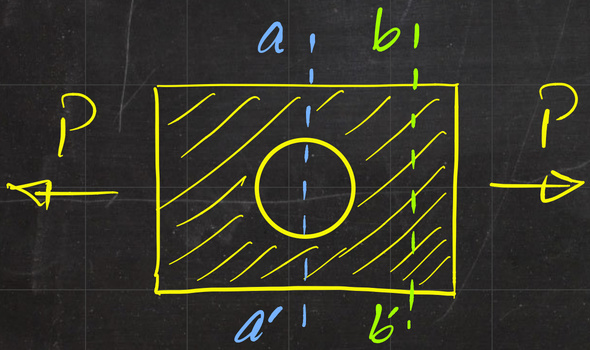




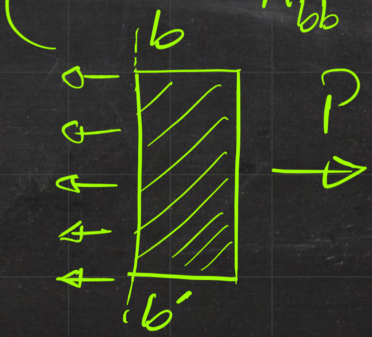
$$\sigma = \frac{P}{A}$$

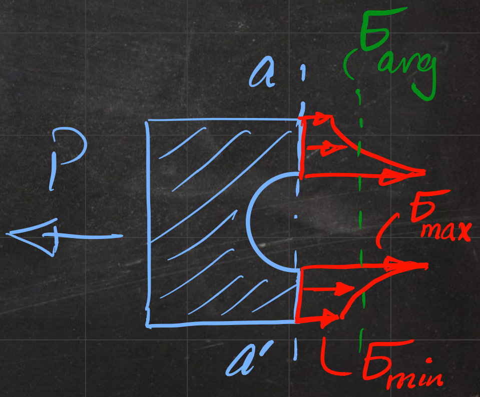
$$\sigma_{avg} = \frac{P}{A}$$

PLATE WITH HOLE:

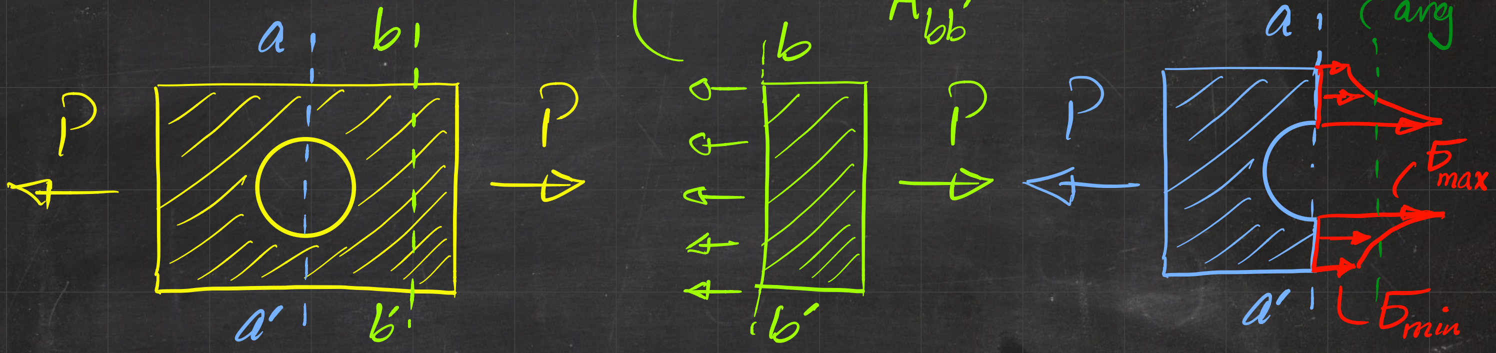


$$\sigma = \sigma_{avg} = \frac{P}{A_{bb'}}$$





$$\sigma = \sigma_{avg} = \frac{P}{A_{bb'}}$$



Equilibrium: holds for any cross section

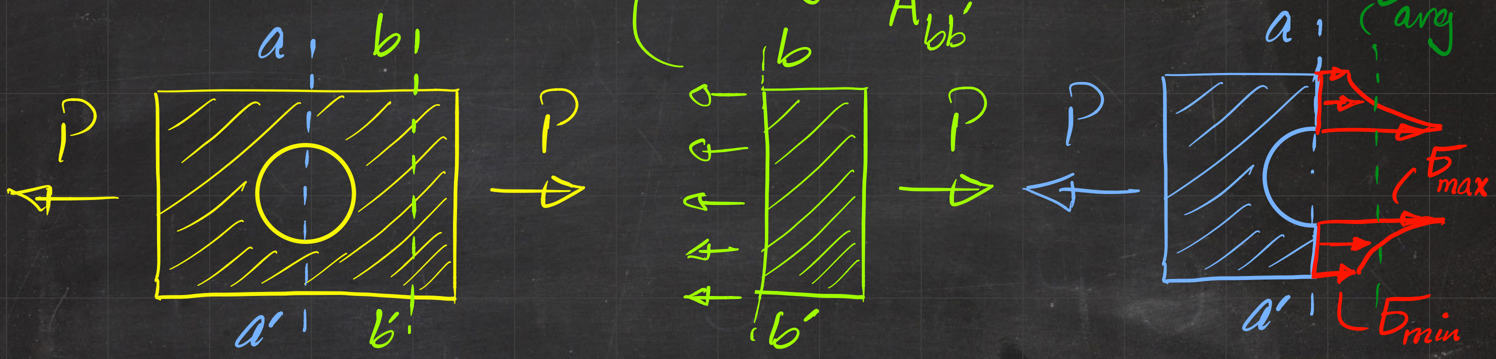
$$P = \int_A \sigma \, dA$$

$$\sigma_{avg} = \frac{P}{A_{aa'}}$$

higher than σ_{avg} at bb'

$$\sigma_{min} < \sigma_{avg} < \sigma_{max}$$

$$\sigma = \sigma_{avg} = \frac{P}{A_{bb'}}$$



We are interested in σ_{max} for design

$$\sigma_{avg} = \frac{P}{A_{aa'}}$$

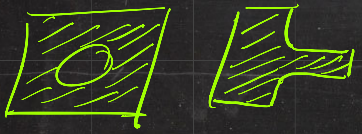
STRESS CONCENTRATION FACTOR

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

$$\sigma_{min} < \sigma_{avg} < \sigma_{max}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} > 1$$

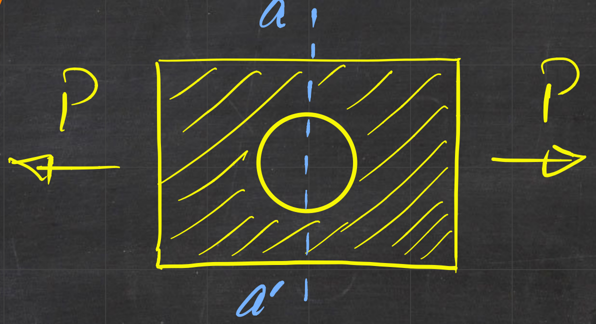
depends on geometry and loading

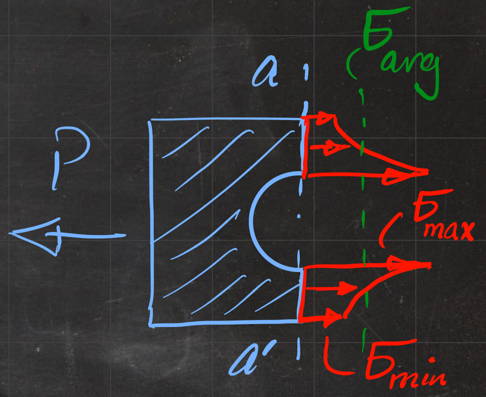


bending

axial loading

torsion





$$\sigma_{avg} = \frac{P}{A_{aa'}}$$

$$\sigma_{min} < \sigma_{avg} < \sigma_{max}$$

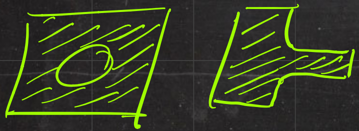
$$K = \frac{\sigma_{max}}{\sigma_{avg}} > 1$$

does NOT depend on (assumptions)

material nor magnitude of loading

St, Al — 100, 200

depends on geometry and loading

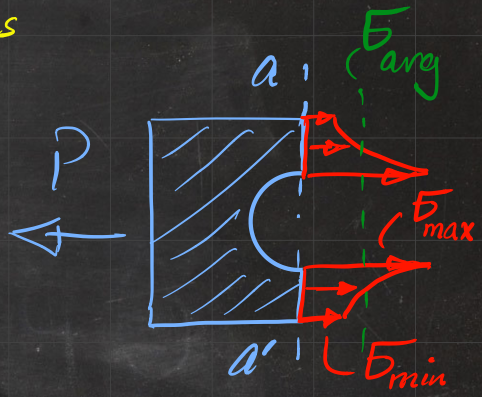


bending

axial loading

torsion

K IS TABULATED !



$$\sigma_{avg} = \frac{P}{A_{aa'}}$$

$$\sigma_{min} < \sigma_{avg} < \sigma_{max}$$

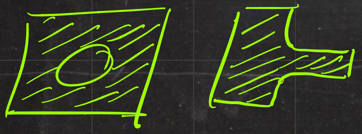
$$K = \frac{\sigma_{max}}{\sigma_{avg}} > 1$$

does NOT depend on (assumptions)

material nor magnitude of loading

St, Al — 100, 200

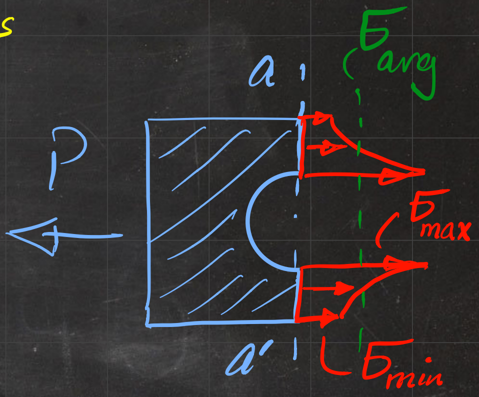
depends on geometry and loading



bending

axial loading

torsion



Step 1: Read K

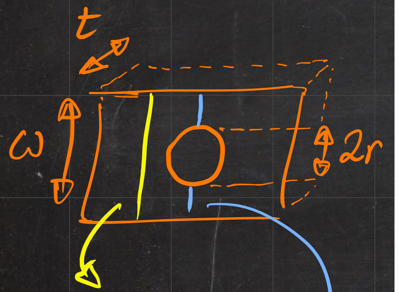
Step 2: Compute σ_{avg}

$\sigma_{avg} = P/A$ — AREA OF CROSS SECTION

Step3: $\sigma_{max} = K \sigma_{avg}$

$$\sigma_{avg} = \frac{P}{A_{aa'}}$$

$$\sigma_{avg} = \frac{P}{A}$$



$$A = \omega t$$

$$A = (\omega - 2r)t$$

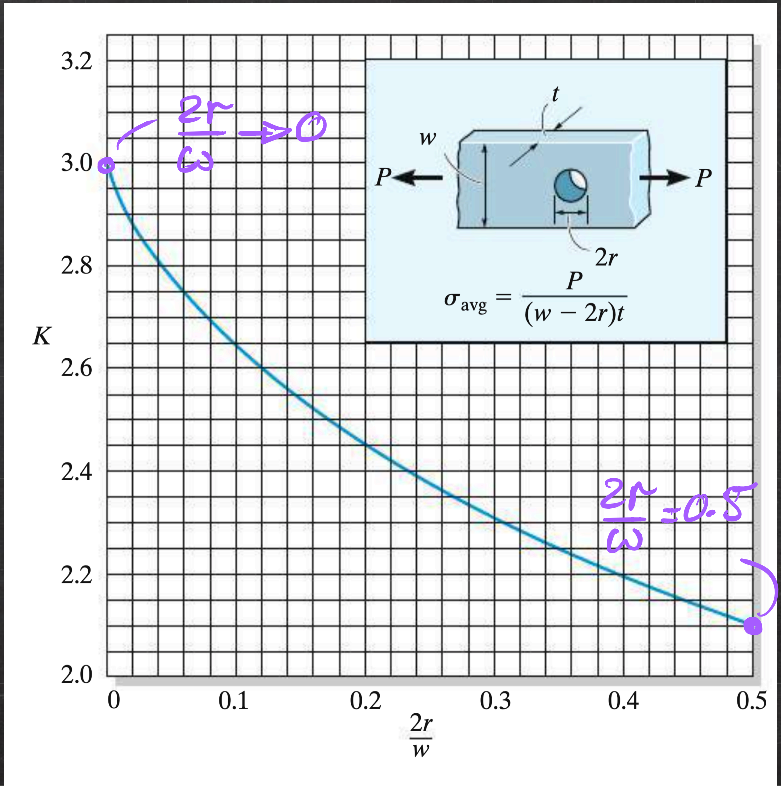
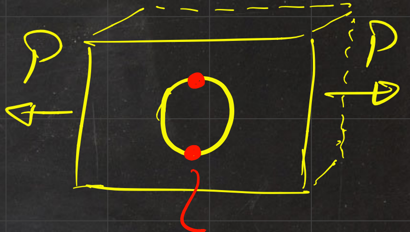


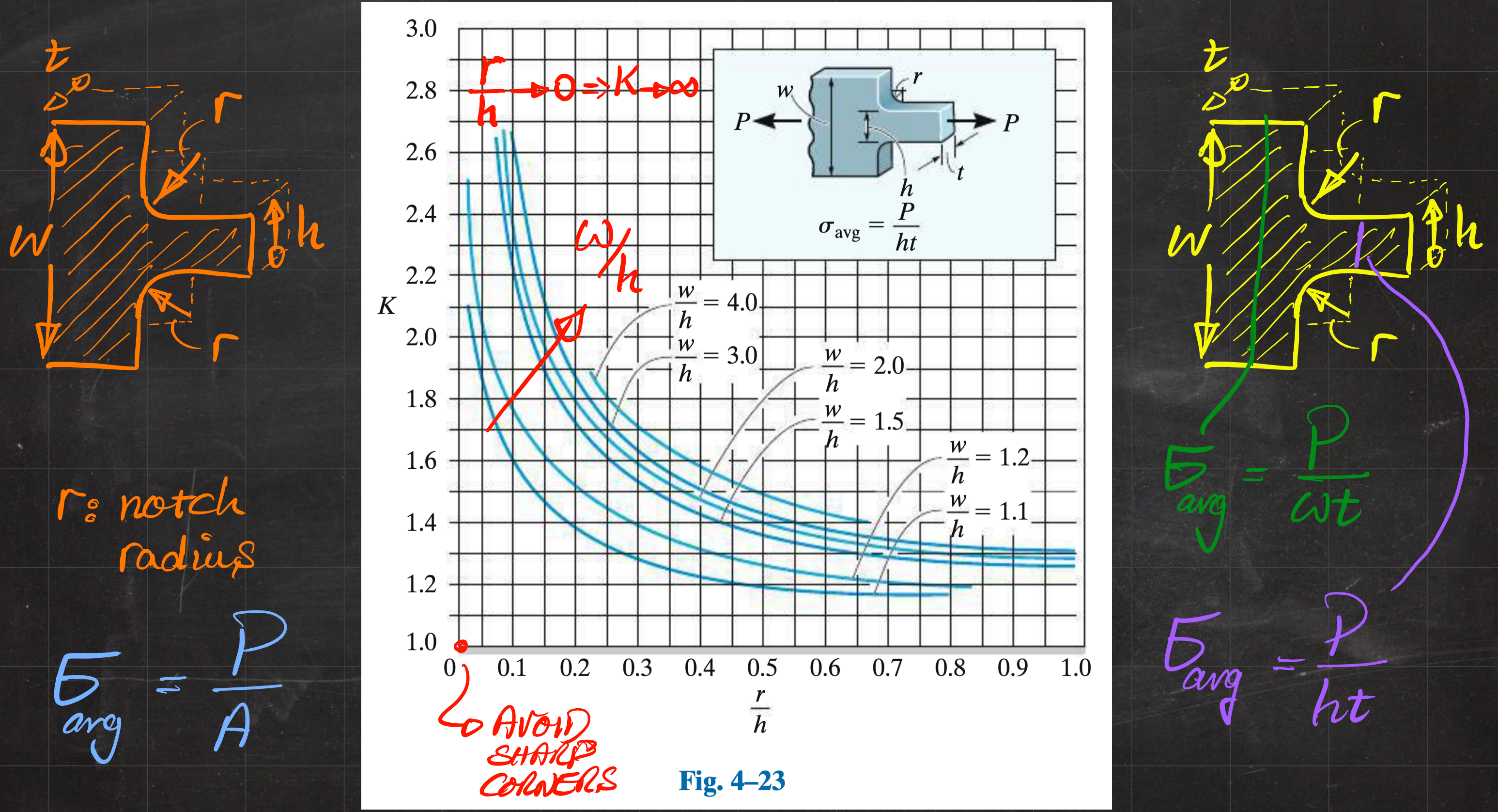
Fig. 4–24



σ_{max}

$$\sigma_{max} = K \sigma_{avg}$$

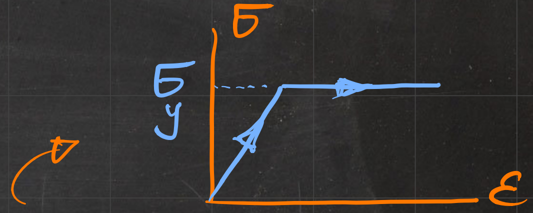
$$\sigma_{avg} = \frac{P}{(\omega - 2r)t}$$



t
w
r
h
r
r: notch radius
σavg = P/A
r/h → 0 ⇒ K → ∞
w/h
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
K
P
w
r
h
t
P
σavg = P/ht
w/h = 4.0
w/h = 3.0
w/h = 2.0
w/h = 1.5
w/h = 1.2
w/h = 1.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r/h
AVOID SHARP CORNERS
σavg = P/wt
σavg = P/ht

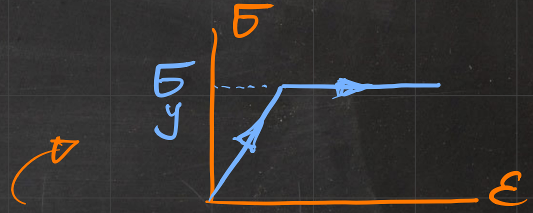
Fig. 4–23

INELASTIC AXIAL DEFORMATION

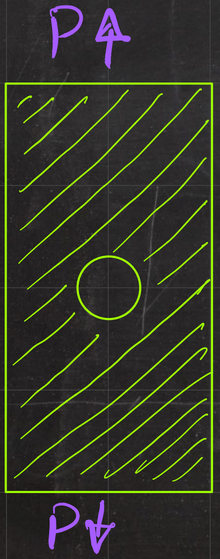


- Assume a ductile material that is "elastic perfectly plastic"

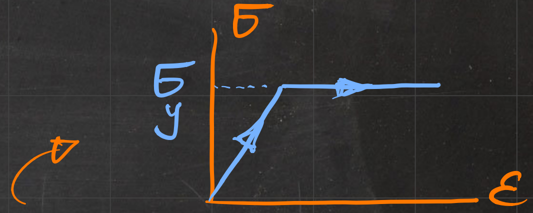
INELASTIC AXIAL DEFORMATION



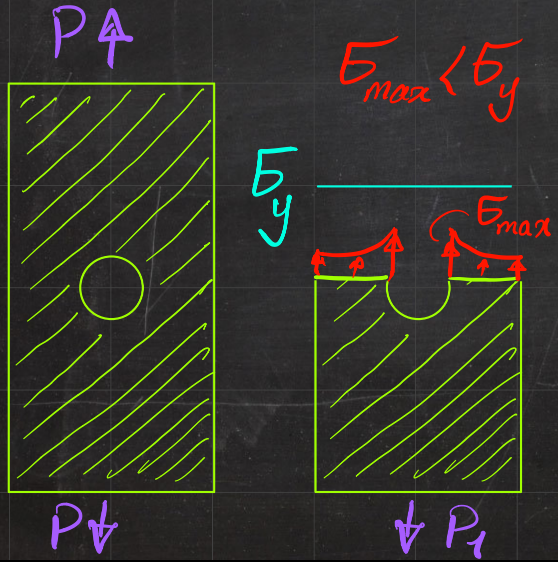
Assume a ductile material that is "elastic perfectly plastic"



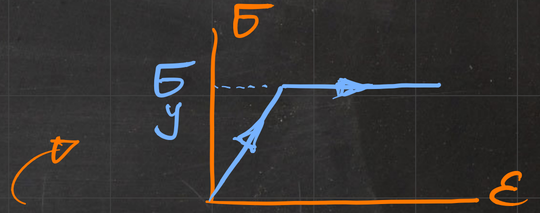
INELASTIC AXIAL DEFORMATION



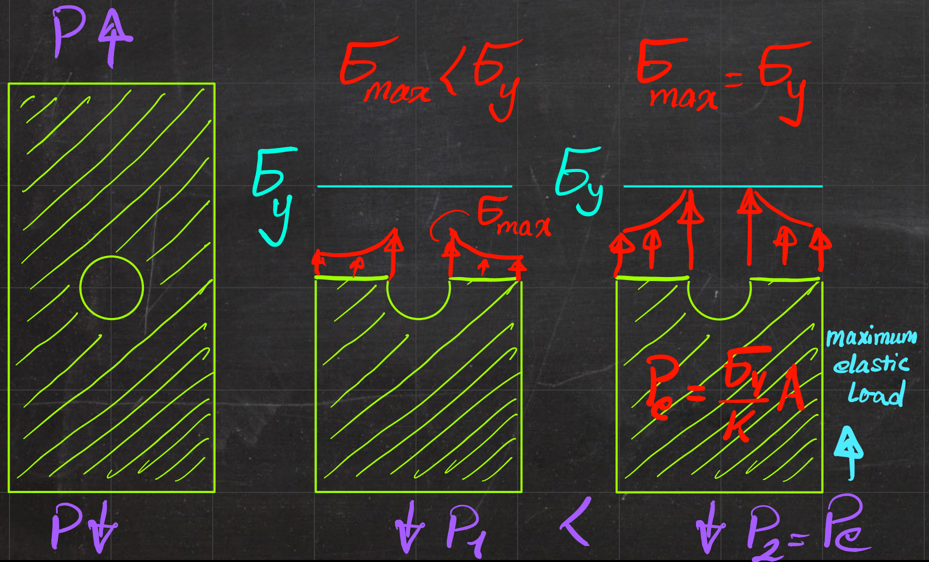
↳ Assume a ductile material that is "elastic perfectly plastic"



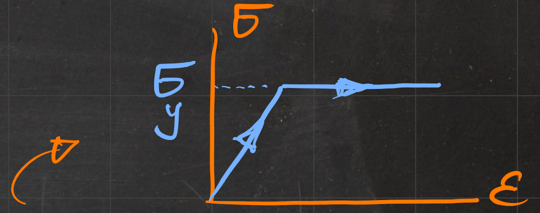
INELASTIC AXIAL DEFORMATION



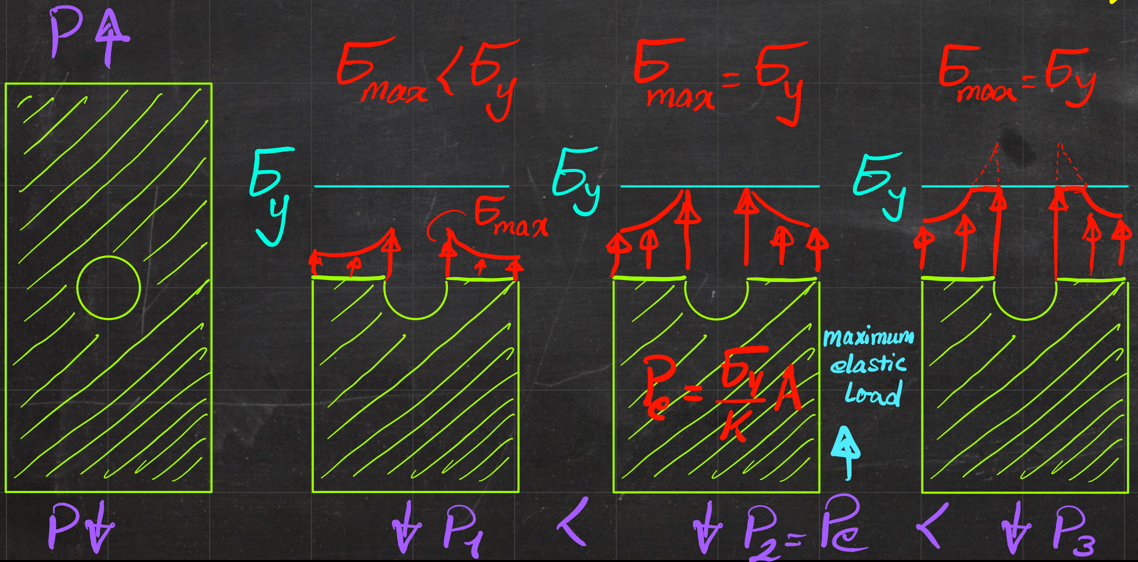
Assume a ductile material that is "elastic perfectly plastic"



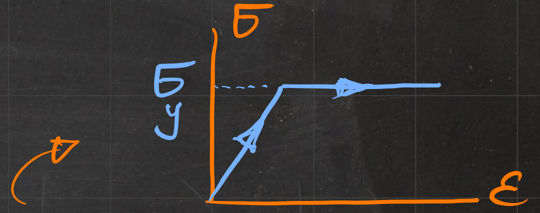
INELASTIC AXIAL DEFORMATION



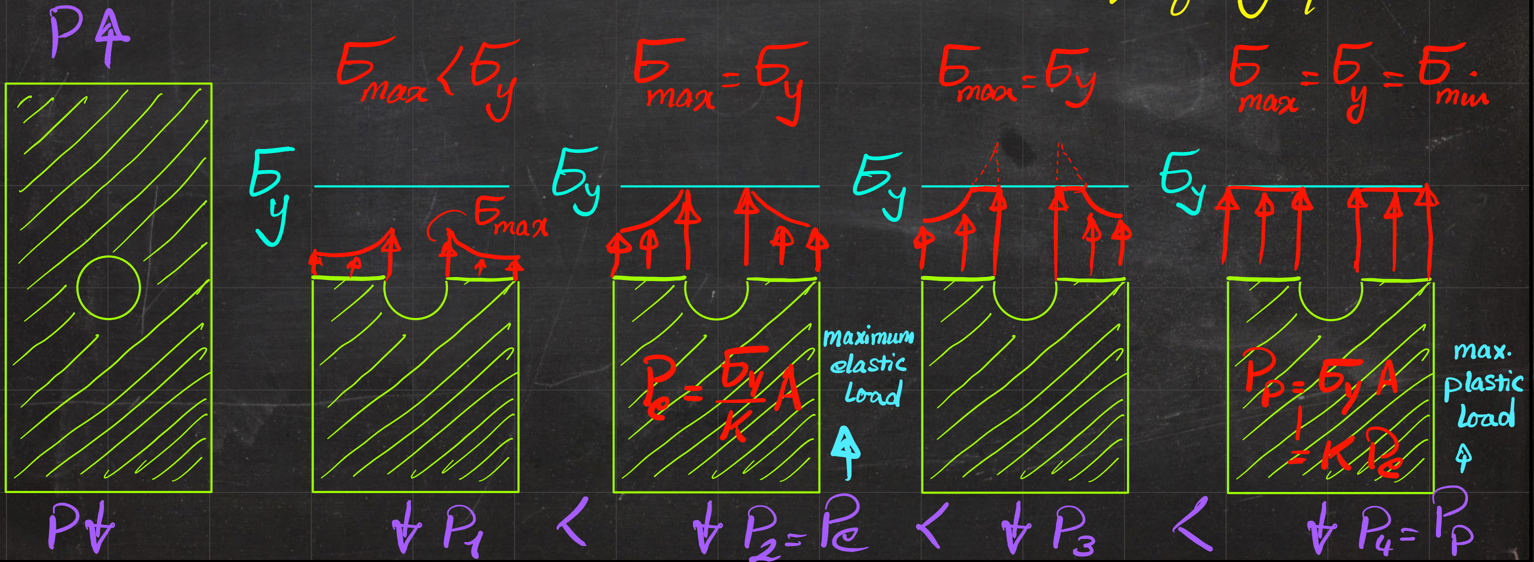
Assume a ductile material that is "elastic perfectly plastic"



INELASTIC AXIAL DEFORMATION

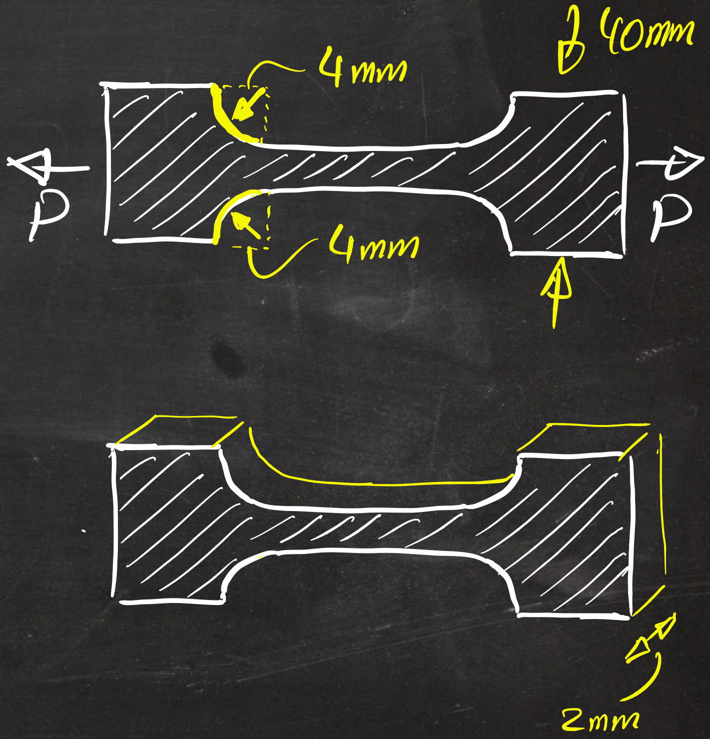


Assume a ductile material that is "elastic perfectly plastic"

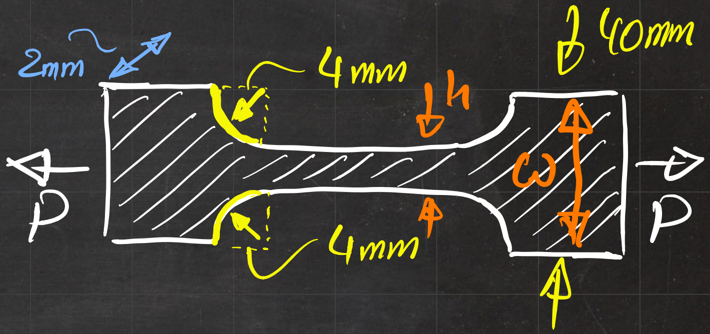


Exercise 2 . [similar to … P. 167 … 4.12]

THE BAR IN THE FIGURE IS MADE OF STEEL AND IS ASSUMED TO BE ELASTIC PERFECTLY PLASTIC WITH $\sigma_y = 250$ MPa. DETERMINE (a) MAXIMUM P WITHOUT CAUSING THE STEEL TO YIELD AND (b) MAXIMUM P THAT THE BAR CAN SUPPORT.



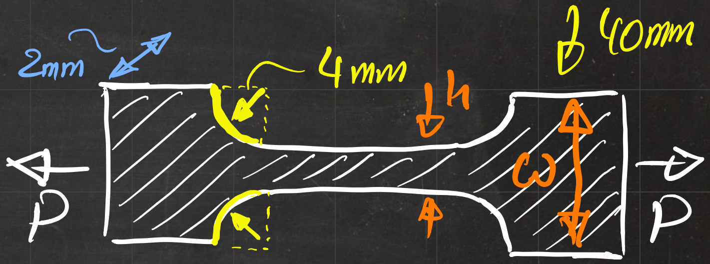
$$h = 40 - 2r = 32\ mm$$



$$\frac{r}{h} = \frac{4}{32} = 0.125, \quad \frac{w}{h} = \frac{40}{32} = 1.25$$

$$h = 40 - 2r = 32\,mm$$

$$\frac{r}{h} = \frac{4}{32} = 0.125\,, \quad \frac{w}{h} = \frac{40}{32} = 1.25$$



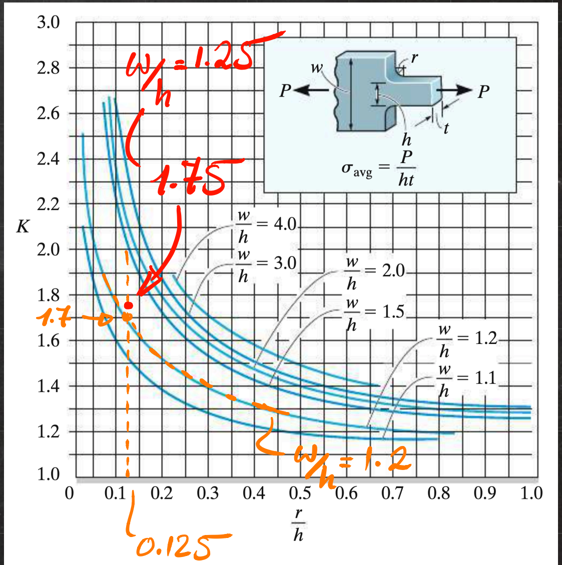
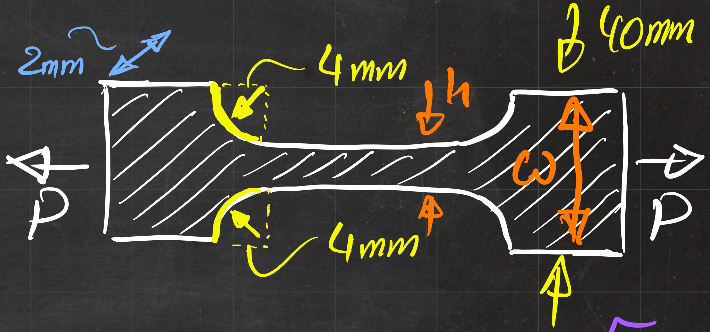


Fig. 4–23

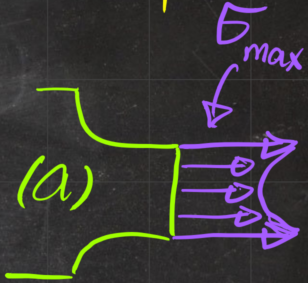
$$h = 40 - 2r = 32\ \text{mm}$$

$$\frac{r}{h} = \frac{4}{32} = 0.125\,, \quad \frac{w}{h} = \frac{40}{32} = 1.25$$



$$K = 1.75$$

$$\sigma_{avg} = \frac{P}{A}\,, \quad \sigma_{max} = K\,\sigma_{avg}$$

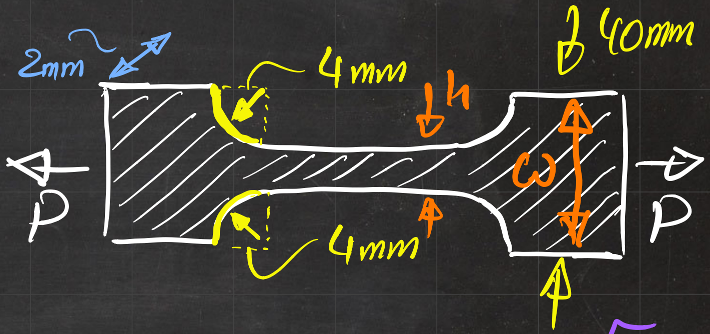


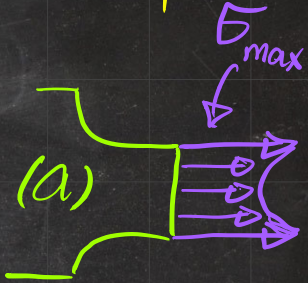
$h = 40 - 2r = 32\ mm$

$\frac{r}{h} = \frac{4}{32} = 0.125, \quad \frac{\omega}{h} = \frac{40}{32} = 1.25$

$K = 1.75$

$\sigma_{avg} = \frac{P}{A}, \quad \sigma_{max} = K\,\sigma_{avg}$

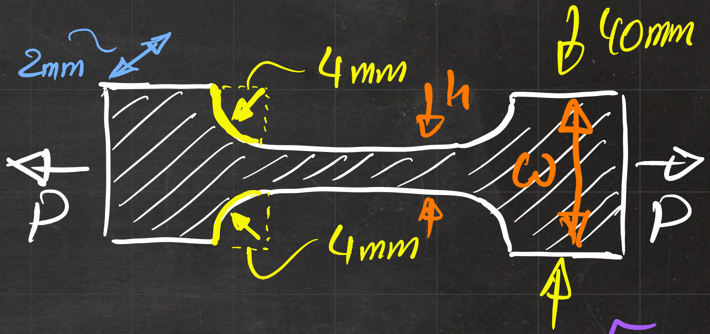




a) $\sigma_{max} = \sigma_y \Rightarrow K\,\sigma_{avg} = \sigma_y \Rightarrow K\,\frac{P}{A} = \sigma_y$

$A = 0.002 \times 0.032$

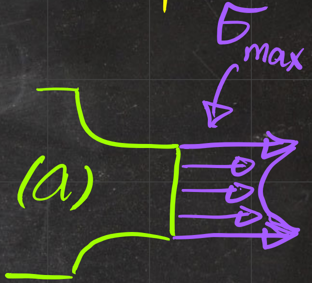
$h = 40 - 2r = 32\ mm$



$\frac{r}{h} = \frac{4}{32} = 0.125$, $\frac{\omega}{h} = \frac{40}{32} = 1.25$

$K = 1.75$

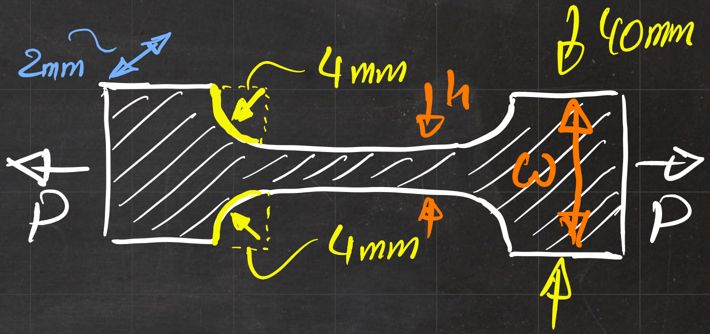
$\sigma_{avg} = \frac{P}{A}$, $\sigma_{max} = K\sigma_{avg}$



a) $\sigma_{max} = \sigma_y \Rightarrow K\sigma_{avg} = \sigma_y \Rightarrow K\frac{P}{A} = \sigma_y$

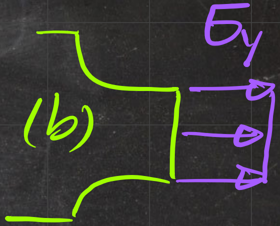
MAXIMUM ELASTIC LOAD $\Rightarrow P_e = 9.14\,kN$ (0.002×0.032

$h = 40 - 2r = 32\ mm$



$\frac{r}{h} = \frac{4}{32} = 0.125, \quad \frac{\omega}{h} = \frac{40}{32} = 1.25$

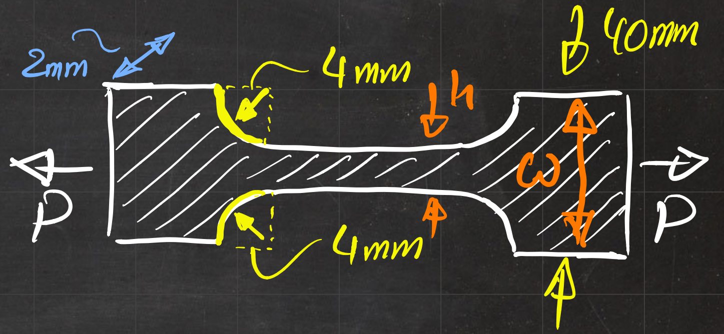
$K = 1.75 \qquad \sigma_{avg} = \frac{P}{A}, \quad \sigma_{max} = K\,\sigma_{avg}$



b) $\sigma = \sigma_y \Rightarrow \frac{P}{A} = \sigma_y$

$A = 0.002 \times 0.032$

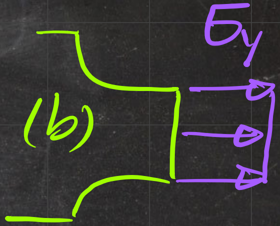
$$h = 40 - 2r = 32\,mm$$



$$\frac{r}{h} = \frac{4}{32} = 0.125, \quad \frac{\omega}{h} = \frac{40}{32} = 1.25$$

$$K = 1.75$$

$$\sigma_{avg} = \frac{P}{A}, \quad \sigma_{max} = K\,\sigma_{avg}$$



$$b)\ \sigma = \sigma_y \Rightarrow \frac{P}{A} = \sigma_y \Rightarrow P_p = 16\ kN$$

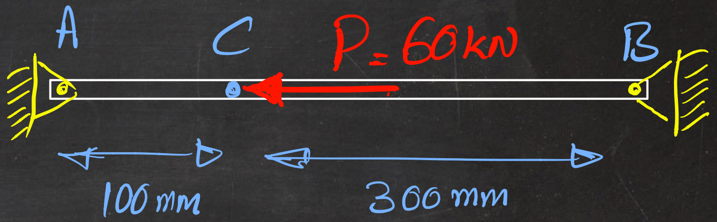
$A = 0.002 \times 0.032$

MAXIMUM PLASTIC LOAD

$$\Rightarrow P_p = K\,P_e$$

(1.75, 9.14)

The rod shown in the figure has a radius of 5mm and is made of an elastic perfectly plastic material for which $\sigma_y = 420$ MPa, $E = 70$ GPa.

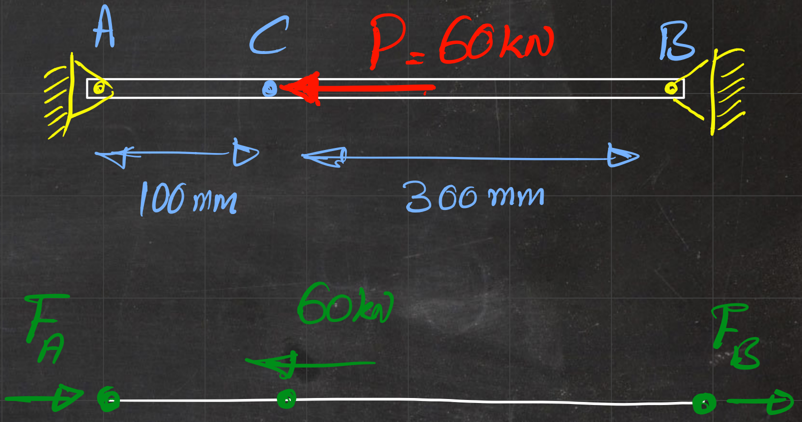


If a force of P = 60 kN is applied to the rod, determine supports reactions and stress in each section.

$$F_A + F_B - 60 = 0$$

$$AC \times F_A = CB \times F_B$$

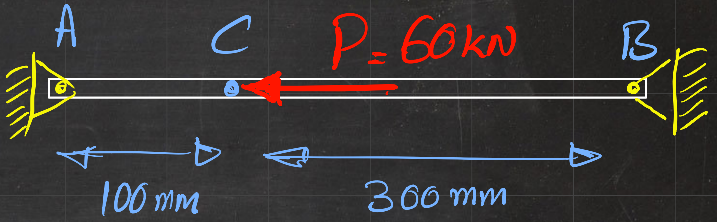
$$\Rightarrow \quad F_A = 45\ \text{kN}, \quad F_B = 15\ \text{kN}$$



$$F_A + F_B - 60 = 0$$

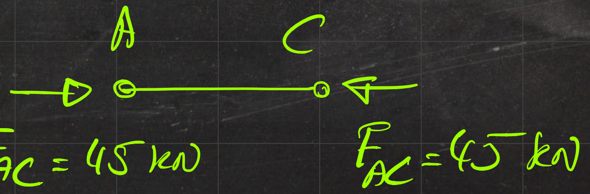
$$AC \times F_A = CB \times F_B$$

$$\Rightarrow F_A = 45\ \text{kN}, \quad F_B = 15\ \text{kN}$$



$$\sigma_{AC} = \frac{45\ \text{kN}}{\pi r^2} = 573\ \text{MPa}\ (C)$$

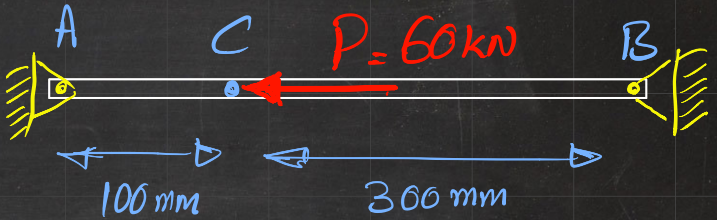
$$r = 5\ \text{mm} = 0.005$$



$$F_A + F_B - 60 = 0$$

$$AC \times F_A = CB \times F_B$$

$$\Rightarrow F_A = 45\ kN, \quad F_B = 15\ kN$$



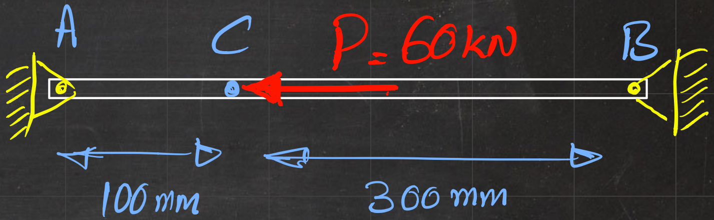
$$\sigma_{AC} = \frac{45\ kN}{\pi r^2} = 573\ MPa\ (C)$$

$$\sigma_{CB} = \frac{15\ kN}{\pi r^2} = 191\ MPa\ (T)$$

$$F_A + F_B - 60 = 0$$

$$AC \times F_A = CB \times F_B$$

$$\Rightarrow F_A = 45\ \text{kN}, \quad F_B = 15\ \text{kN}$$

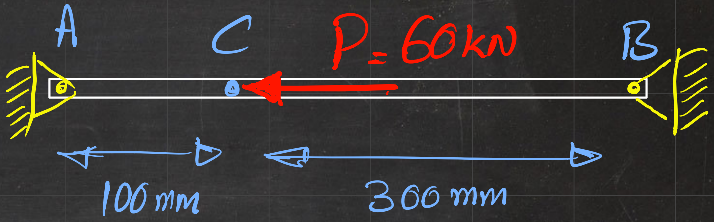


$$\sigma_{AC} = \frac{45\ \text{kN}}{\pi r^2} = 573\ \text{MPa (C)} > \sigma_y = 420\ \text{MPa} \ !$$

$$\Downarrow$$

$$\sigma_{AC} = 420\ \text{MPa}$$

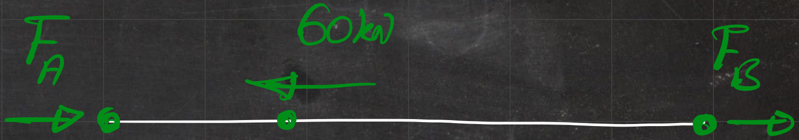
$$\sigma_{CB} = \frac{15\ \text{kN}}{\pi r^2} = 191\ \text{MPa (T)}$$



$$F_A + F_B - 60 = 0$$

$$AC \times F_A = CB \times F_B$$

$$\Rightarrow F_A = 45 \text{ kN}, \quad F_B = 15 \text{ kN}$$



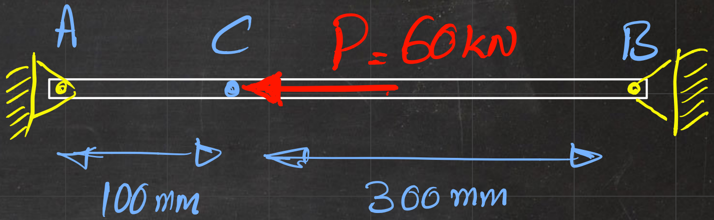
$$\sigma_{AC} = \frac{45 \text{ kN}}{\pi r^2} = \cancel{573 \text{ MPa}} \ (C)$$

$$\sigma_{CB} = \frac{15 \text{ kN}}{\pi r^2} = 191 \text{ MPa} \ (T)$$

$$\Rightarrow \sigma_{AC} = 420 \text{ MPa} = \sigma_y$$

$$\Downarrow$$

$$F_A = \sigma_{AC} \times A \ (\pi r^2) = 33 \text{ kN}$$



$$F_A + F_B - 60 = 0$$

$$AC \times F_A = CB \times F_B$$

$$\Rightarrow F_A = 45 \text{ kN}, \quad F_B = 15 \text{ kN}$$

$$\sigma_{AC} = \frac{45 \text{ kN}}{\pi r^2} = ~~573 \text{ MPa (C)}~~$$

$$\Rightarrow F_A = 33 \text{ kN}, \quad \sigma_{AC} = 420 \text{ MPa}$$

$$\sigma_{CB} = \frac{15 \text{ kN}}{\pi r^2} = ~~191 \text{ MPa (T)}~~$$

$$F_B = 27 \text{ kN} \Rightarrow \sigma_{CB} = \frac{27}{\pi r^2} = 344 \text{ MPa} \quad (< \sigma_y \checkmark)$$

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Axial loading

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