MECHANICS AND MATERIALS I

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19

Ali Javili

Bilkent University

MECHANICS AND MATERIALS I

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Bending iv

Sections ... 6.5 ... 6.9 – 6.10

Chap. 6

[Hibbeler 9th edition]

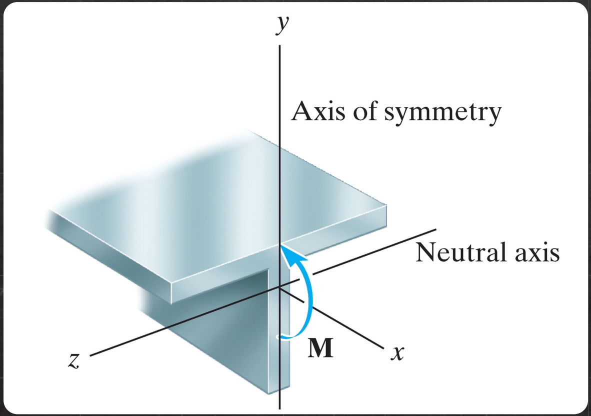
UNSYMMETRIC BENDING

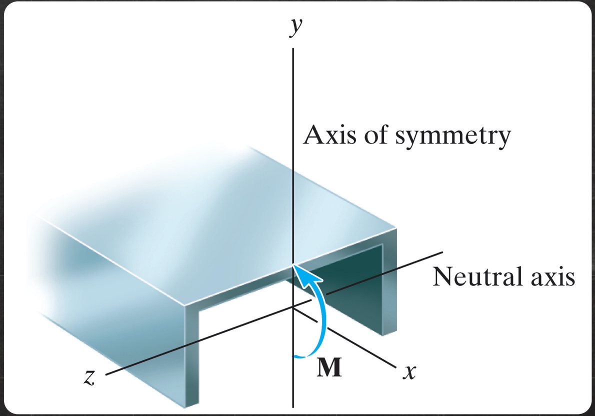
UNSYMMETRIC BENDING

↳ previously, cross-sectional area was

symmetric about an axis perpendicular to the neutral axis

and the resultant internal moment $\mathbb{M}$ acted along the neutral axis.



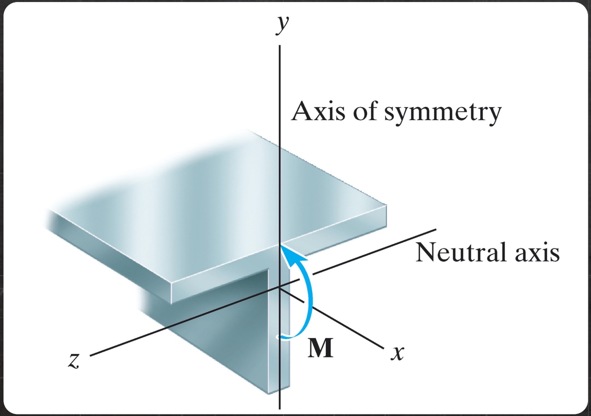


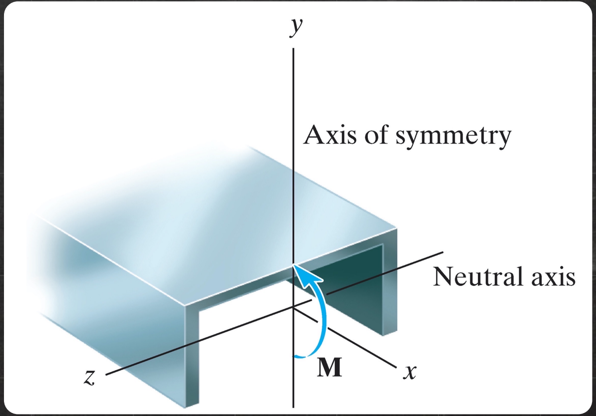
UNSYMMETRIC BENDING

FLEXURE FORMULA

↳ $\sigma = -\dfrac{My}{I}$

↳ previously, cross-sectional area was symmetric about an axis perpendicular to the neutral axis and the resultant internal moment M acted along the neutral axis.

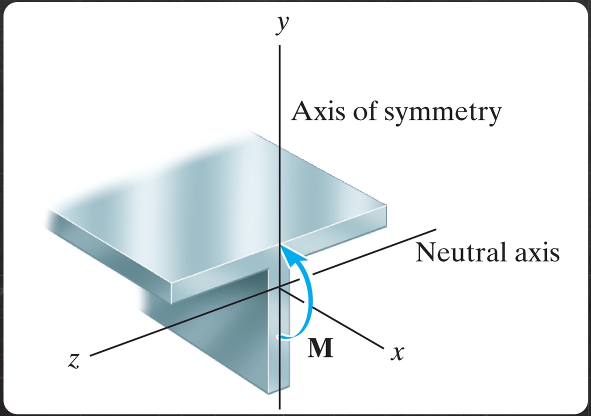


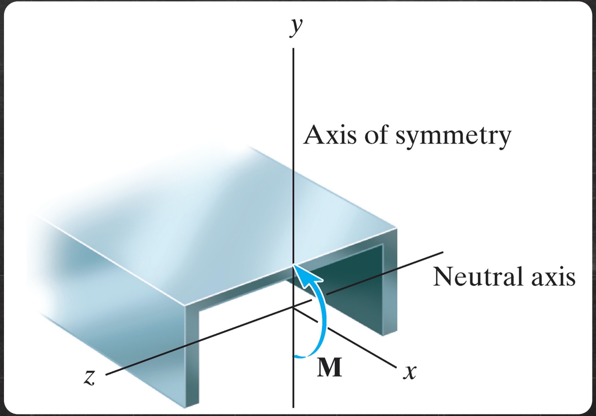


UNSYMMETRIC BENDING

$\Rightarrow$ FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION

$\Rightarrow$ previously, cross-sectional area was symmetric about an axis perpendicular to the neutral axis and the resultant internal moment $\mathbf{M}$ acted along the neutral axis.





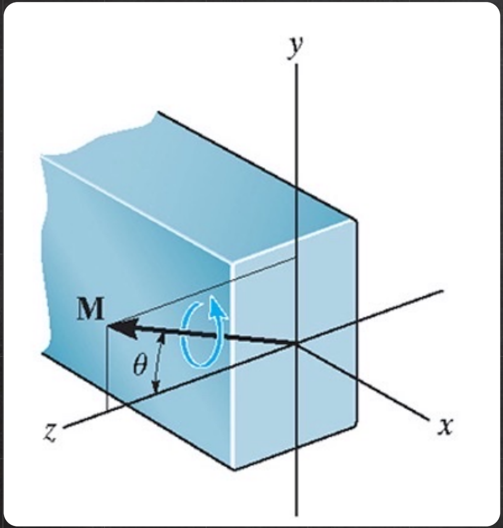
UNSYMMETRIC BENDING

m → FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION

↳ y-axis and z-axis are the principal axes of the cross-section!

↳ We assume we know them for simplicity

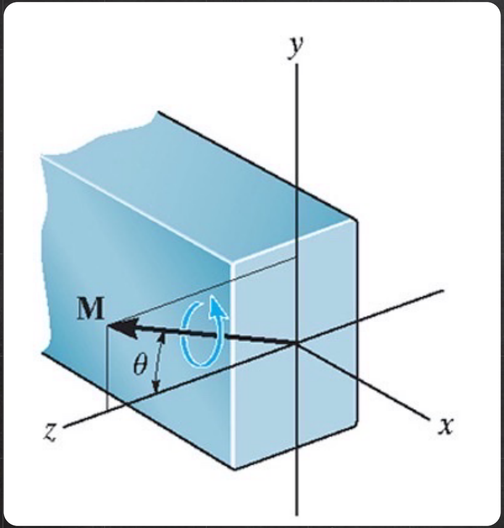
↳ otherwise, we must calculate them first!



↳ ARBITRARILY APPLIED MOMENT

UNSYMMETRIC BENDING

$\Rightarrow$ FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION

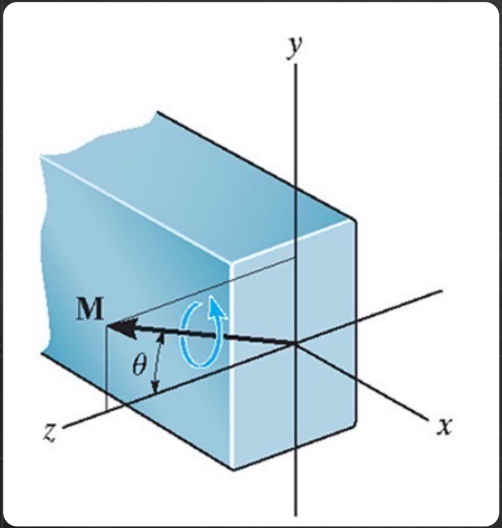


↳ ARBITRARILY APPLIED MOMENT

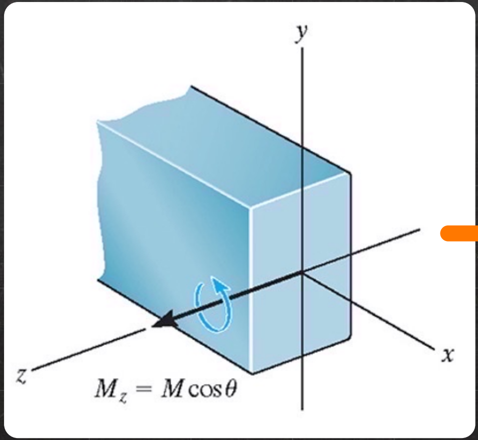
FLEXURE FORMULA: $\sigma = -\dfrac{My}{I}$

UNSYMMETRIC BENDING $\Rightarrow$ FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION

FLEXURE FORMULA : $\sigma = -\dfrac{My}{I}$



↳ ARBITRARILY APPLIED MOMENT



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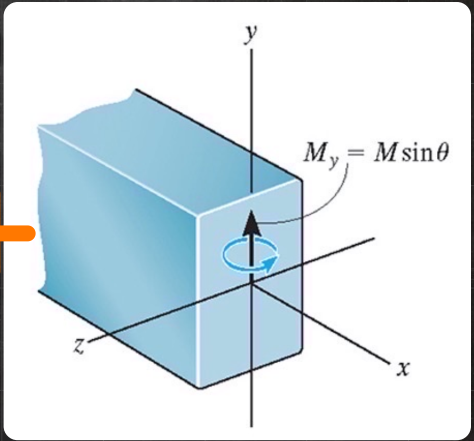
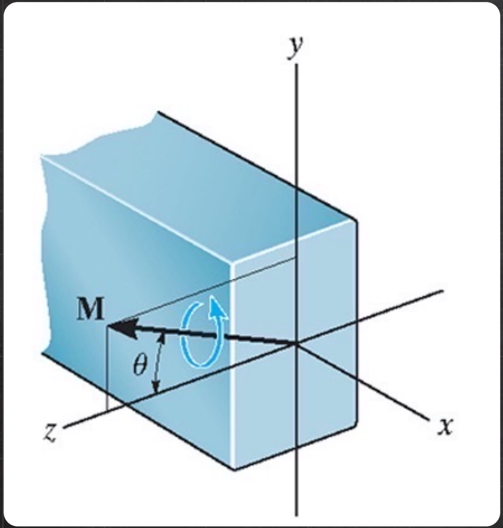
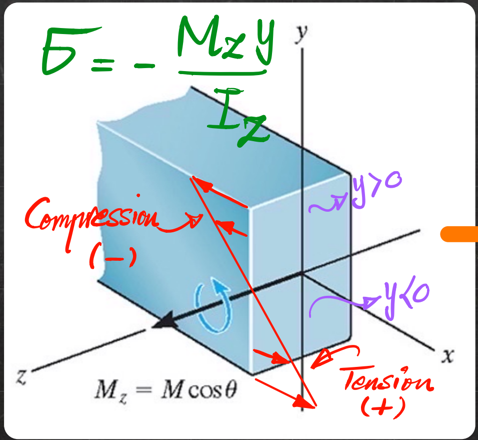


UNSYMMETRIC BENDING $\rightsquigarrow$ FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION



& ARBITRARILY APPLIED MOMENT

FLEXURE FORMULA : $\sigma = -\dfrac{M y}{I}$



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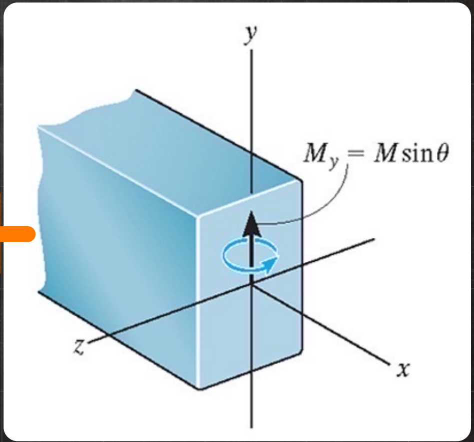
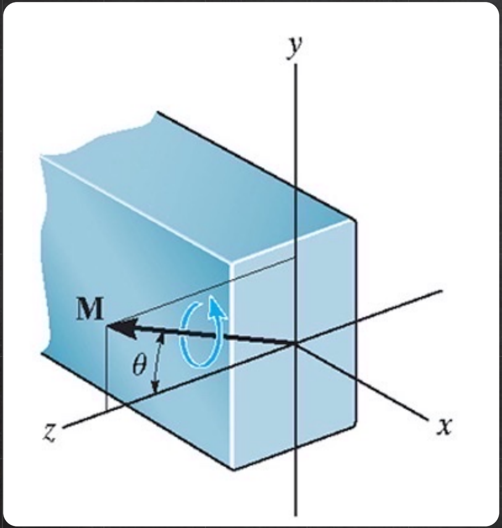


UNSYMMETRIC BENDING $\rightarrow$ FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION

FLEXURE FORMULA : $\sigma = -\dfrac{My}{I}$



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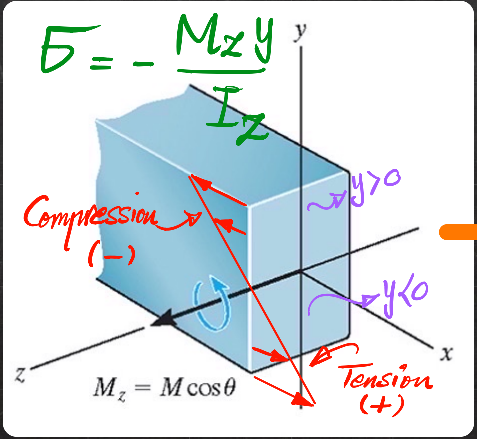

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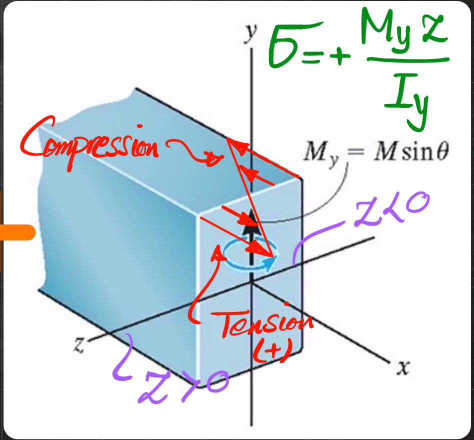
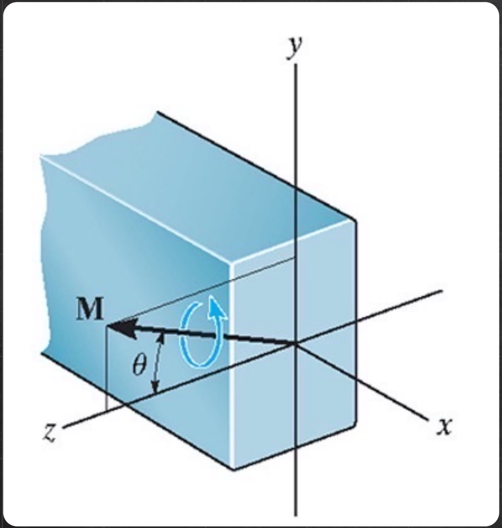


UNSYMMETRIC BENDING

→ FLEXURE-FORMULA IS STILL VALID BUT IN CONJUNCTION WITH THE PRINCIPLE OF SUPERPOSITION

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



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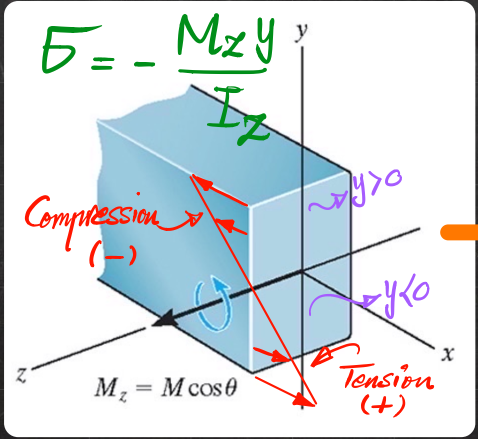

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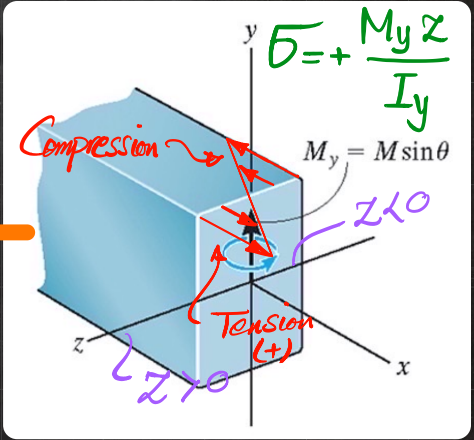
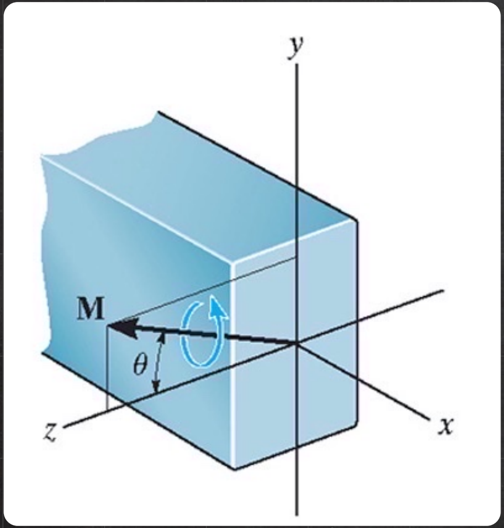


UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



$$M_z = M \cos\theta$$

$$M_y = M \sin\theta$$

UNSYMMETRIC BENDING

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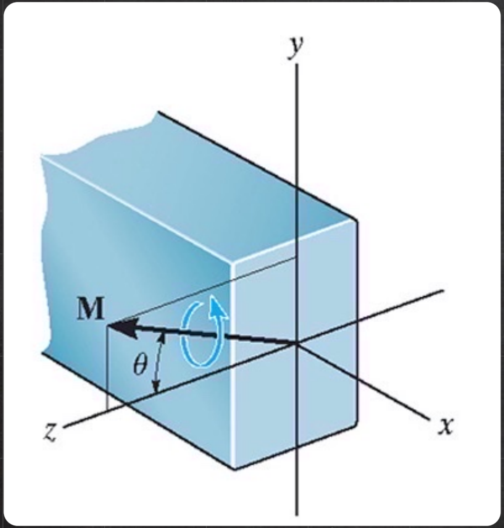


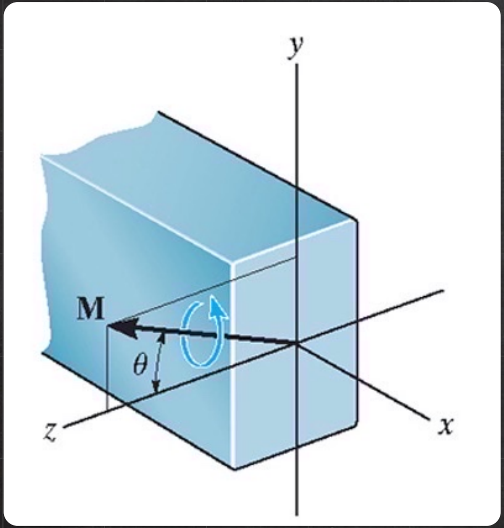
How about neutral axis ?

$$M_z = M \cos\theta$$

$$M_y = M \sin\theta$$

UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



$M_z = M \cos\theta$

$M_y = M \sin\theta$

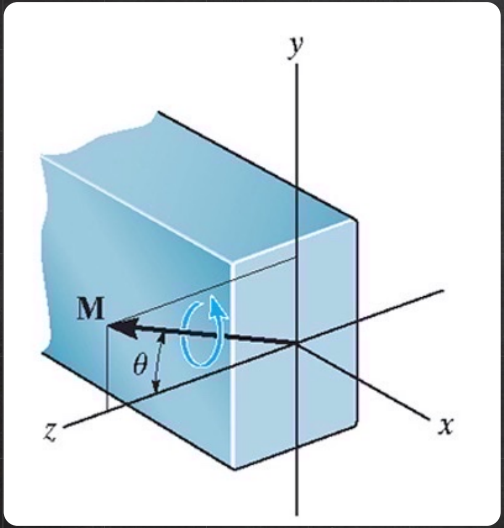
HOW ABOUT NEUTRAL AXIS ?

↳ as before, it passes through centroid

↳ $\int \sigma \, dA = 0$

Unsymmetric Bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



$M_z = M \cos\theta$

$M_y = M \sin\theta$

How about neutral axis?

↳ as before, it passes through centroid

↳ $\int \sigma \, dA = 0$

↳ its direction follows from its definition

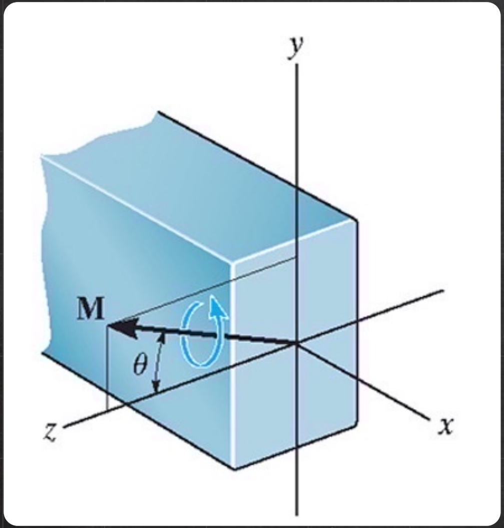
Neutral axis

$\varepsilon = 0$

↳ $\sigma_x = 0$

UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



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$M_y = M \sin\theta$

HOW ABOUT NEUTRAL AXIS ?

↳ as before, it passes through centroid

↳ $\int \sigma \, dA = 0$

↳ its direction follows from its definition

$$\sigma_x = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_x} \overset{!}{=} 0$$

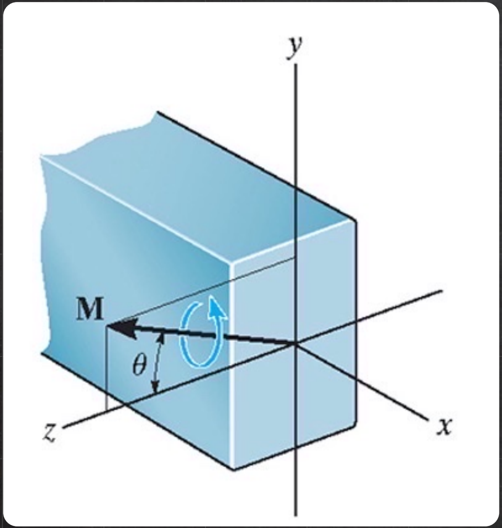
Neutral axis

$\varepsilon = 0$

↳ $\sigma_x = 0$

UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



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$$M_y = M \sin\theta$$

HOW ABOUT NEUTRAL AXIS ?

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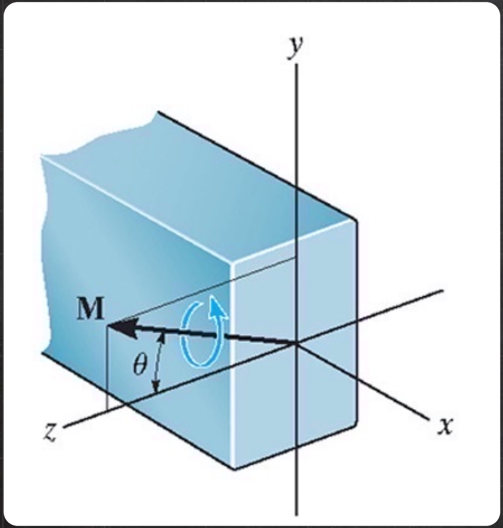
$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_x} \overset{!}{=} 0$$

$$\Rightarrow \frac{M_z y}{I_z} = \frac{M_y z}{I_x}$$

Unsymmetric Bending

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



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How about neutral axis?

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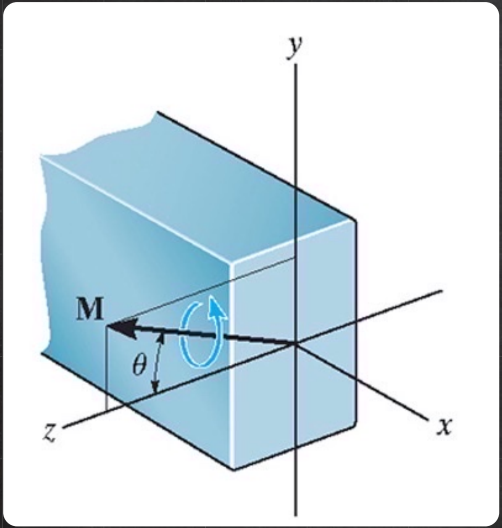
$$\sigma_x = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_x} \overset{!}{=} 0$$

$$\Rightarrow \frac{M_z\, y}{I_z} = \frac{M_y\, z}{I_x} \Rightarrow y = \left(\frac{I_z}{I_y}\frac{M_y}{M_z}\right) z$$

Unsymmetric Bending

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



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How about neutral axis?

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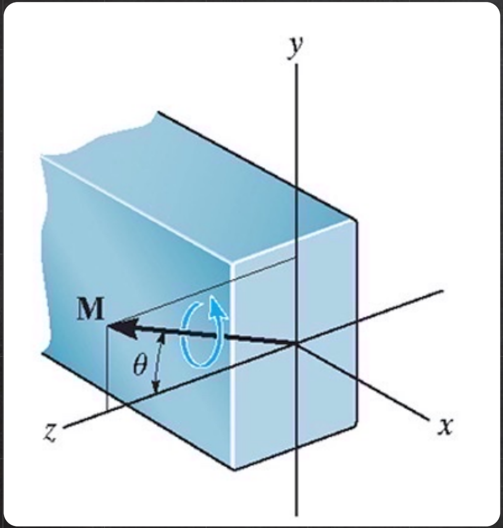
Neutral Axis Equation

$$y = \left(\frac{I_z}{I_y}\,\frac{M_y}{M_z}\right) z$$

Unsymmetric Bending

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$



$$M_z = M \cos\theta$$

$$M_y = M \sin\theta$$

How about neutral axis?

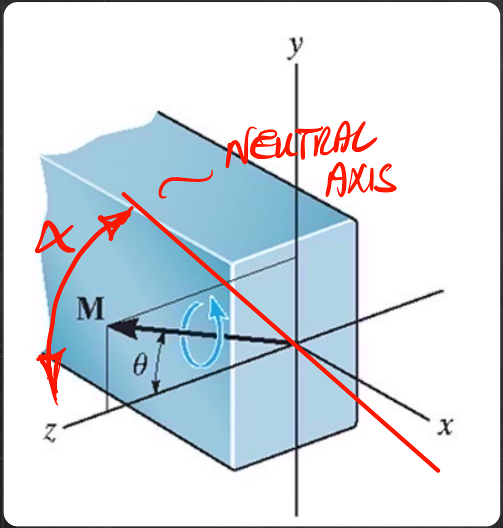
↳ as before, it passes through centroid

$$y = \left(\frac{I_z}{I_y}\frac{M_y}{M_z}\right) z \qquad \frac{M_y}{M_z} = \tan\theta$$

Neutral Axis Equation

UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



$M_z = M \cos\theta$

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HOW ABOUT NEUTRAL AXIS ?

as before, it passes through centroid

Neutral Axis Equation

$$y = \left(\frac{I_z}{I_y}\frac{M_y}{M_z}\right) z$$

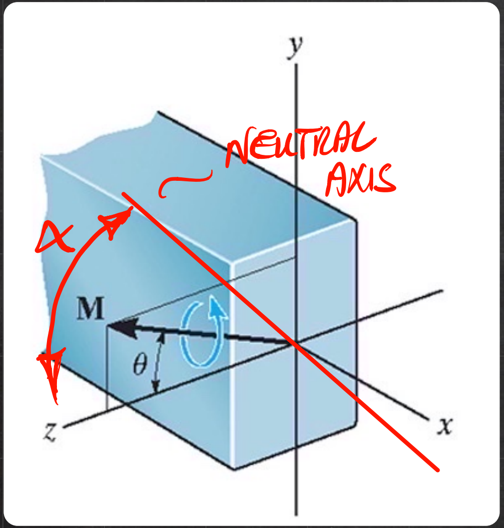
$$\frac{M_y}{M_z} = \tan\theta$$

$$\Downarrow$$

$$y = (\tan\alpha)\, z$$

UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



$$M_z = M \cos\theta$$

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HOW ABOUT NEUTRAL AXIS ?

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Neutral Axis Equation

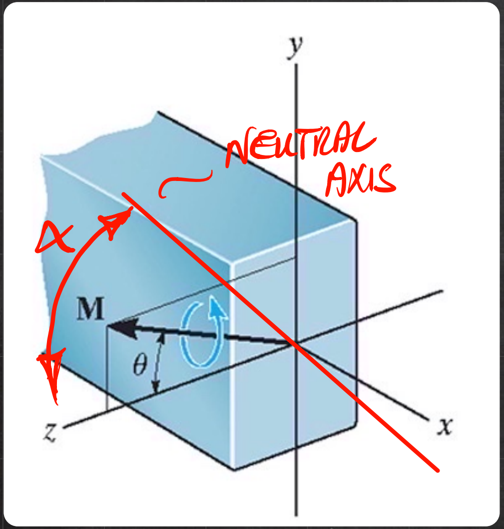
$$y = \left(\frac{I_z}{I_y}\frac{M_y}{M_z}\right) z \quad \frac{M_y}{M_z} = \tan\theta$$

$$\Downarrow$$

$$y = (\tan\alpha)\, z \quad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$

Unsymmetric Bending

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

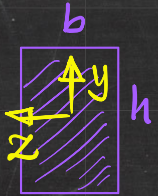


Neutral Axis Equation

$$y = \left(\frac{I_z}{I_y}\frac{M_y}{M_z}\right) z \qquad \frac{M_y}{M_z} = \tan\theta$$

$$\Downarrow$$

$$y = (\tan\alpha)\, z \qquad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$



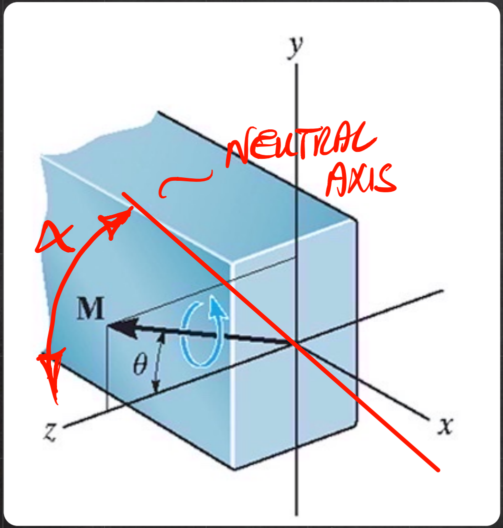
$$M_z = M\cos\theta$$

$$M_y = M\sin\theta$$

$$\frac{I_z}{I_y} = \left(\frac{h}{b}\right)^2$$

UNSYMMETRIC BENDING

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$



$$M_z = M \cos\theta$$

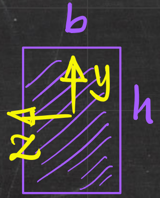
$$M_y = M \sin\theta$$

Neutral Axis Equation

$$y = \left(\frac{I_z}{I_y}\frac{M_y}{M_z}\right) z \qquad \frac{M_y}{M_z} = \tan\theta$$

$$\Downarrow$$

$$y = (\tan\alpha)\, z \qquad \tan\alpha = \frac{I_z}{I_y}\tan\theta$$



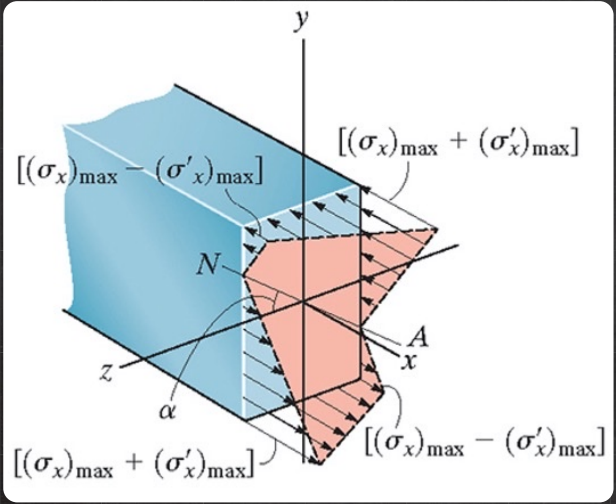
e.g. $I_z = I_y \Rightarrow \alpha = \theta$

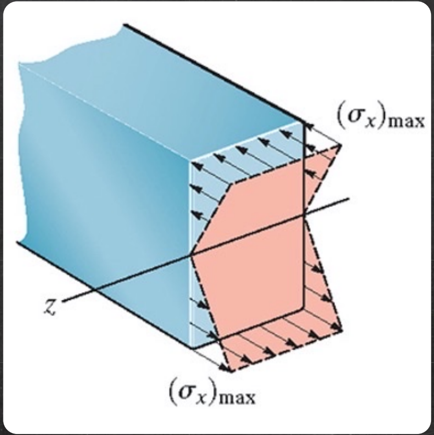
$$\frac{I_z}{I_y} = \left(\frac{h}{b}\right)^2 \qquad I_z > I_y \Rightarrow \alpha > \theta$$

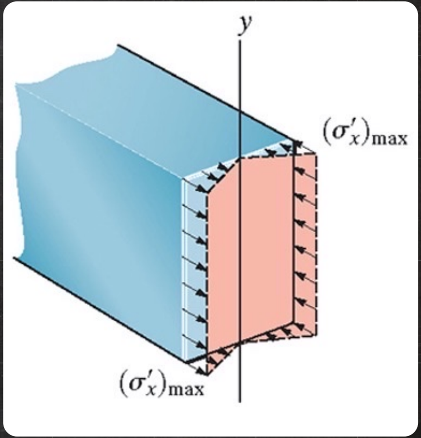
NEUTRAL AXIS TURNS TOWARDS LONGER SIDE

UNSYMMETRIC BENDING

$$\sigma_x = -\underbrace{\frac{M_z\, y}{I_z}}_{(\sigma_x)_{max}} + \underbrace{\frac{M_y\, z}{I_y}}_{(\sigma_x')_{max}}$$







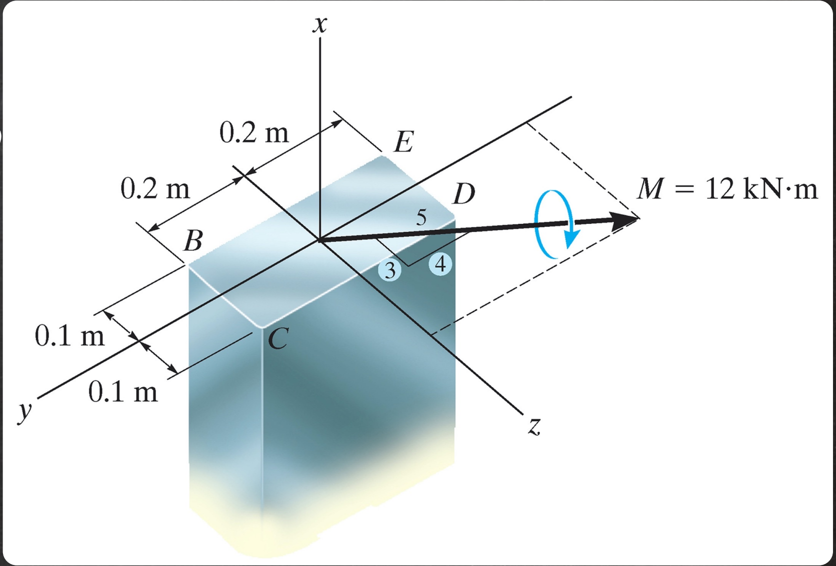
↳ NEUTRAL AXIS:

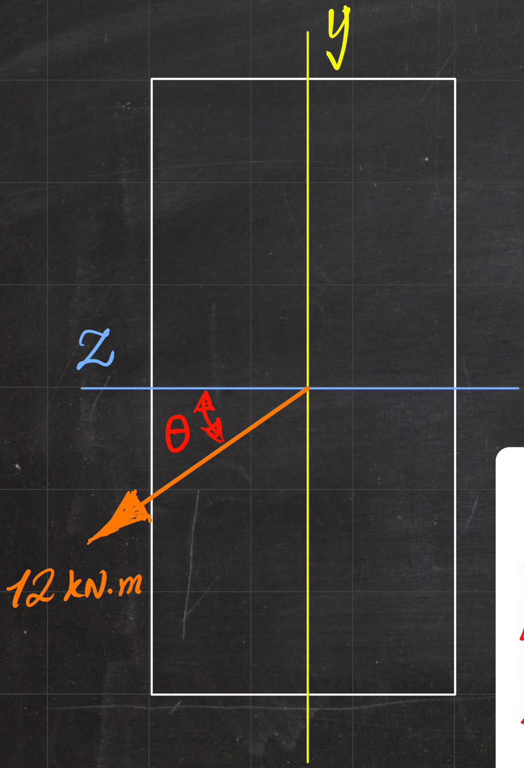
$$y = (\tan\alpha)\, z$$

Exercise 1 . [similar to … P. 310 … 6.15]

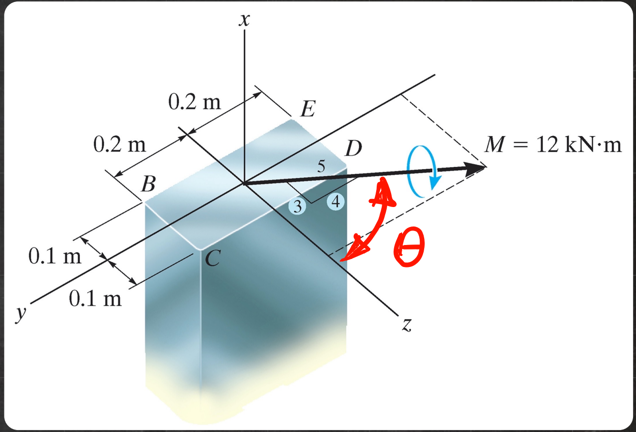
THE RECTANGULAR CROSS SECTION SHOWN IN THE FIGURE IS SUBJECTED TO THE BENDING MOMENT M=12 kN.m AS SHOWN.

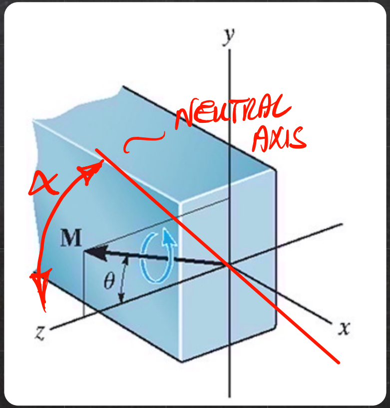
DETERMINE THE NORMAL STRESS DEVELOPED AT EACH CORNER OF THE SECTION, AND SPECIFY THE ORIENTATION OF THE NEUTRAL AXIS.





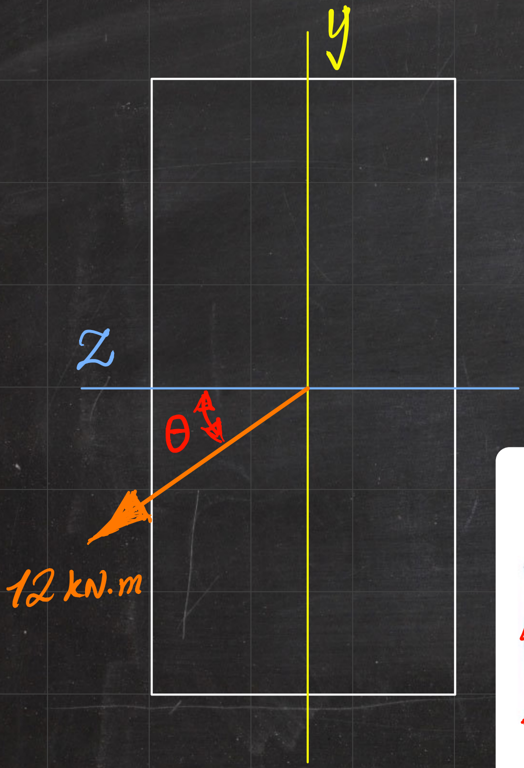
$\theta = -53.1^\circ$





RECALL:

WE MEASURED θ FROM Z-AXIS TOWARDS Y-AXIS



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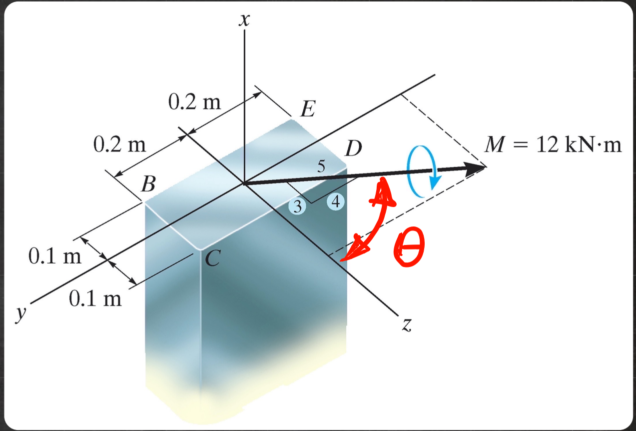
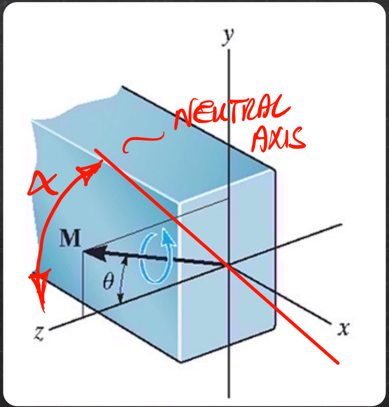
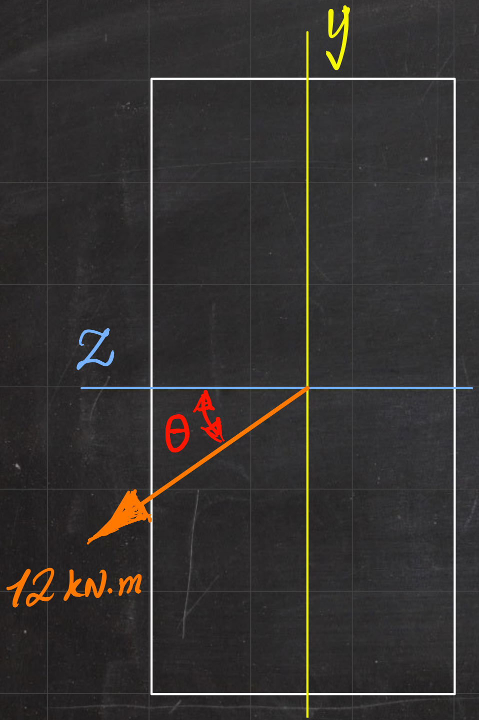




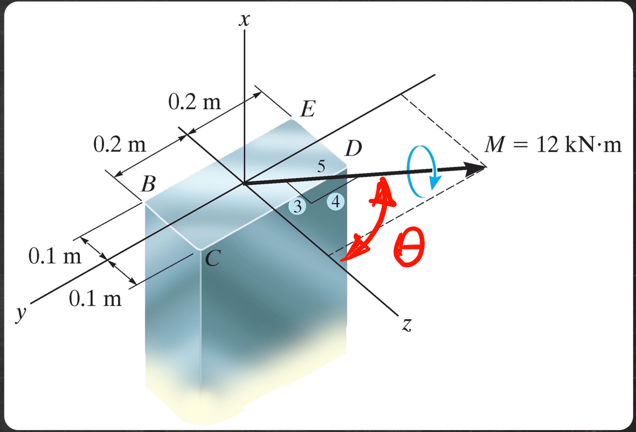
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$M_z = M \cos\theta$$

$$M_y = M \sin\theta$$



$\theta = -53.1^\circ \Leftarrow$



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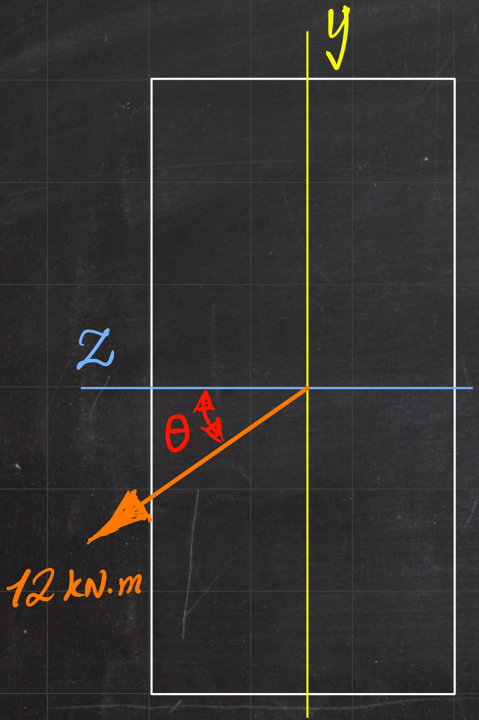
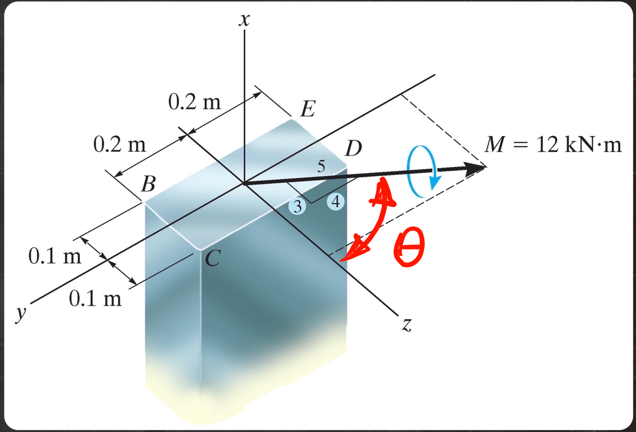


$$\theta = -53.1^\circ \Leftarrow$$

$$\cos\theta = \frac{3}{5}$$

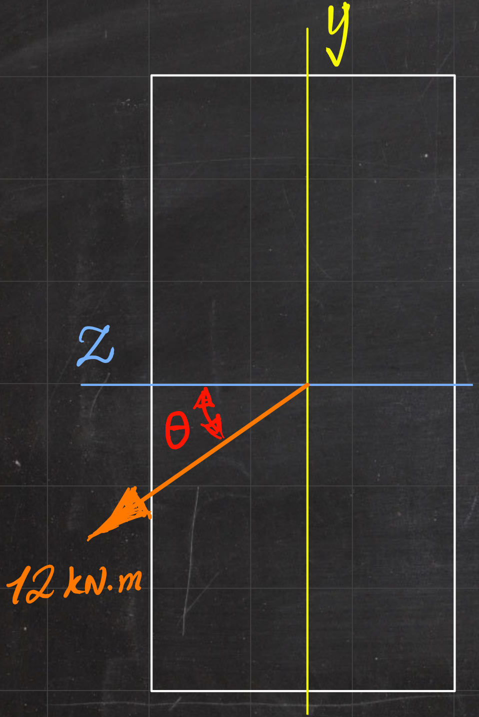
$$\sin\theta = -\frac{4}{5}$$



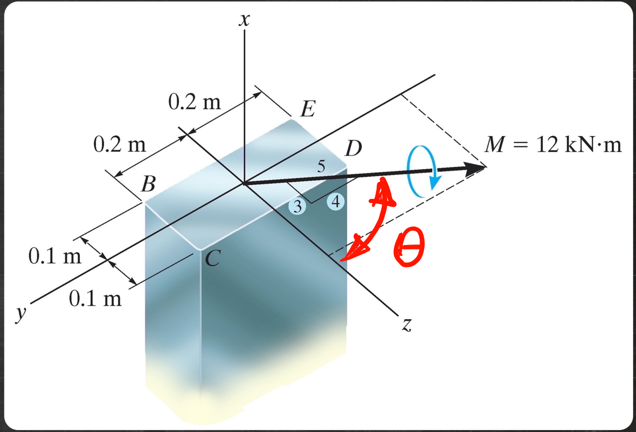
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$$M_z = M\cos\theta$$

$$M_y = M\sin\theta$$



$$\theta = -53.1^\circ$$



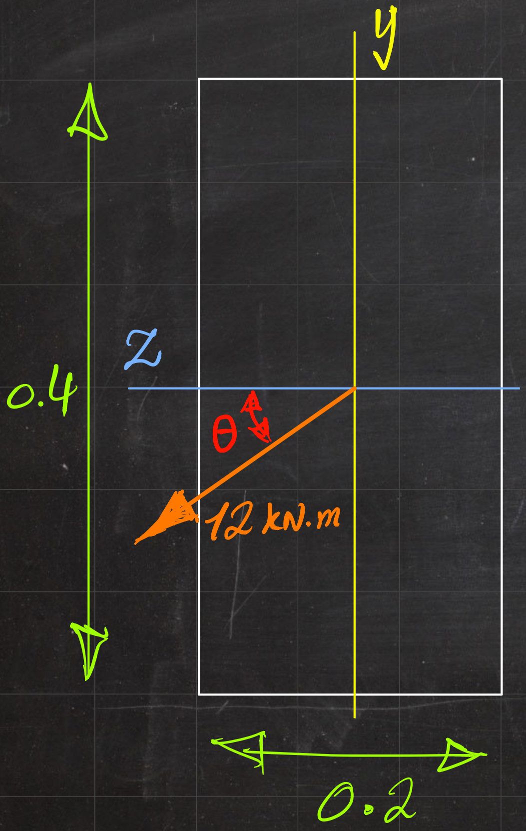
$$\cos\theta = \frac{3}{5}$$

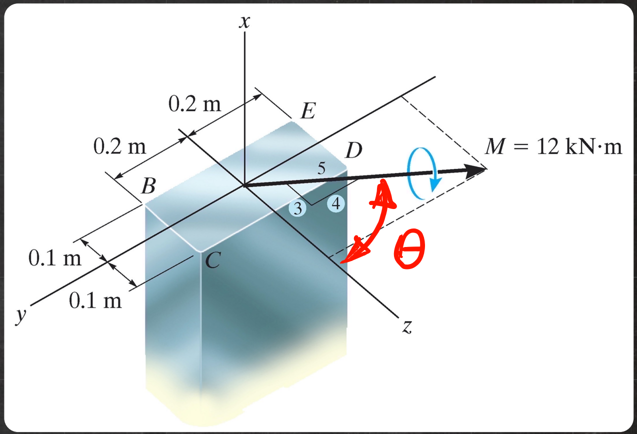
$$\sin\theta = -\frac{4}{5}$$

$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$$M_z = 12 \times \frac{3}{5} = 7.2 \text{ kN.m} \qquad M_z = M\cos\theta$$

$$M_y = 12 \times \left(-\frac{4}{5}\right) = -9.6 \text{ kN.m} \qquad M_y = M\sin\theta$$

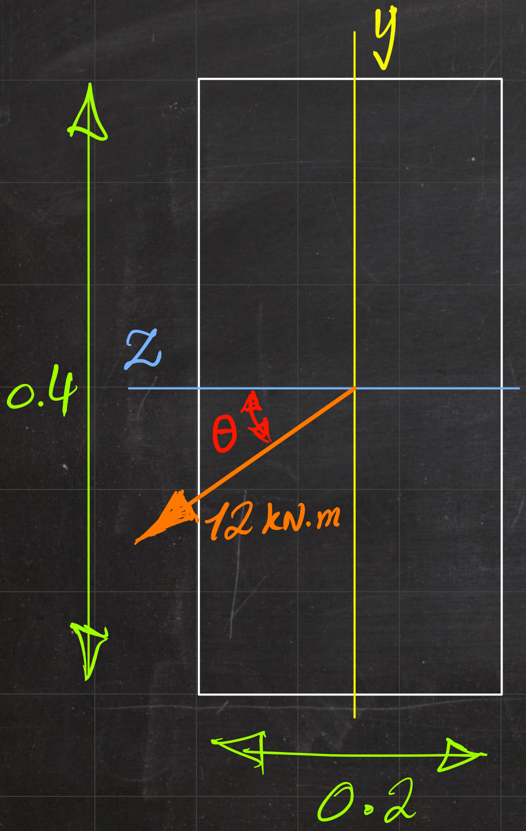




$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$$M_z = 12 \times \frac{3}{5} = 7.2 \text{ kN.m} \qquad M_z = M \cos\theta$$

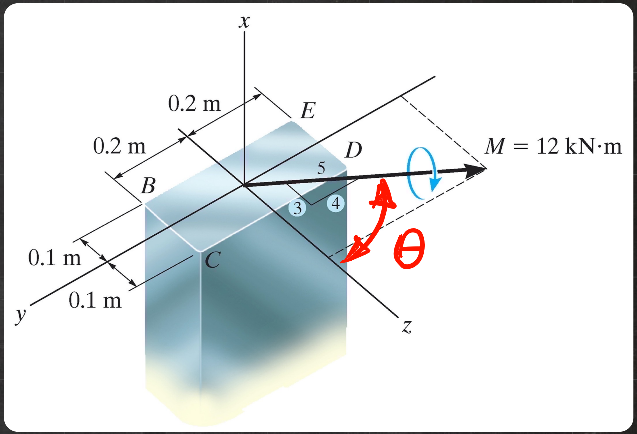
$$M_y = 12 \times \left(-\frac{4}{5}\right) = -9.6 \text{ kN.m} \qquad M_y = M \sin\theta$$



$$I_y = \frac{1}{12} \times 0.4 \times 0.2^3$$

$$\Downarrow$$

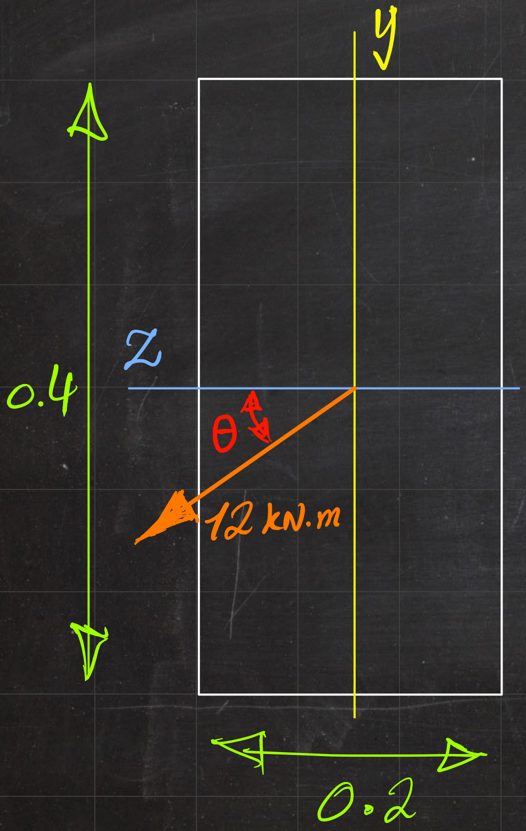
$$I_y = 0.267 \times 10^{-3} \ m^4$$



$$\sigma = -\frac{M_z \, y}{I_z} + \frac{M_y \, z}{I_y}$$

$$M_z = 12 \times \frac{3}{5} = 7.2 \ kN.m \qquad M_z = M \cos\theta$$

$$M_y = 12 \times \left(-\frac{4}{5}\right) = -9.6 \ kN.m \qquad M_y = M \sin\theta$$



$$I_y = \frac{1}{12} \times 0.4 \times 0.2^3$$

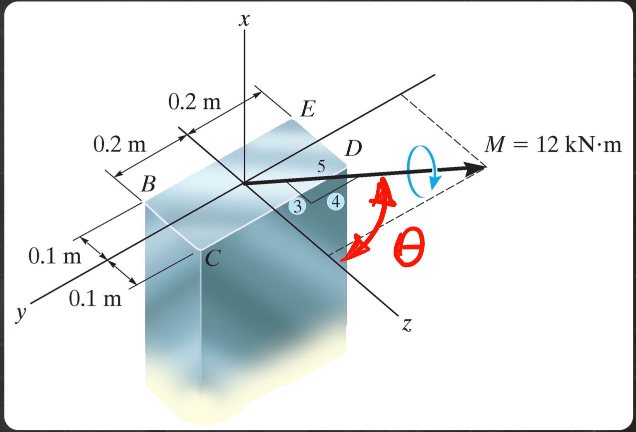
$$\Downarrow$$

$$I_y = 0.267 \times 10^{-3} \ m^4$$

$$I_z = \frac{1}{12} \times 0.2 \times 0.4^3$$

$$\Downarrow$$

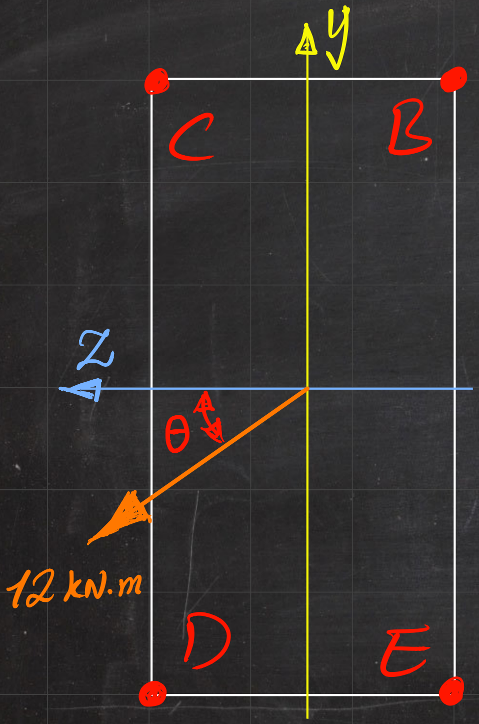
$$I_z = 1.067 \times 10^{-3} \ m^4$$



$$\sigma = -\frac{M_z \, y}{I_z} + \frac{M_y \, z}{I_y}$$

$$M_z = 12 \times \frac{3}{5} = 7.2 \text{ kN.m} \qquad M_z = M \cos\theta$$

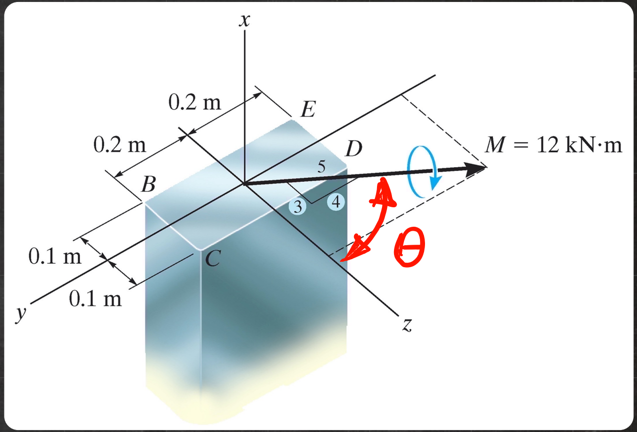
$$M_y = 12 \times \left(-\frac{4}{5}\right) = -9.6 \text{ kN.m} \qquad M_y = M \sin\theta$$



B :

$y_B = +0.2$

$z_B = -0.1$



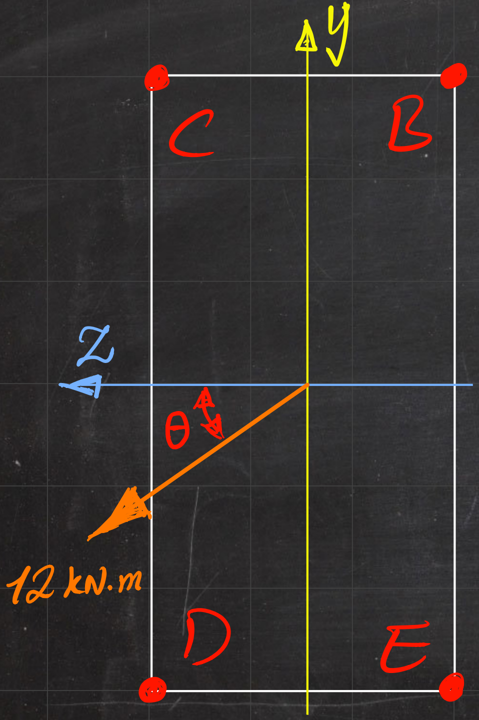
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$I_z = 1.067 \times 10^{-3}\ m^4$

$I_y = 0.267 \times 10^{-3}\ m^4$

$M_z = 7.2$ kN.m

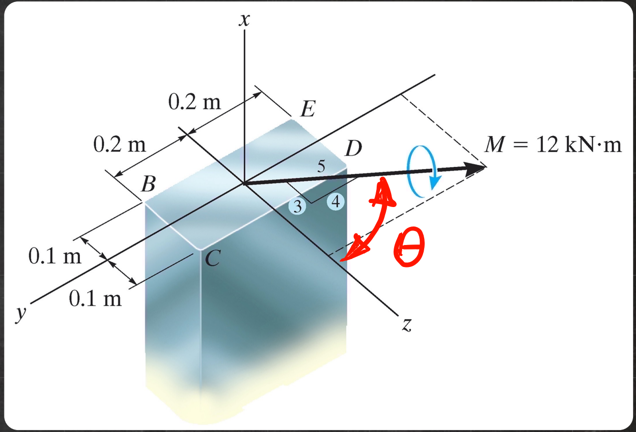
$M_y = -9.6$ kN.m



B :

$y_B = +0.2$

$z_B = -0.1$



$\sigma_B = 2.25 \text{ MPa}$

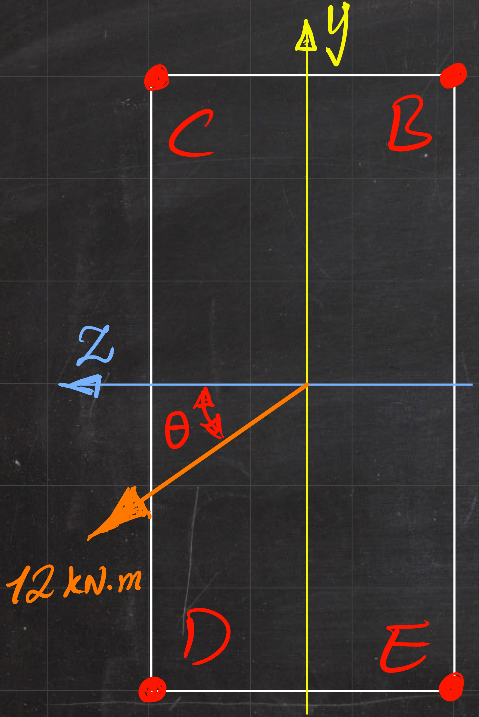
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$I_z = 1.067 \times 10^{-3} \text{ m}^4$

$I_y = 0.267 \times 10^{-3} \text{ m}^4$

$M_z = 7.2 \text{ kN.m}$

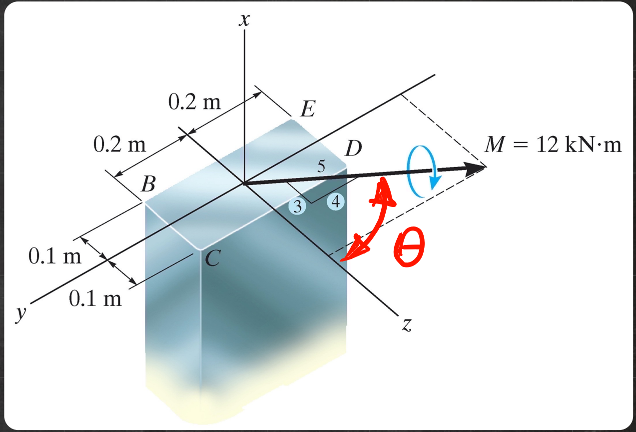
$M_y = -9.6 \text{ kN.m}$



C :

$$y_c = +0.2$$

$$z_c = +0.1$$



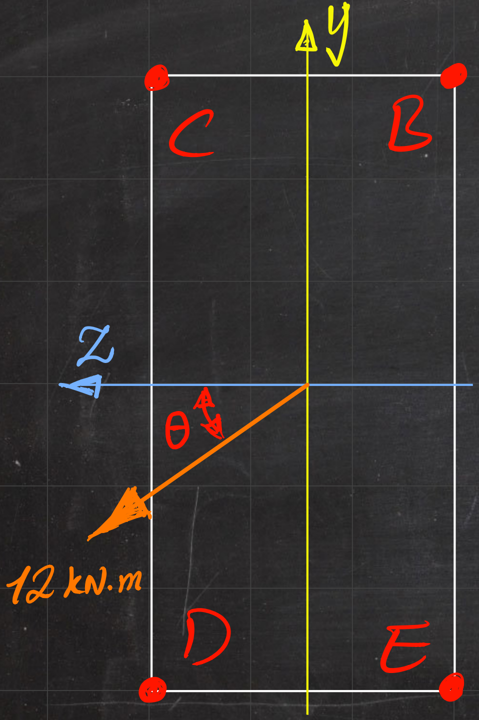
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$$I_z = 1.067 \times 10^{-3}\ m^4$$

$$I_y = 0.267 \times 10^{-3}\ m^4$$

$$M_z = 7.2\ kN.m$$

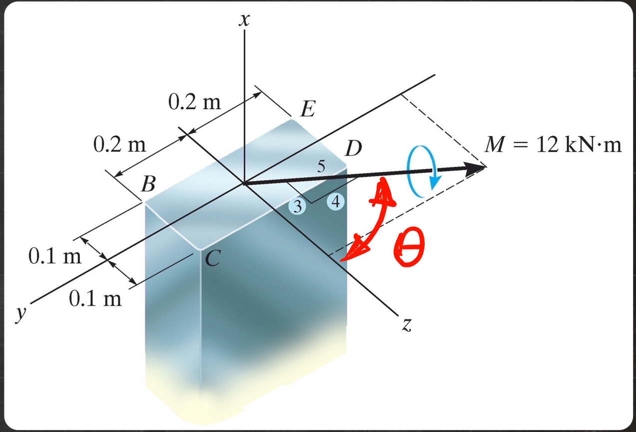
$$M_y = -9.6\ kN.m$$



C:

$y_c = +0.2$

$z_c = +0.1$



$\sigma_c = -4.95\ \text{MPa}$

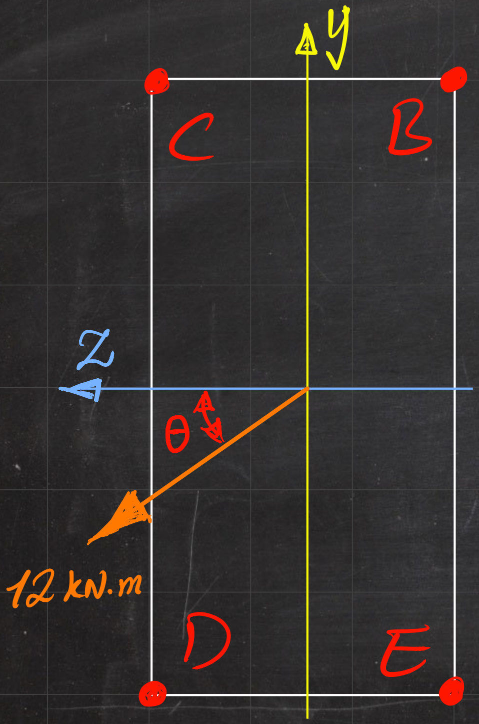
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$I_z = 1.067 \times 10^{-3}\ \text{m}^4$

$I_y = 0.267 \times 10^{-3}\ \text{m}^4$

$M_z = 7.2\ \text{kN.m}$

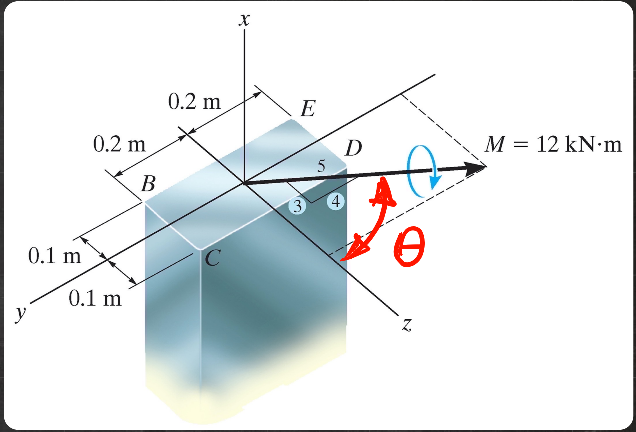
$M_y = -9.6\ \text{kN.m}$



D :

$y_D = -0.2$

$z_D = +0.1$



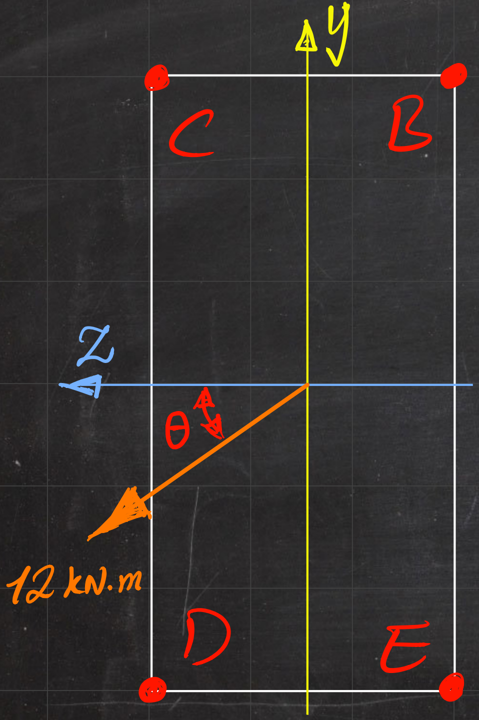
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

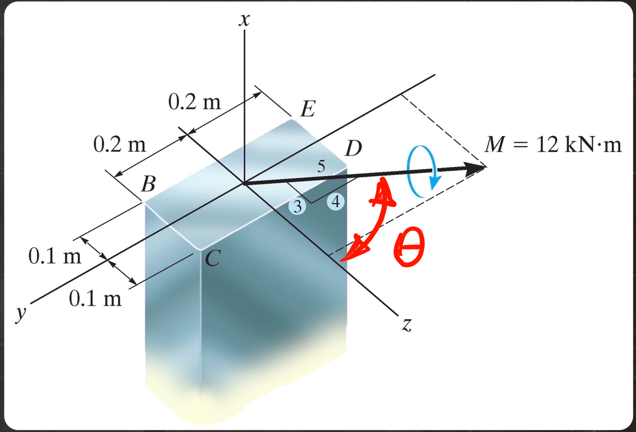
$I_z = 1.067 \times 10^{-3}\ m^4$

$I_y = 0.267 \times 10^{-3}\ m^4$

$M_z = 7.2\ kN.m$

$M_y = -9.6\ kN.m$





D :

$y_D = -0.2$

$z_D = +0.1$

$\sigma_D = -2.25$ MPa

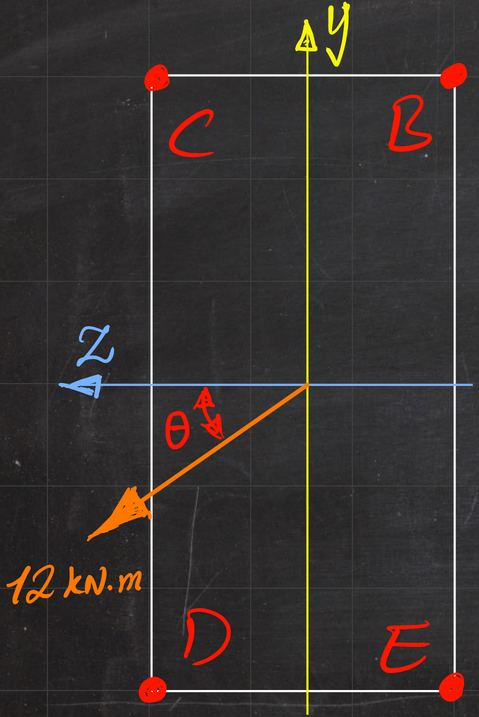
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$I_z = 1.067 \times 10^{-3}\ m^4$

$I_y = 0.267 \times 10^{-3}\ m^4$

$M_z = 7.2$ kN.m

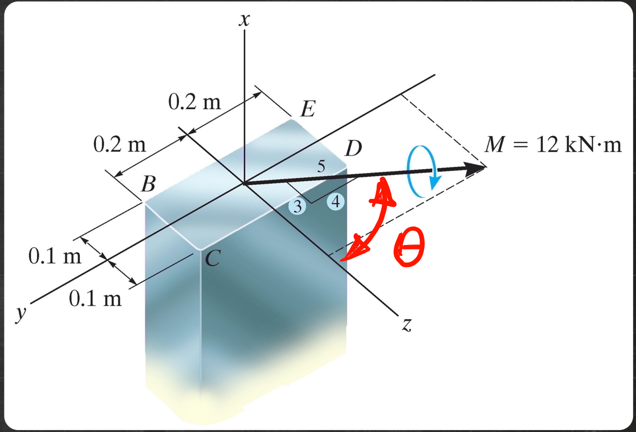
$M_y = -9.6$ kN.m



$E:$

$y_E = -0.2$

$z_E = -0.1$



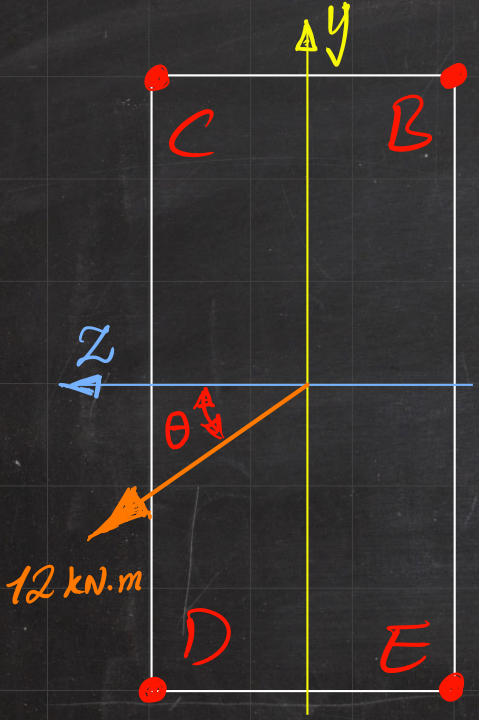
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$I_z = 1.067 \times 10^{-3} \ m^4$$

$$I_y = 0.267 \times 10^{-3} \ m^4$$

$$M_z = 7.2 \ kN.m$$

$$M_y = -9.6 \ kN.m$$

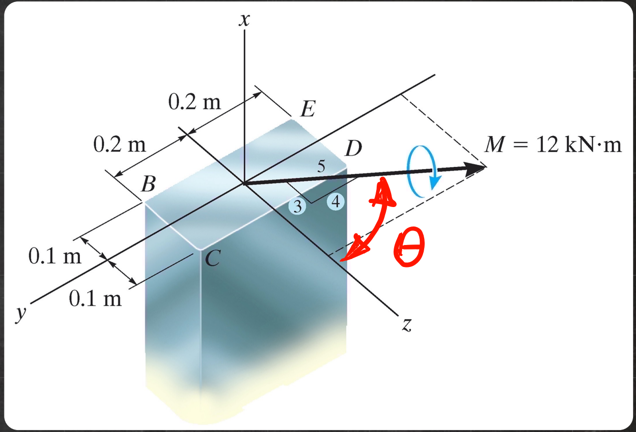


E:

$$y_E = -0.2$$

$$z_E = -0.1$$

$$\sigma_E = 4.95\ \text{MPa}$$



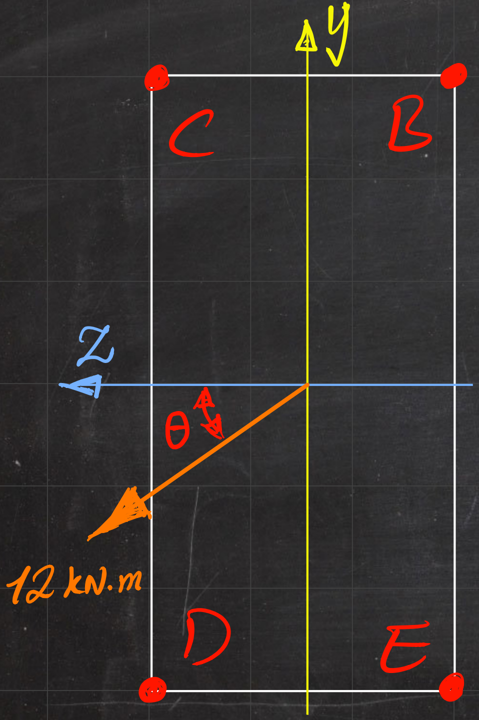
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$$I_z = 1.067 \times 10^{-3}\ \text{m}^4$$

$$I_y = 0.267 \times 10^{-3}\ \text{m}^4$$

$$M_z = 7.2\ \text{kN.m}$$

$$M_y = -9.6\ \text{kN.m}$$

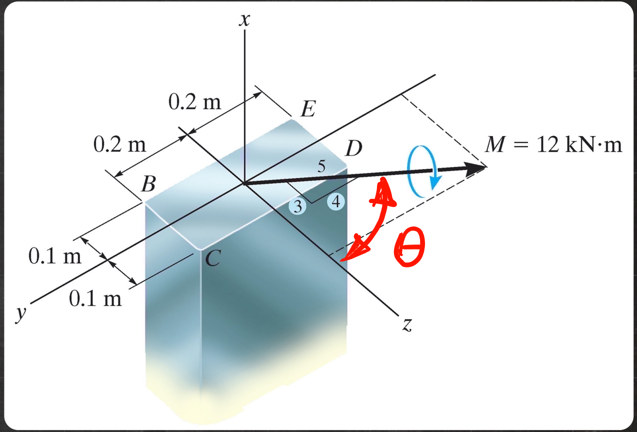


$\sigma_B = 2.25 \text{ MPa}$

$\sigma_C = -4.95 \text{ MPa}$

$\sigma_D = -2.25 \text{ MPa}$

$\sigma_E = 4.95 \text{ MPa}$



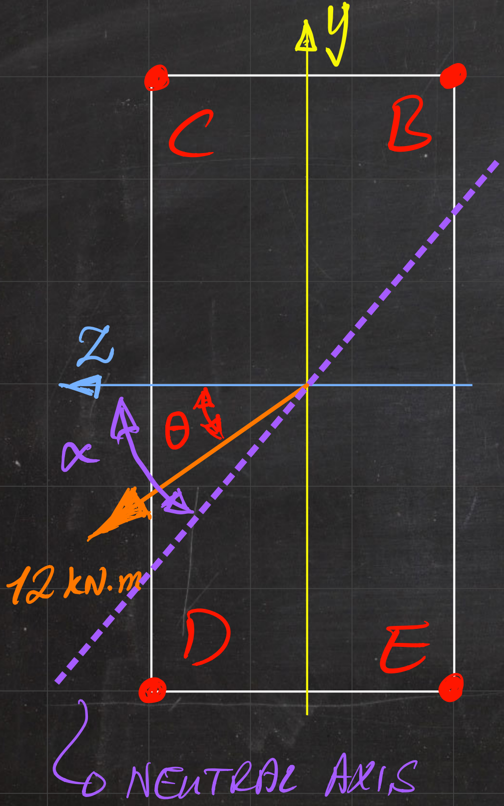
$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$I_z = 1.067 \times 10^{-3} \text{ m}^4$

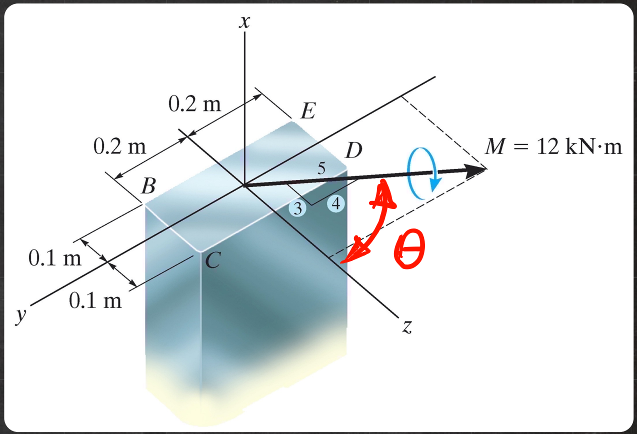
$I_y = 0.267 \times 10^{-3} \text{ m}^4$

$M_z = 7.2 \text{ kN.m}$

$M_y = -9.6 \text{ kN.m}$



$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$



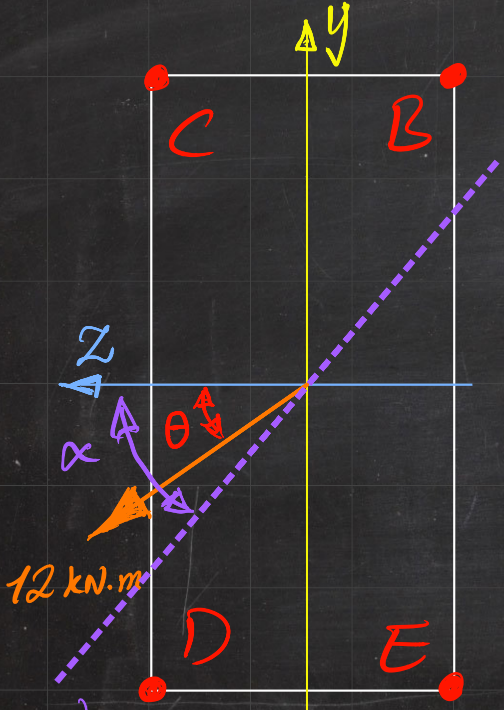
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

$$I_z = 1.067\times 10^{-3}\ m^4$$

$$I_y = 0.267\times 10^{-3}\ m^4$$

$$M_z = 7.2\ kN.m$$

$$M_y = -9.6\ kN.m$$

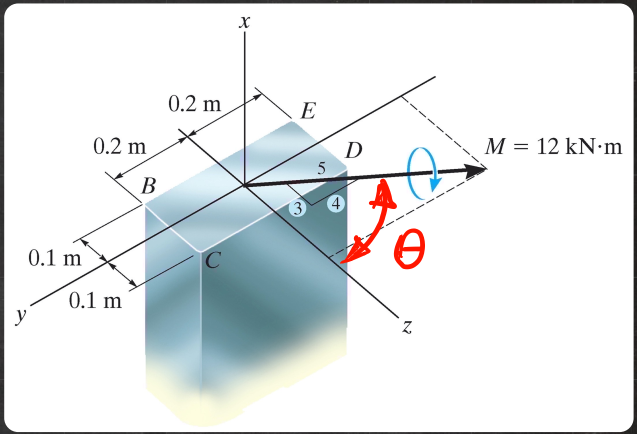


NEUTRAL AXIS

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta$$

$$\Downarrow$$

$$\alpha = -79.4^\circ$$



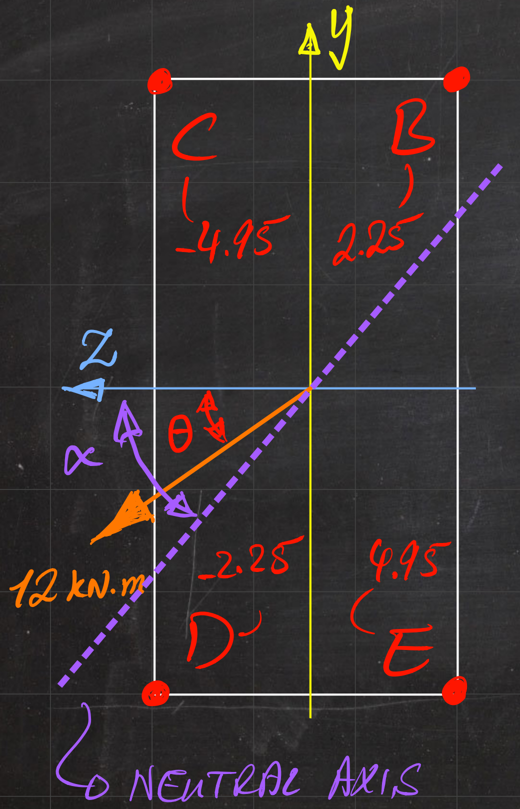
$$\sigma = -\frac{M_z\, y}{I_z} + \frac{M_y\, z}{I_y}$$

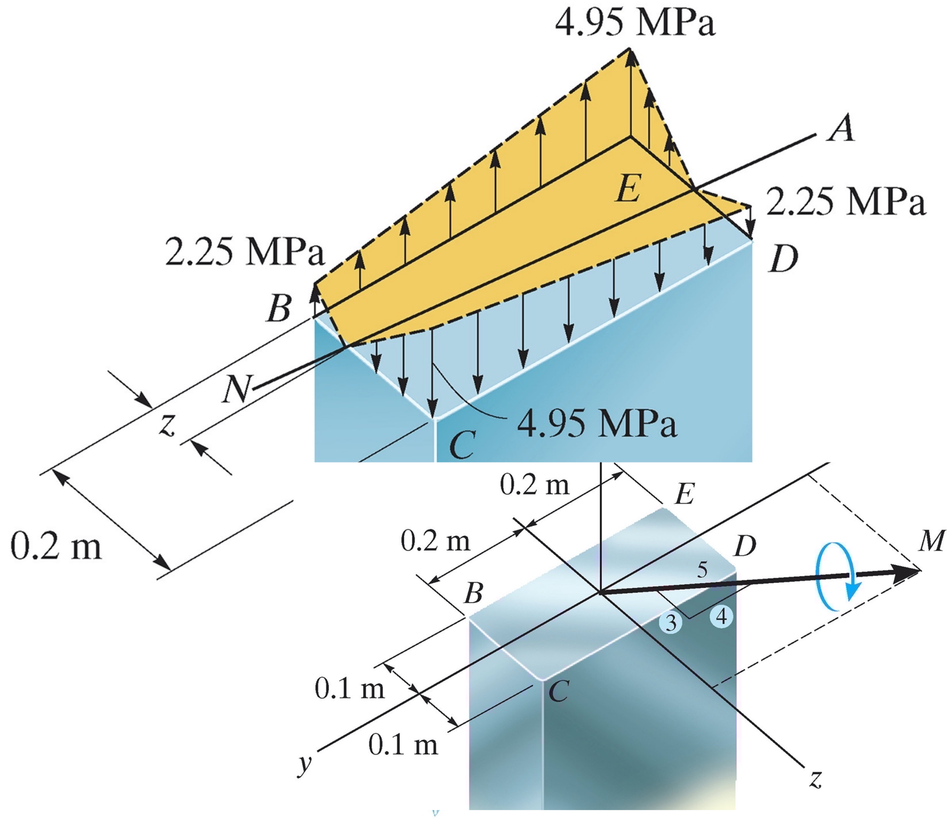
$$I_z = 1.067\times10^{-3}\ \mathrm{m}^4$$

$$I_y = 0.267\times10^{-3}\ \mathrm{m}^4$$

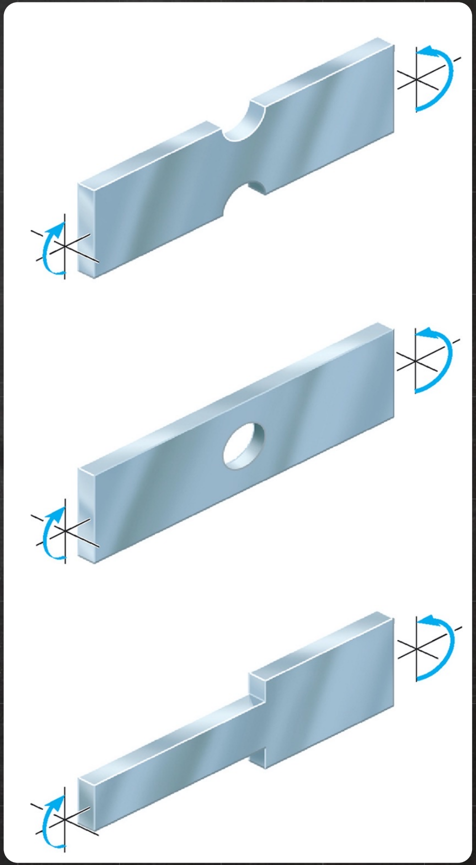
$$M_z = 7.2\ \mathrm{kN.m}$$

$$M_y = -9.6\ \mathrm{kN.m}$$


y
C
B
-4.95
2.25
z
θ
α
-2.25
4.95
12 kN.m
D
E
NEUTRAL AXIS

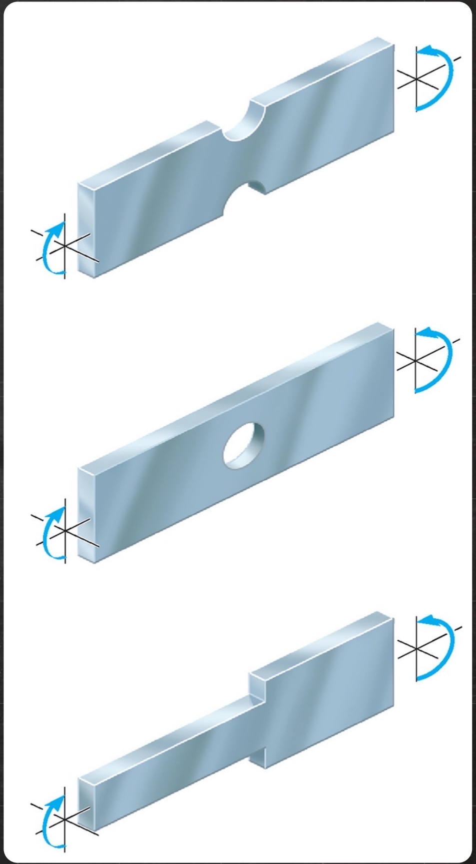

4.95 MPa
A
E
2.25 MPa
2.25 MPa
D
B
N
z
4.95 MPa
C
0.2 m
0.2 m
E
0.2 m
D
M
5
B
3
4
0.1 m
C
0.1 m
y
z

STRESS CONCENTRATION



STRESS CONCENTRATION

Occurs due to geometrical imperfections,
such as change of cross-sectional area



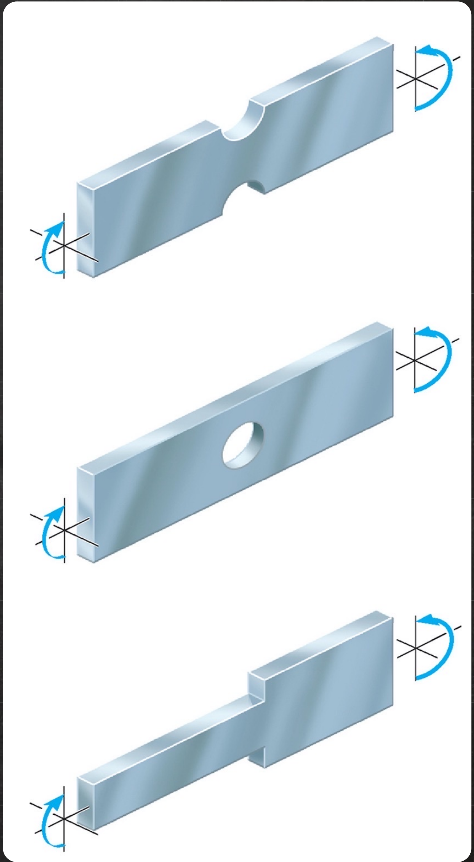
STRESS CONCENTRATION

Occurs due to geometrical imperfections,

such as change of cross-sectional area

$$\sigma_{max} = \frac{Mc}{I}$$

RECALL : $\sigma = -\frac{My}{I}$ ← linear distribution if no stress concentration



STRESS CONCENTRATION

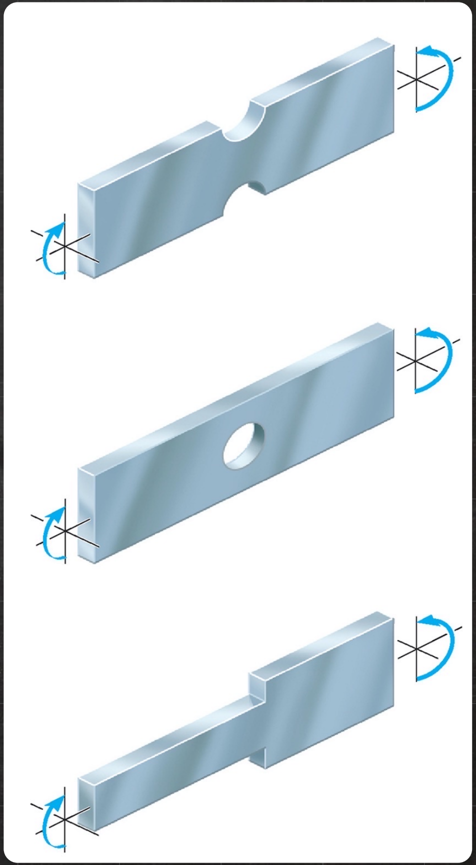
Occurs due to geometrical imperfections,

such as change of cross-sectional area

$$\sigma_{max} = K \frac{Mc}{I}$$

$$\sigma_{max} = \frac{Mc}{I}$$

RECALL : $\sigma = -\frac{My}{I}$ ⟵ linear distribution if no stress concentration



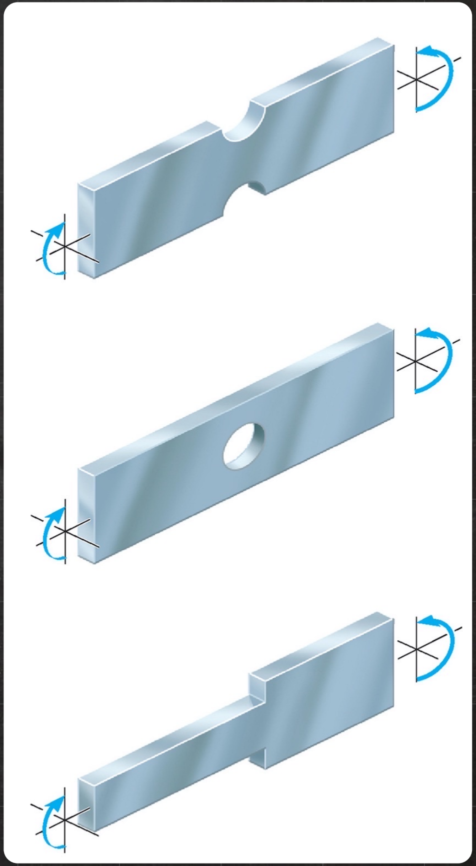
Stress Concentration

Occurs due to geometrical imperfections,
such as change of cross-sectional area

$$\sigma_{max} = K \frac{Mc}{I}$$

e.g.

$$\frac{c}{I} \equiv \frac{h/2}{\frac{1}{12} b h^3} = \frac{6}{bh^2} \Rightarrow \frac{c}{I} \propto \frac{1}{h^2}$$



Stress Concentration

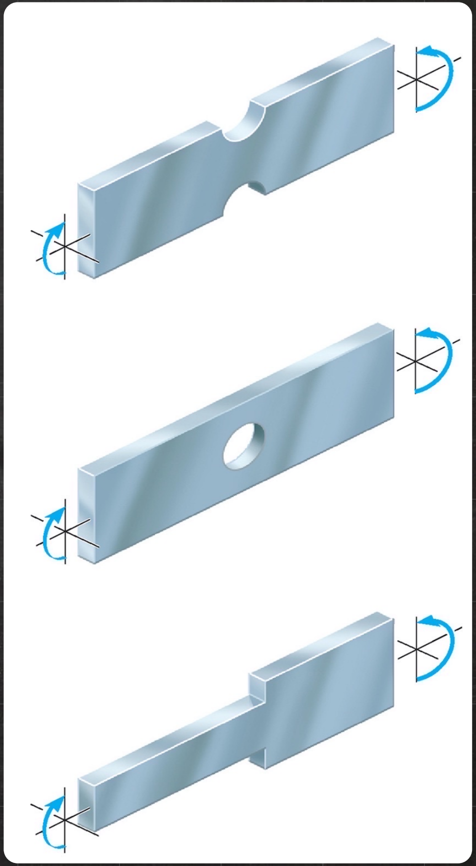
Occurs due to geometrical imperfections,
such as change of cross-sectional area

$$\sigma_{max} = K \frac{Mc}{I}$$

e.g.

$$\frac{c}{I}\uparrow \Leftarrow h\downarrow$$

$$\frac{c}{I} = \frac{h/2}{\frac{1}{12} b h^3} = \frac{6}{bh^2} \Rightarrow \frac{c}{I} \propto \frac{1}{h^2}$$



Stress Concentration

Occurs due to geometrical imperfections,

such as change of cross-sectional area

$$\sigma_{max} = K\,\frac{Mc}{I}$$

THE MAXIMUM NORMAL STRESS OCCURS AT THE SECTION WITH THE SMALLEST CROSS-SECTIONAL AREA!

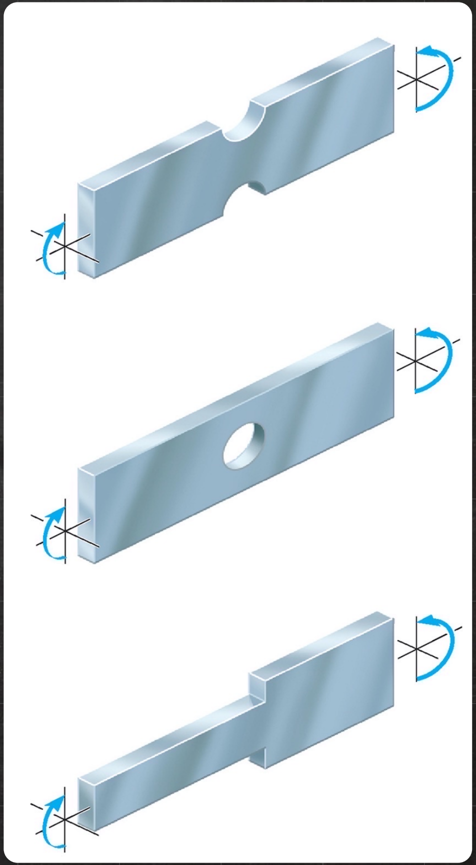
$\frac{c}{I}\uparrow \Leftarrow h\downarrow$ ⇑

e.g.

h

b

$$\frac{c}{I} \equiv \frac{h/2}{\frac{1}{12} b h^3} = \frac{6}{bh^2} \Rightarrow \frac{c}{I} \propto \frac{1}{h^2}$$

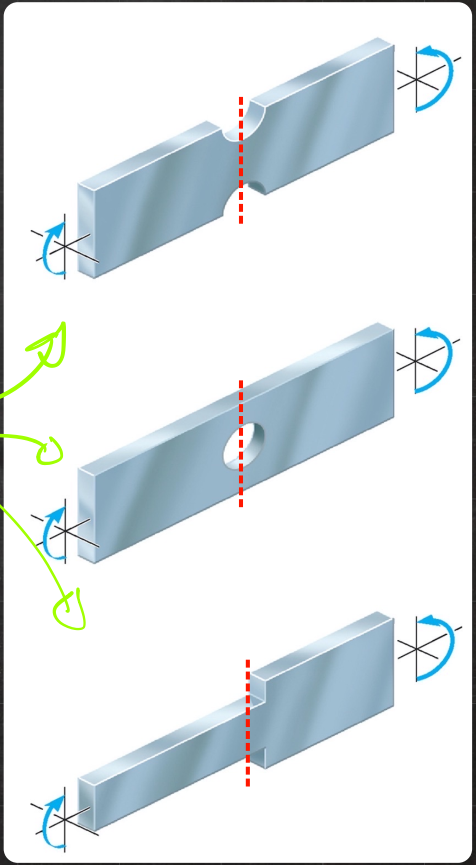


STRESS CONCENTRATION

Occurs due to geometrical imperfections,
such as change of cross-sectional area

$$\sigma_{max} = K \frac{Mc}{I}$$

THE MAXIMUM NORMAL STRESS OCCURS AT THE SECTION WITH THE SMALLEST CROSS-SECTIONAL AREA !

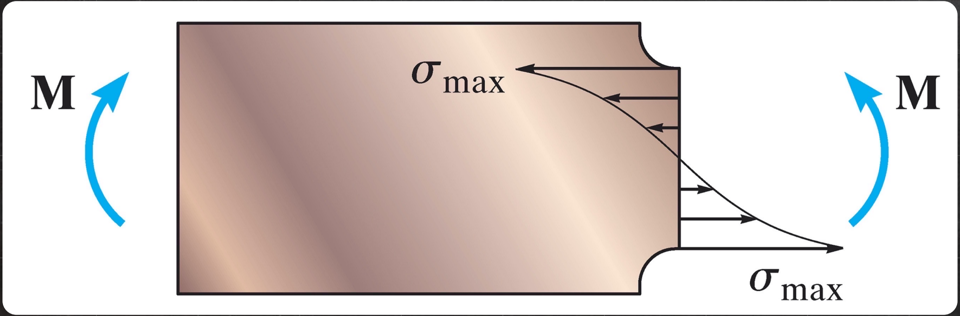


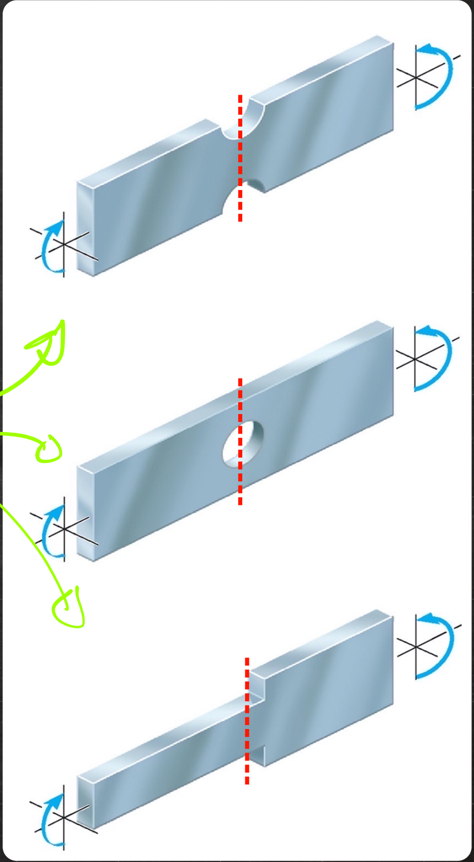
STRESS CONCENTRATION

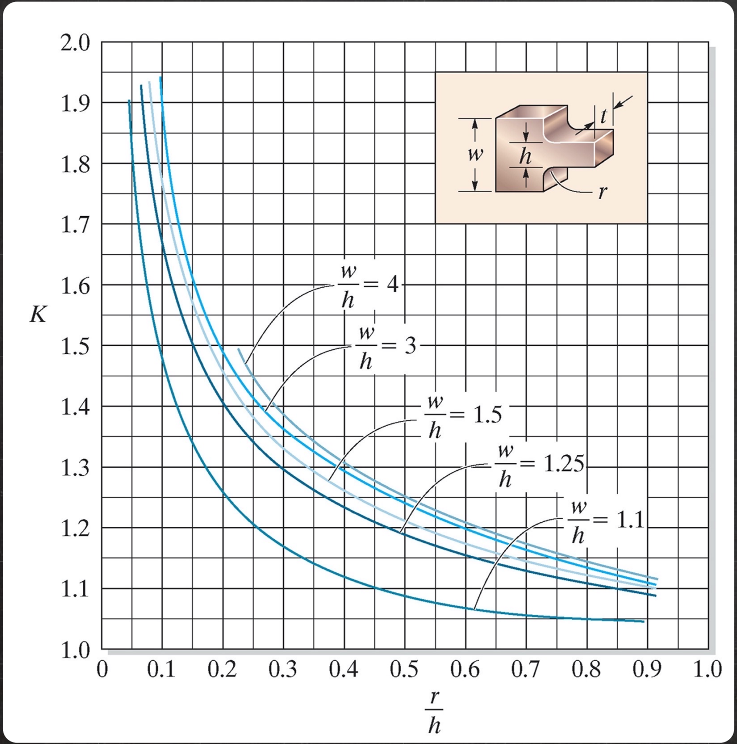
Occurs due to geometrical imperfections,
such as change of cross-sectional area

$$\sigma_{max} = K \frac{Mc}{I}$$

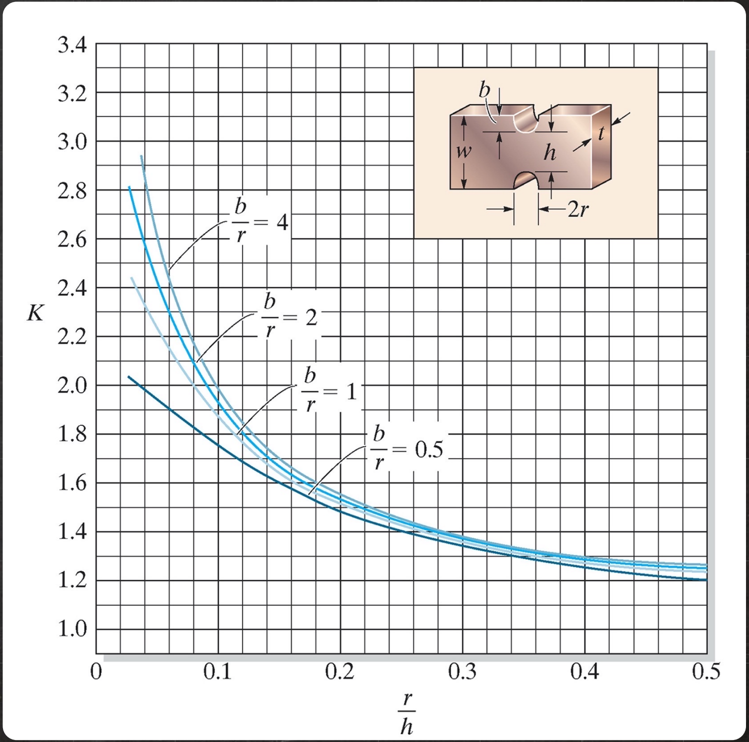
THE MAXIMUM NORMAL STRESS OCCURS AT THE SECTION WITH THE SMALLEST CROSS-SECTIONAL AREA !







2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
K
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
r/h
w/h = 4
w/h = 3
w/h = 1.5
w/h = 1.25
w/h = 1.1
w
h
t
r

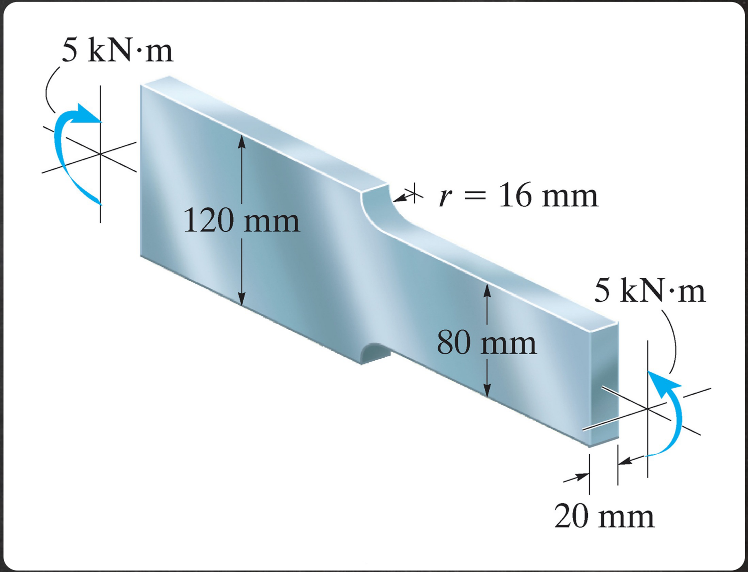


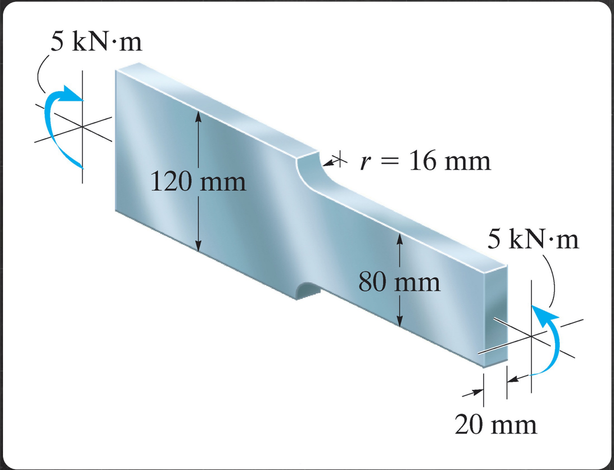
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
K
0
0.1
0.2
0.3
0.4
0.5
r/h
b/r = 4
b/r = 2
b/r = 1
b/r = 0.5
b
w
h
t
2r

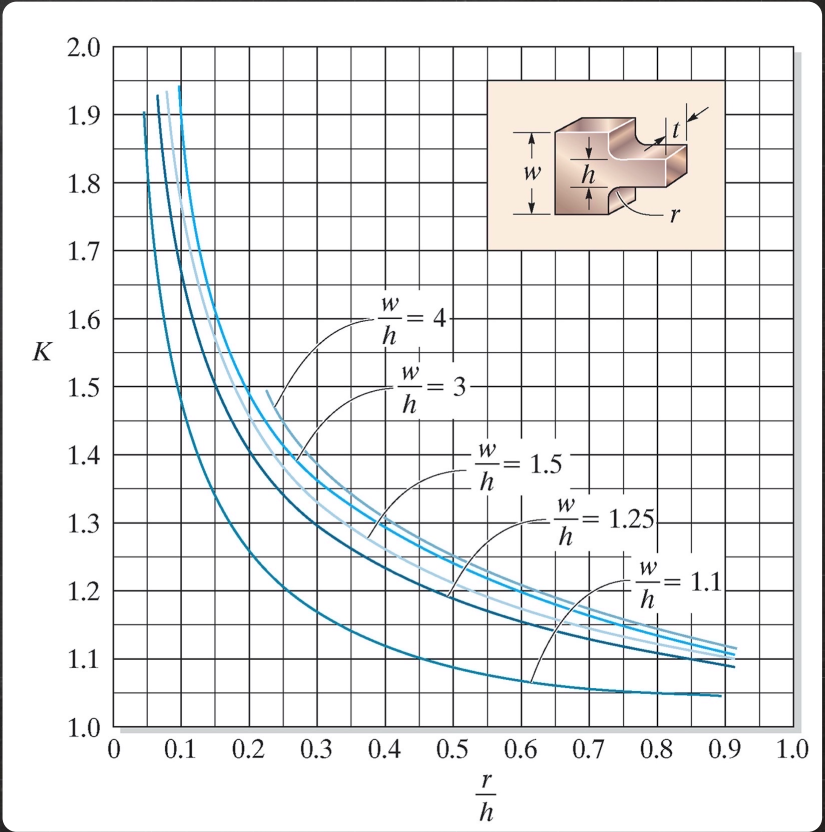
Exercise 2 . [similar to … P. 332 … 6.20]

THE BAR SHOWN IN THE FIGURE IS SUBJECTED TO A BENDING MOMENT OF 5 kN.m AND IS MADE OF STEEL WITH E_y = 500 MPa.

DETERMINE THE MAXIMUM NORMAL STRESS DEVELOPED IN THE BAR.

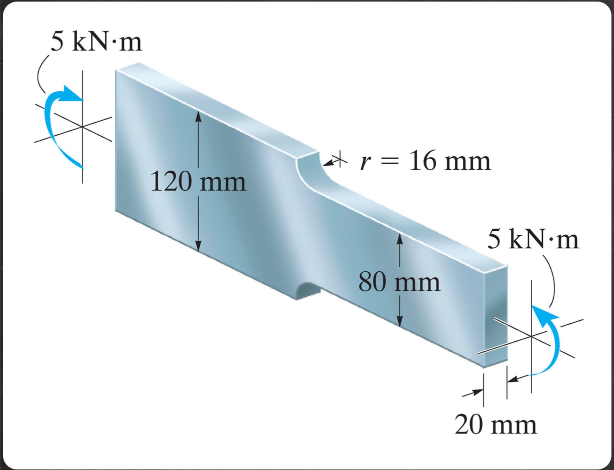


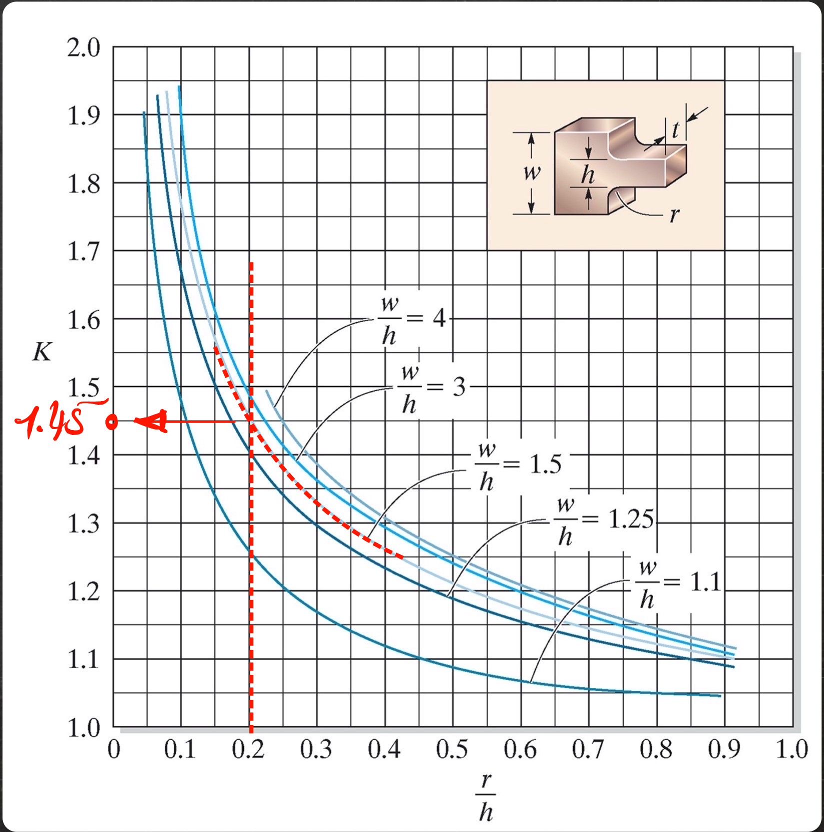




$$\frac{r}{h} = \frac{16}{80} = 0.2$$

$$\frac{w}{h} = \frac{120}{80} = 1.5$$





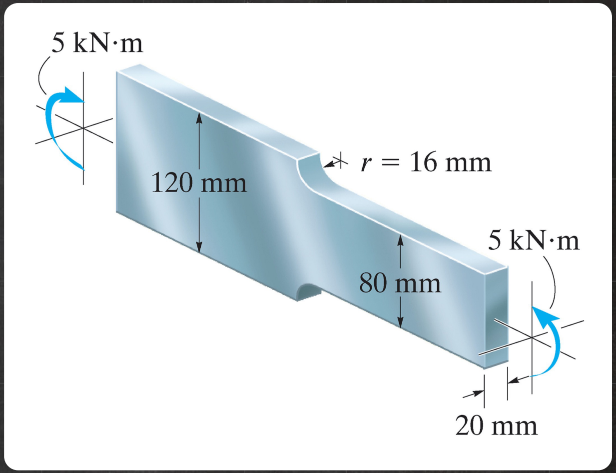
$$\frac{r}{h} = \frac{16}{80} = 0.2$$

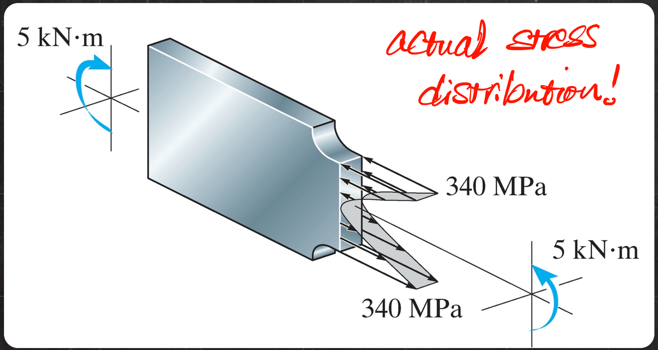
$$\Rightarrow K = 1.45$$

$$\frac{w}{h} = \frac{120}{80} = 1.5$$

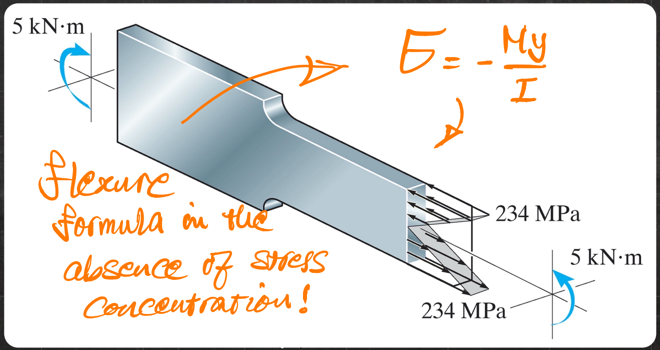
$$\sigma_{max} = \underbrace{K}_{1.45} \frac{\overbrace{M}^{5\times10^3}\,\overbrace{c}^{0.04}}{\underbrace{I}_{\frac{1}{12}\times0.02\times0.08^3}}$$

$$= 1.45 \times 234 = 340 < \sigma_y \checkmark$$





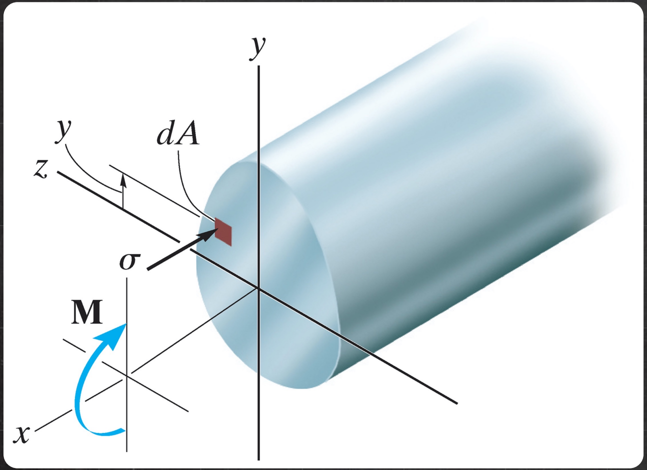
actual stress distribution!

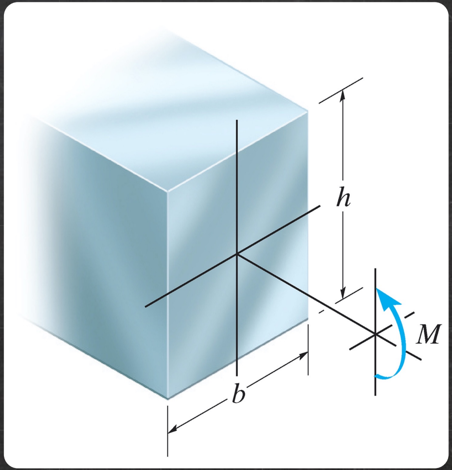


$$\sigma = -\frac{My}{I}$$

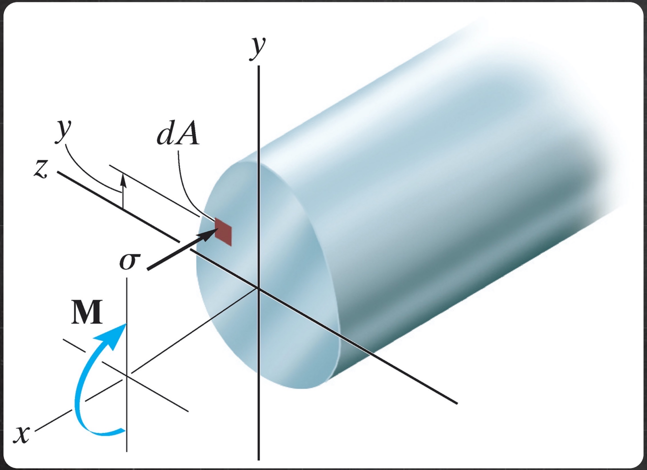
Flexure formula in the absence of stress concentration!

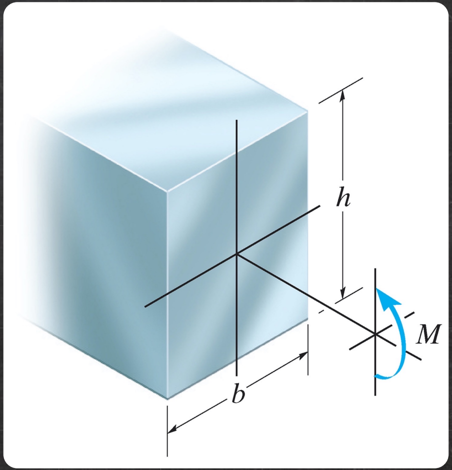
INELASTIC BENDING





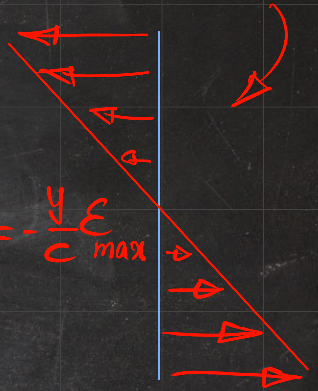
INELASTIC BENDING





geometrical definition

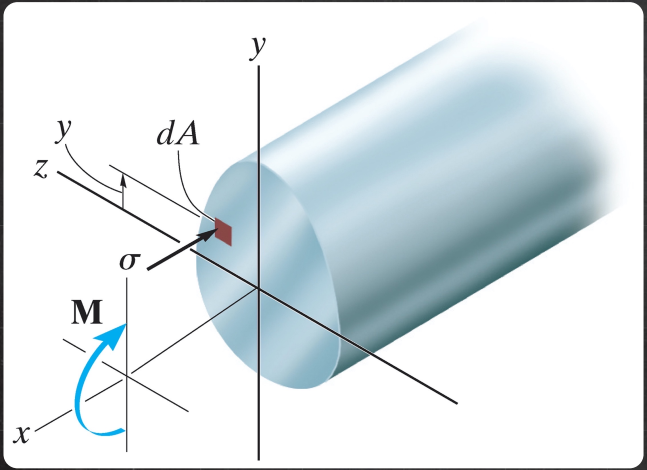
STRAIN DISTRIBUTION

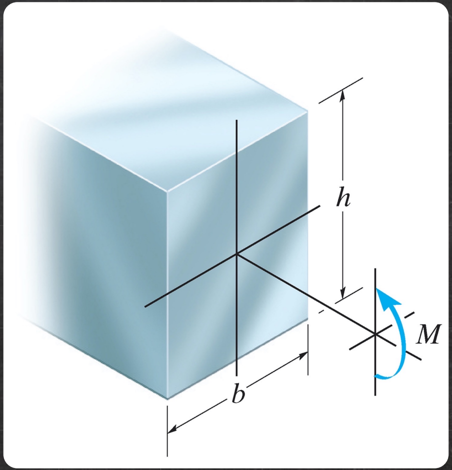


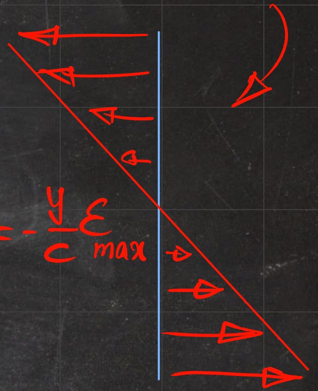
$$\varepsilon = -\frac{y}{c}\varepsilon_{max}$$

INELASTIC BENDING

geometrical definition ← STRAIN DISTRIBUTION

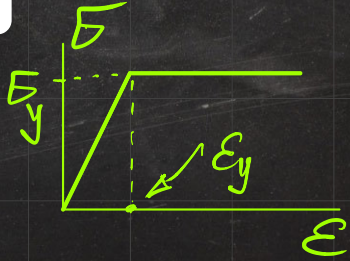




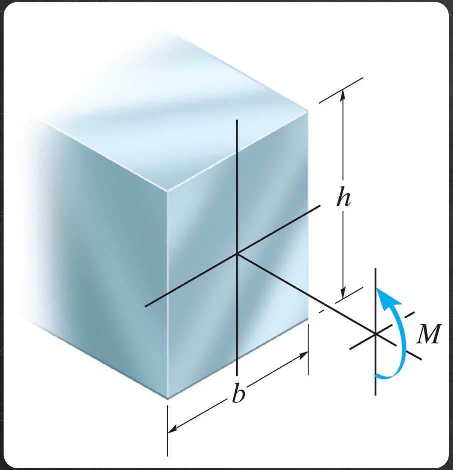


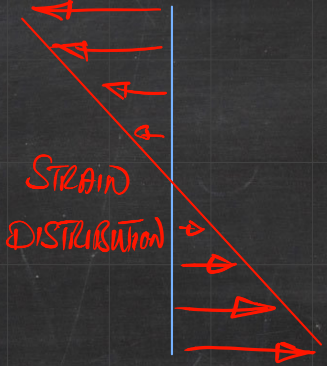
$$\varepsilon = -\frac{y}{c}\,\varepsilon_{max}$$

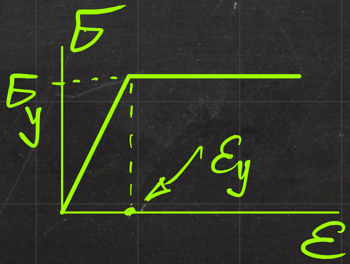
$$M = \int_A y(\sigma\, dA) \longrightarrow \sigma = ? \longrightarrow$$



MAXIMUM ELASTIC MOMENT



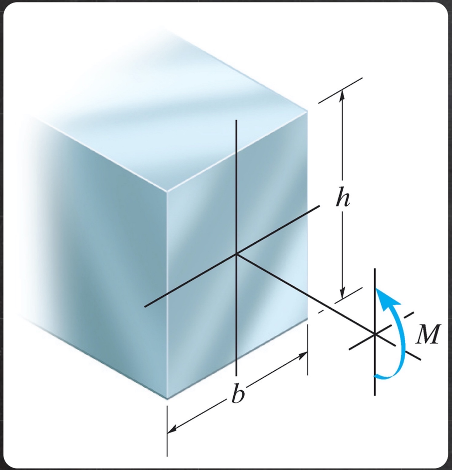


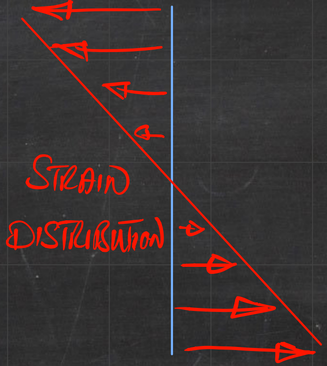


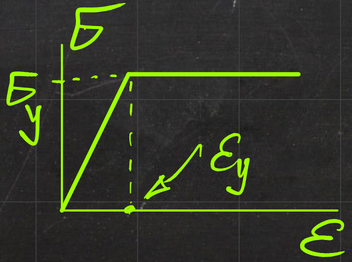
$M \leq M_{el.}$

MAXIMUM ELASTIC MOMENT

$$M = \int_A y\,(\sigma\, dA)$$







$$M \leq M_{el.}$$

MAXIMUM ELASTIC MOMENT

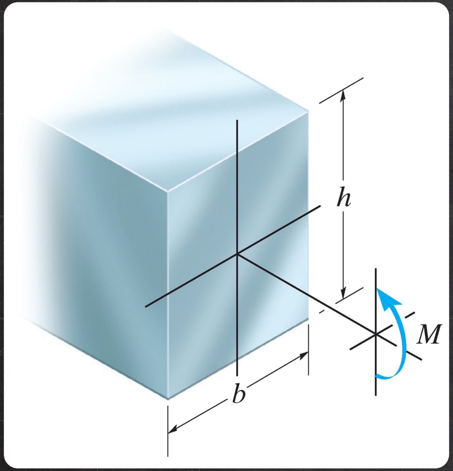
$$M = \int_A y\,(\sigma\, dA)$$

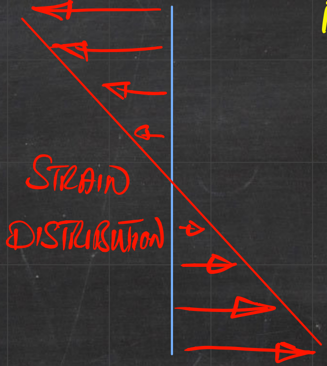
FLEXURE FORMULA

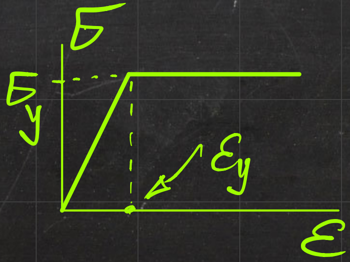
WE DID THIS BEFORE!

$$M = \frac{\sigma_{max}\, I}{c}$$

$$\sigma_{max} = \sigma_y$$



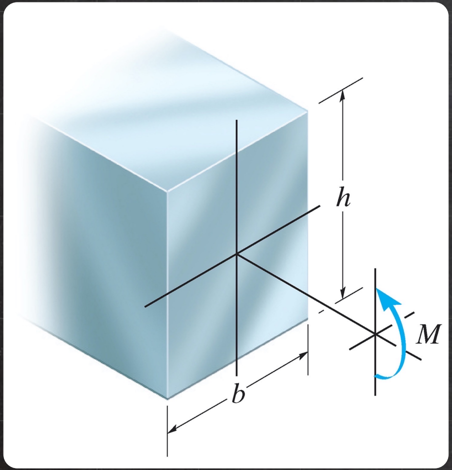


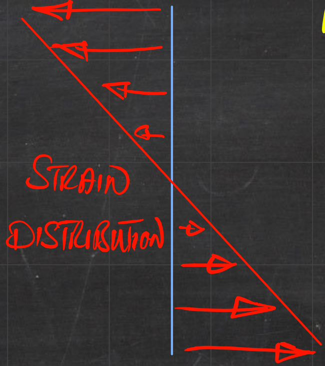


$$M \leq M_{el.}$$

MAXIMUM ELASTIC MOMENT

$$M = \int_A y\,(\sigma\, dA)$$





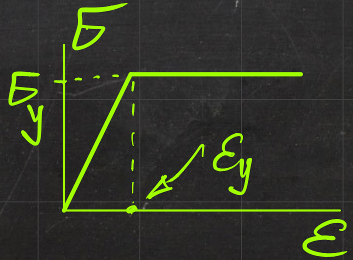
FLEXURE FORMULA

$\Downarrow$ WE DID THIS BEFORE!

$$M = \frac{\sigma_{max}}{c} I$$

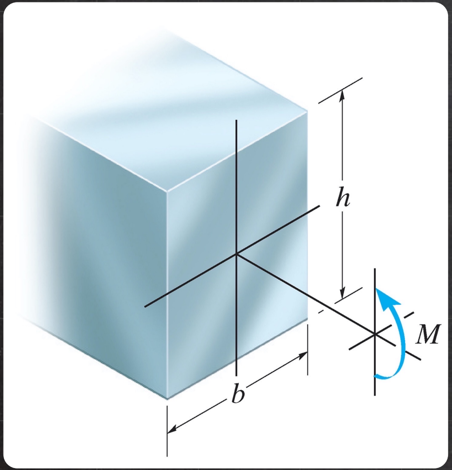
$$\sigma_{max} = \sigma_y$$

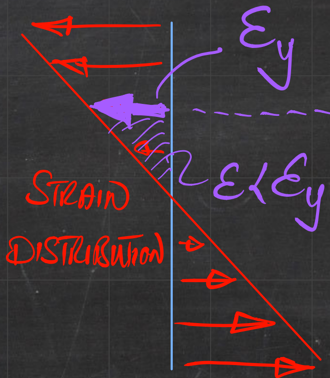
$$\Rightarrow M_{el} = \frac{\sigma_y}{c} I$$

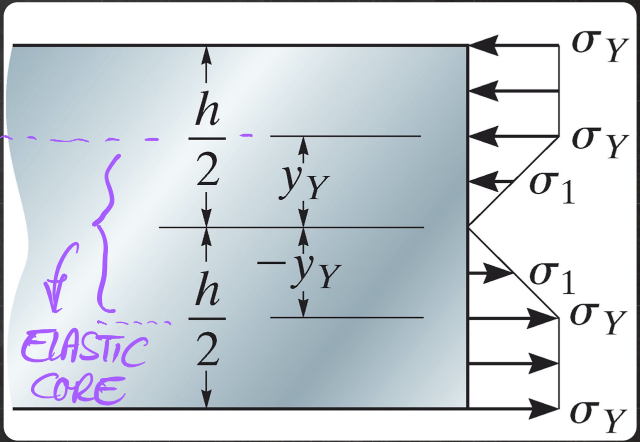


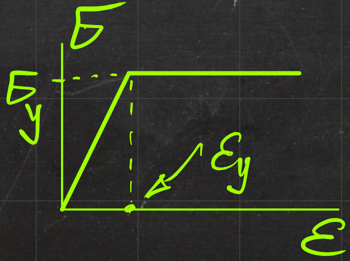
$$M \leq M_{el}$$

ELASTO-PLASTIC MOMENT





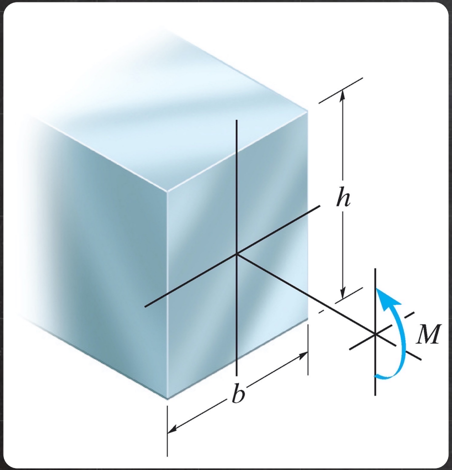


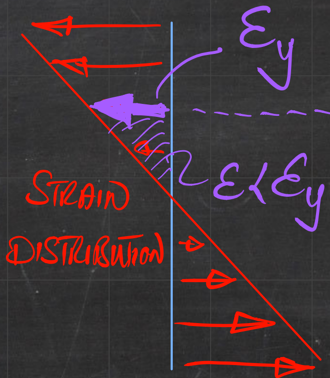


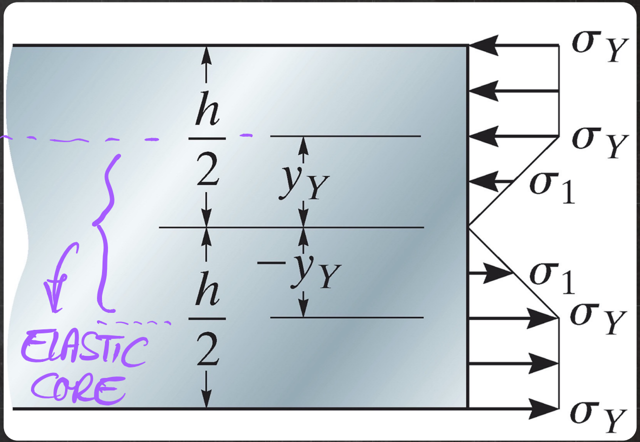
$$M_{el.} \leq M \leq M_{pl.}$$

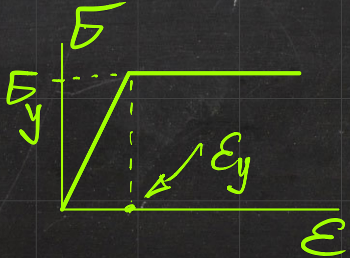
ELASTO-PLASTIC MOMENT

$$M = \int_A y\,(\sigma\, dA)$$





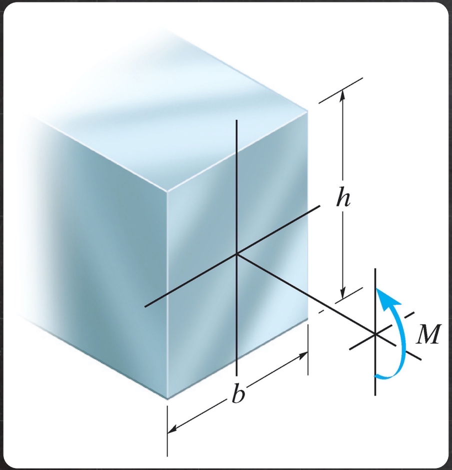


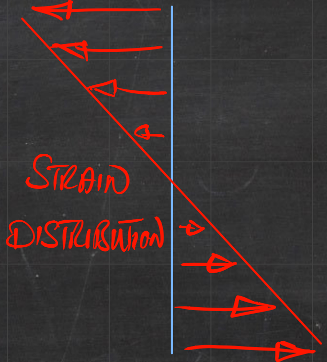


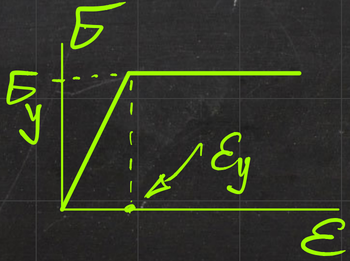
$$M_{el} < M < M_{pl}$$

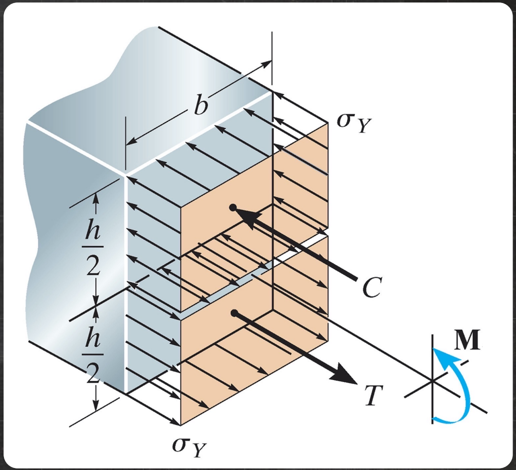
$$M = \int_{A_1} \cdots + \int_{A_2} \cdots$$

Maximum Plastic Moment





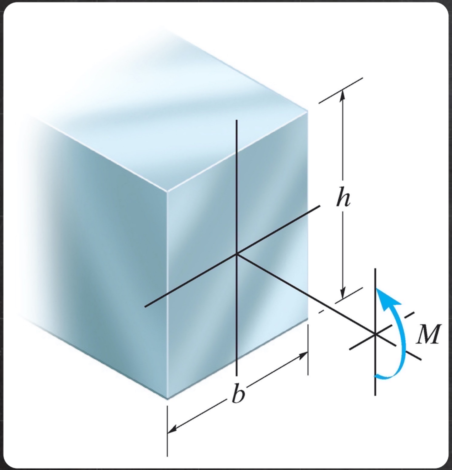


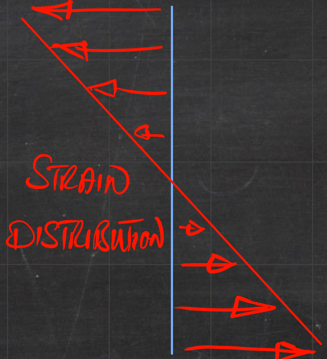


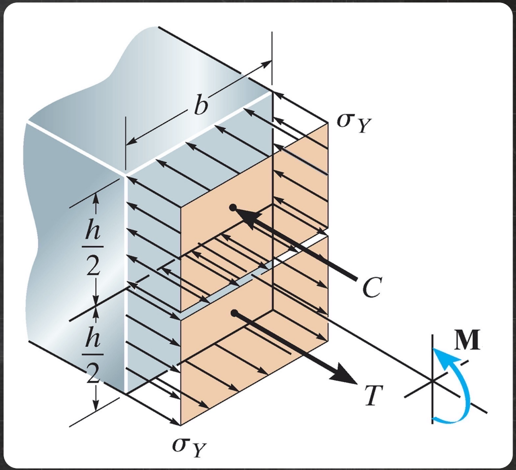
$$M = M_{pl}$$

MAXIMUM PLASTIC MOMENT

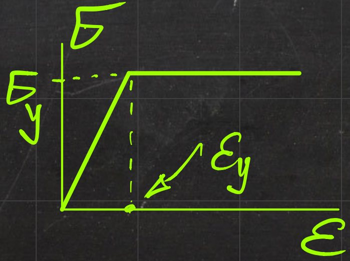
$$M = \int_A y\,(\sigma\, dA)$$







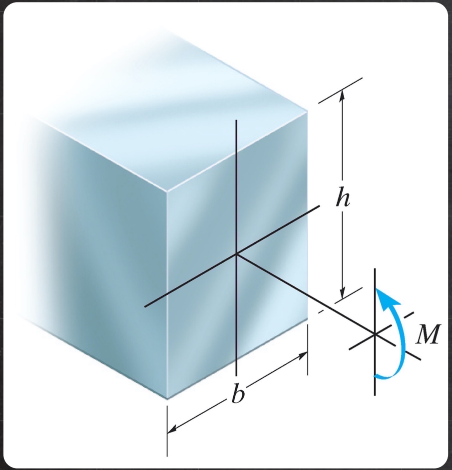
$$M = M_{pl}$$

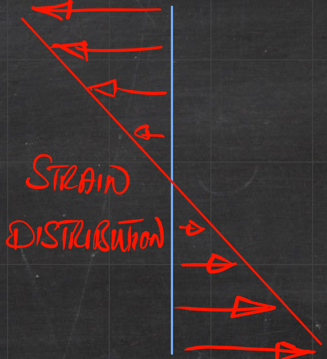


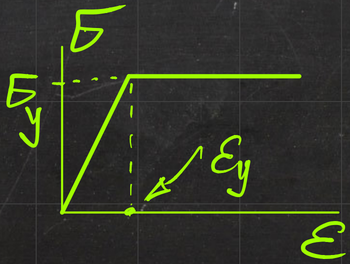
$\hookrightarrow \sigma = \sigma_y$

Maximum Plastic Moment

$$M = \int_A y(\sigma \, dA)$$





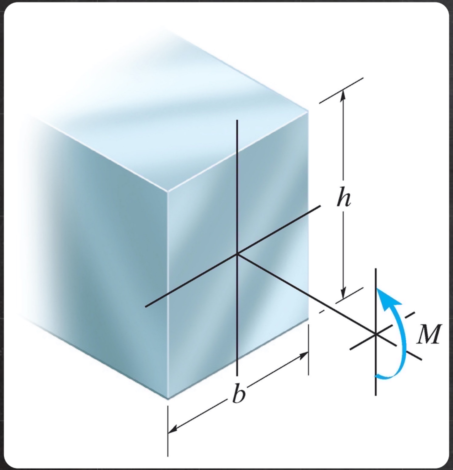


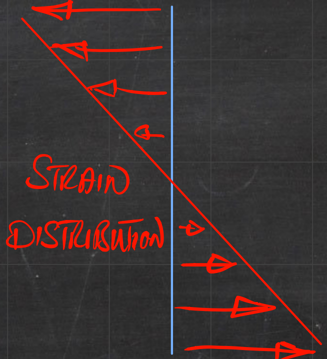
$$M = M_{pl}$$

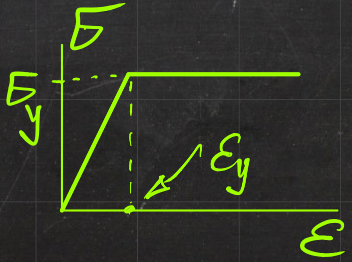
Maximum Plastic Moment

$$M = \int_A y\,(\sigma\, dA)$$

$$= \int_A y\,\sigma_y\, dA$$







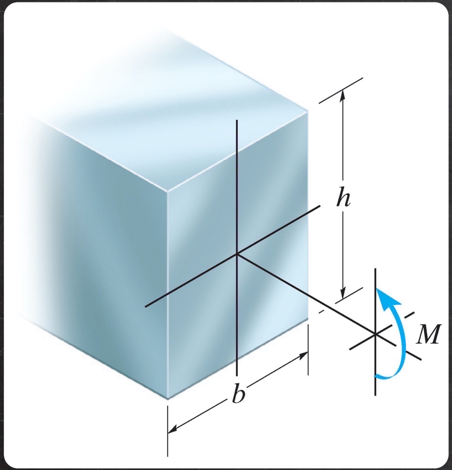
$$M = M_{pl}$$

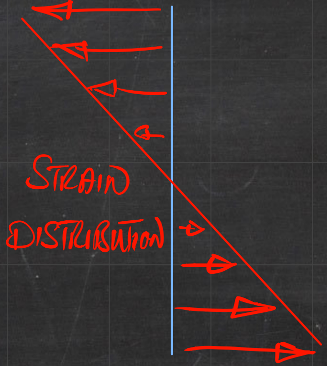
Maximum Plastic Moment

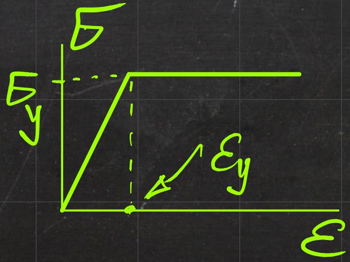
$$M = \int_A y\,(\sigma\, dA)$$

$$= \int_A y\, \sigma_y\, dA$$

$$= \int_{-h/2}^{h/2} y\, \sigma_y\, dA$$

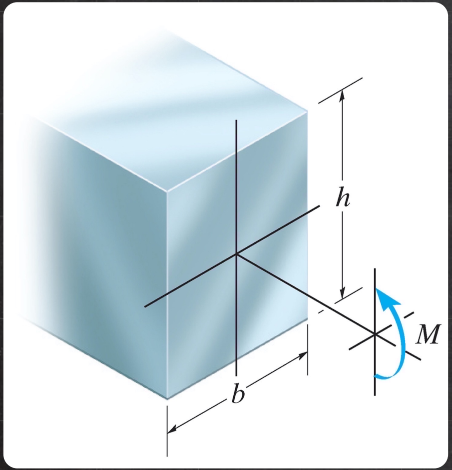


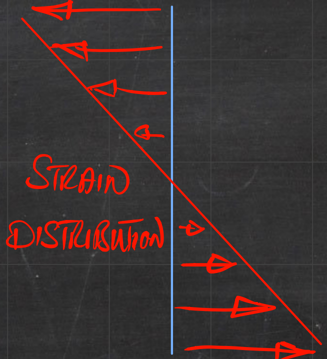




$$M = M_{pl}$$

Maximum Plastic Moment



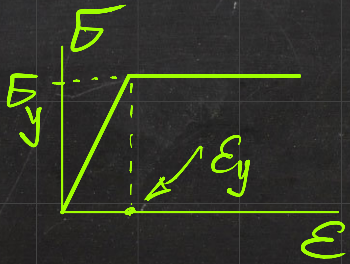


$$M = \int_A y\,(\sigma\, dA)$$

$$= \int_A y\, \sigma_y\, dA$$

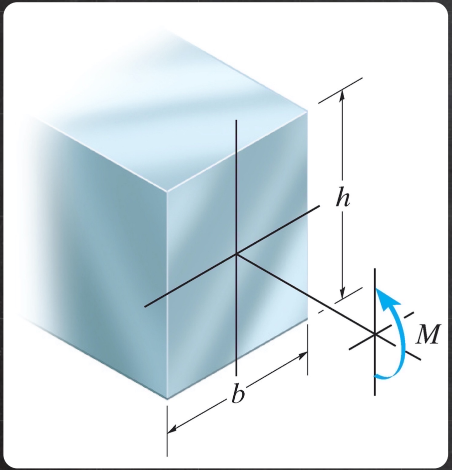
$$= \int_{-h/2}^{h/2} y\, \sigma_y\, dA$$

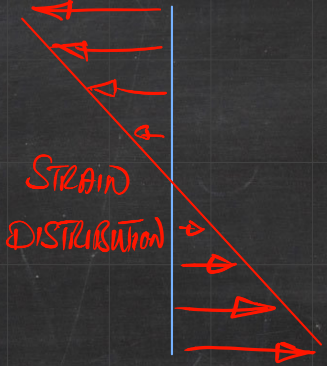
$$= 2\int_0^{h/2} y\, \sigma_y\, b\, dy$$

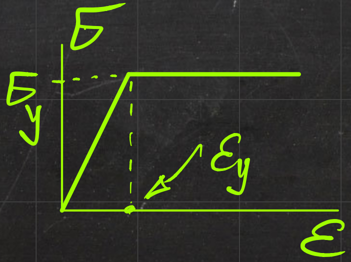


$$M = M_{pl}$$

Maximum Plastic Moment







$$M = \int_A y\,(\sigma\, dA)$$

$$= \int_A y\, \sigma_y\, dA$$

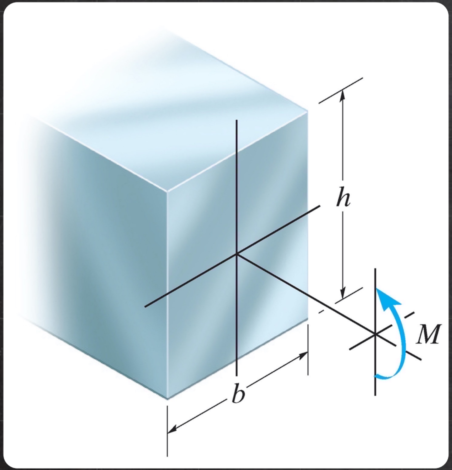
$$= \int_{-h/2}^{h/2} y\, \sigma_y\, dA$$

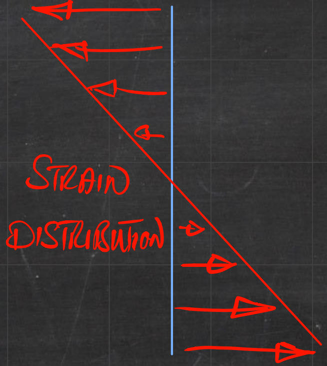
$$= 2\int_0^{h/2} y\, \sigma_y\, b\, dy$$

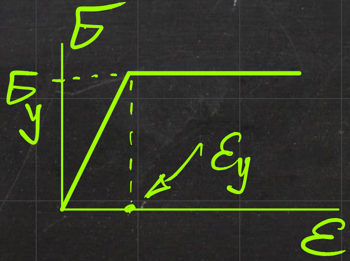
$$= \frac{3}{2}\,\frac{\sigma_y}{c}\, I$$

$$M = M_{pl}$$

Maximum Plastic Moment







$$M = \int_A y\,(\sigma\, dA)$$

$$= \int_A y\, \sigma_y\, dA$$

$$= \int_{-h/2}^{h/2} y\, \sigma_y\, dA$$

$$= 2\int_0^{h/2} y\, \sigma_y\, b\, dy$$

$$= \frac{3}{2}\,\frac{\sigma_y}{c}\, I$$

$$\Rightarrow M_{pl.} = \frac{3}{2} M_{el}$$

$$M = M_{pl}$$

INELASTIC BENDING

$\swarrow$ e.g. rectangular cross-section

ELASTIC $\rightsquigarrow$ $M \leq M_{el.}$ $\qquad M_{el.} = \dfrac{\sigma_y}{c} I$

ELASTO-PLASTIC $\rightsquigarrow$ $M_{el.} \leq M \leq M_{pl.}$

PLASTIC $\rightsquigarrow$ $M = M_{pl.}$ $\qquad M_{pl.} = \dfrac{3}{2} \dfrac{\sigma_y}{c} I$

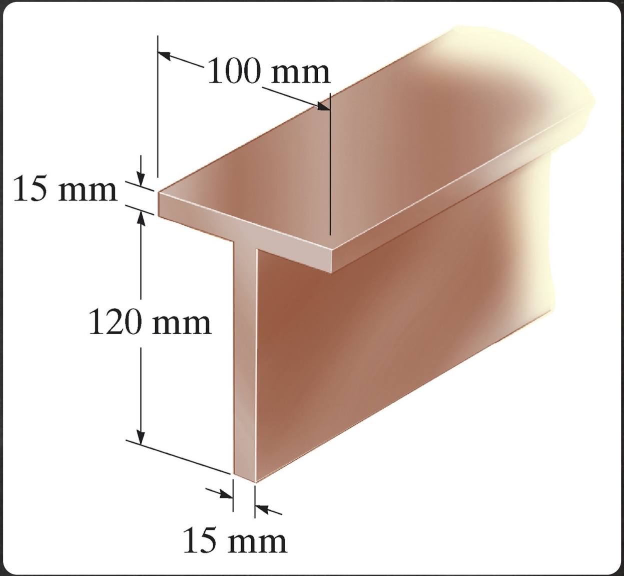
↳ NOTE THAT NEUTRAL AXIS LOCATION FOLLOWS FROM EQUILIBRIUM $\swarrow \int_A \sigma \, dA = 0$

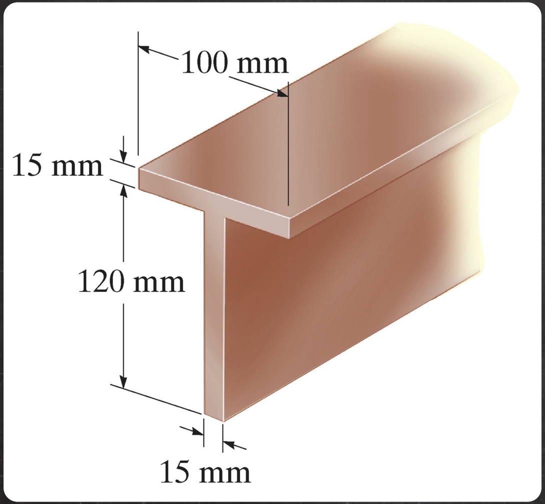
↳ if inelastic, its location changes, unless symmetric cross-section!

↳ See next exercise ...

Exercise 3 . [similar to … P. 346 … 6.22]

A BEAM WITH THE DIMENSIONS SHOWN IN THE FIGURE IS MADE OF AN ELASTIC PERFECTLY PLASTIC MATERIAL HAVING A TENSILE AND COMPRESSIVE YIELD STRESS OF $\sigma_y = 250$ MPa. DETERMINE THE PLASTIC MOMENT THAT CAN BE RESISTED BY THE BEAM.





RECALL:

EQUILIBRIUM $\leadsto \int dF = 0$

$\Rightarrow \int_A \sigma \, dA = 0$

NO LONGER HOLDS!

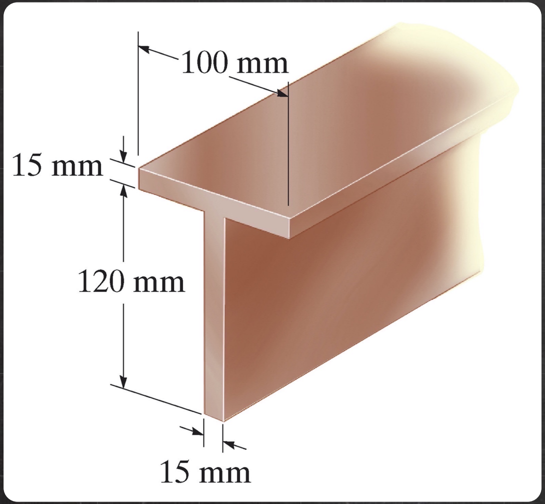
$\sigma = E\varepsilon$

$\sigma = -\frac{y}{c}\sigma_{max}$

$\varepsilon = -\frac{y}{c}\varepsilon_{max}$

$\Rightarrow \int y \, dA = 0$

NEUTRAL AXIS AT CENTROID



RECALL:

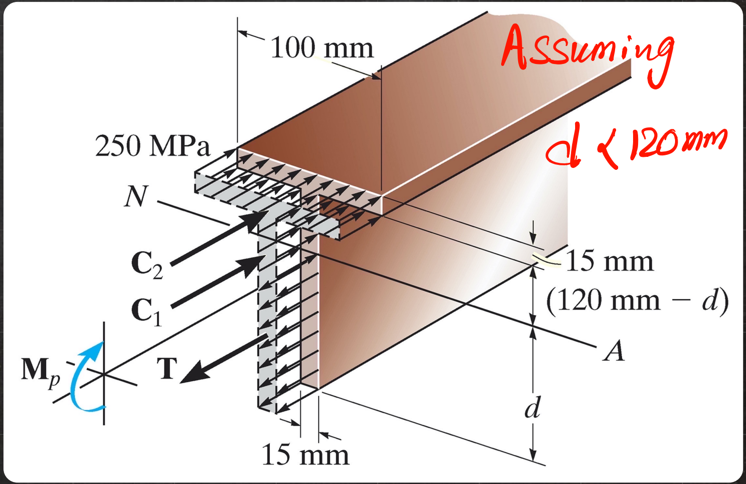
EQUILIBRIUM $\rightsquigarrow \int dF = 0$

$\Rightarrow \int_A \sigma \, dA = 0$

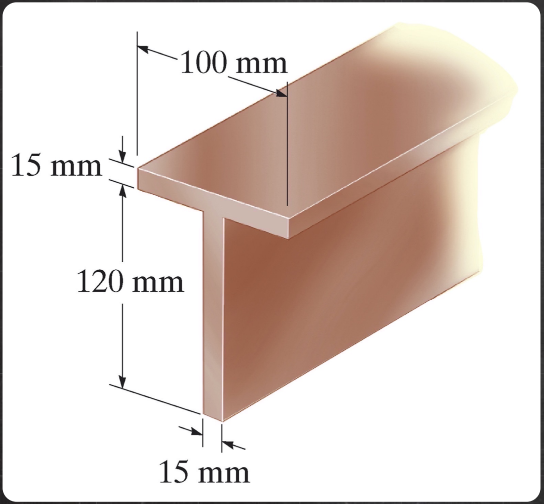
EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma \, dA = \underbrace{T}_{\sigma_y A_t} - \underbrace{C_1}_{\sigma_y A_{C_1}} - \underbrace{C_2}_{\sigma_y A_{C_2}} = 0$$



Assuming $d < 120\,mm$



$\Rightarrow A_t - A_{C_1} - A_{C_2} = 0$

$0.015 \times d$ $\quad 0.015 \times (0.12 - d)$ $\quad 0.1 \times 0.015$ $\Rightarrow d = 110$ mm

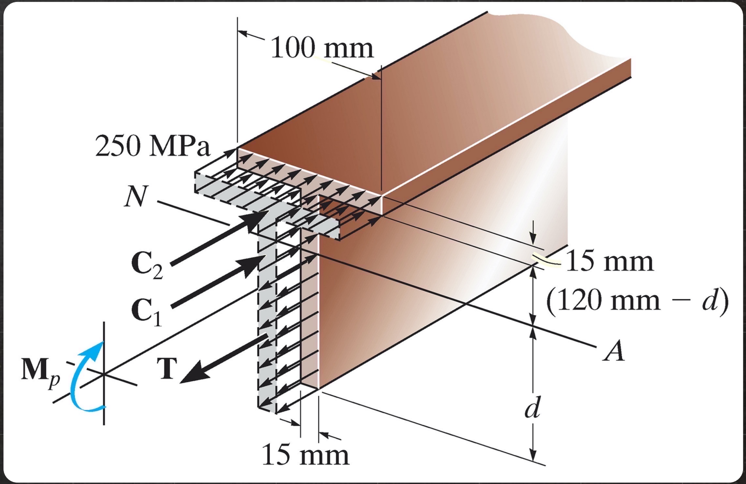
< 120 mm ✓

EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma \, dA = T - C_1 - C_2 = 0$$

$\sigma_y A_t \quad \sigma_y A_{C_1} \quad \sigma_y A_{C_2}$



$$T = 250\times 10^{6}\times 0.015\times 0.11$$

$$\Rightarrow T = 412.5\ kN$$

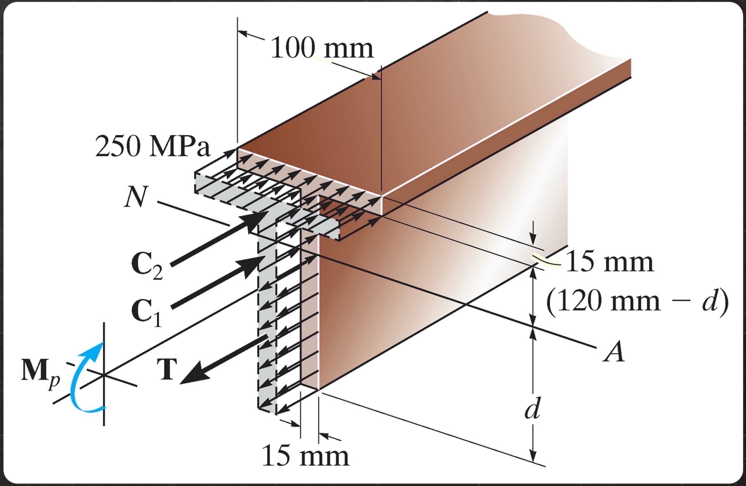
$$\Rightarrow \underbrace{A_t}_{0.015\times d} - \underbrace{A_{C_1}}_{0.015\times(0.12-d)} - \underbrace{A_{C_2}}_{0.1\times 0.015} = 0 \Rightarrow d = 110\ mm$$

$< 120\ mm$ ✓

EQUILIBRIUM
OF
FORCES

$\Downarrow$

$$\int_A \sigma\, dA = \underbrace{T}_{\sigma_y A_t} - \underbrace{C_1}_{\sigma_y A_{C_1}} - \underbrace{C_2}_{\sigma_y A_{C_2}} = 0$$



$$T = 250 \times 10^6 \times 0.015 \times 0.11$$

$$\Rightarrow T = 412.5 \text{ kN}$$

$$C_1 = 250 \times 10^6 \times 0.015 \times 0.01$$

$$\Rightarrow C_1 = 37.5 \text{ kN}$$

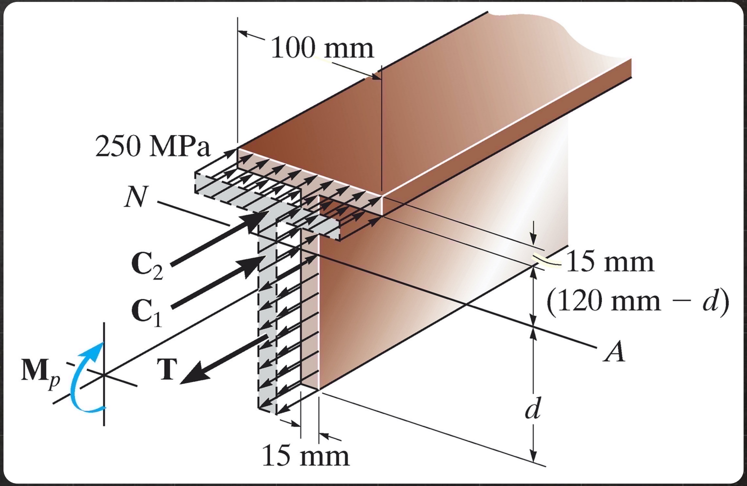
$$\Rightarrow \underbrace{A_t}_{0.015 \times d} - \underbrace{A_{C_1}}_{0.1 \times 0.015} - \underbrace{A_{C_2}}_{0.015 \times (0.12 - d)} = 0 \Rightarrow d = 110 \text{ mm}$$

< 120 mm ✓

EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma \, dA = \underbrace{T}_{\sigma_y A_t} - \underbrace{C_1}_{\sigma_y A_{C_1}} - \underbrace{C_2}_{\sigma_y A_{C_2}} = 0$$



$T = 250 \times 10^6 \times 0.015 \times 0.11$

$\Rightarrow T = 412.5 \text{ kN}$

$C_1 = 250 \times 10^6 \times 0.015 \times 0.01$

$\Rightarrow C_1 = 37.5 \text{ kN}$

$C_2 = 250 \times 10^6 \times 0.1 \times 0.015$

$\Rightarrow C_2 = 375 \text{ kN}$

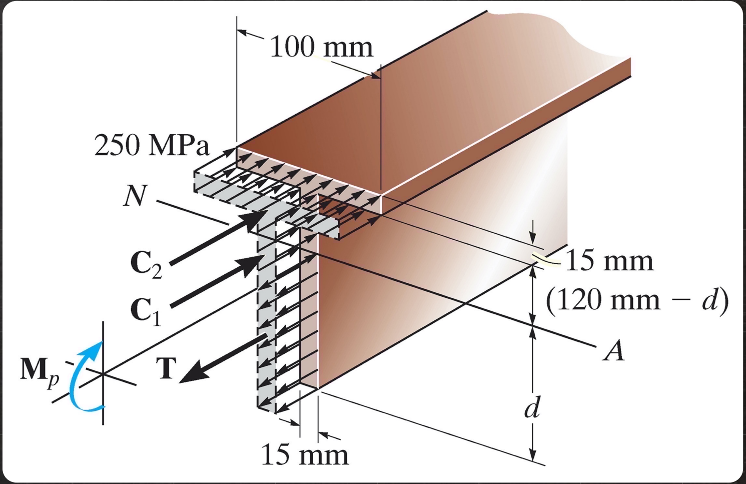
EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma \, dA = \underbrace{T}_{\sigma_y A_t} - \underbrace{C_1}_{\sigma_y A_{C_1}} - \underbrace{C_2}_{\sigma_y A_{C_2}} = 0$$

$$\Rightarrow \underbrace{A_t}_{0.015 \times d} - \underbrace{A_{C_1}}_{0.015 \times (0.12 - d)} - \underbrace{A_{C_2}}_{0.1 \times 0.015} = 0$$

$\Rightarrow d = 110 \text{ mm}$ $< 120 \text{ mm}$ ✓



$$T = 250\times10^6 \times 0.015 \times 0.11$$

$$\Rightarrow T = 412.5\ \text{kN}$$

$$C_1 = 250\times10^6 \times 0.015 \times 0.01$$

$$\Rightarrow C_1 = 37.5\ \text{kN}$$

$$C_2 = 250\times10^6 \times 0.1 \times 0.015$$

$$\Rightarrow C_2 = 375\ \text{kN}$$

EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma\, dA = \underbrace{T}_{412.5} - \underbrace{C_1}_{37.5} - \underbrace{C_2}_{375} = 0$$

$$\Rightarrow A_t - A_{C_1} - A_{C_2} = 0$$

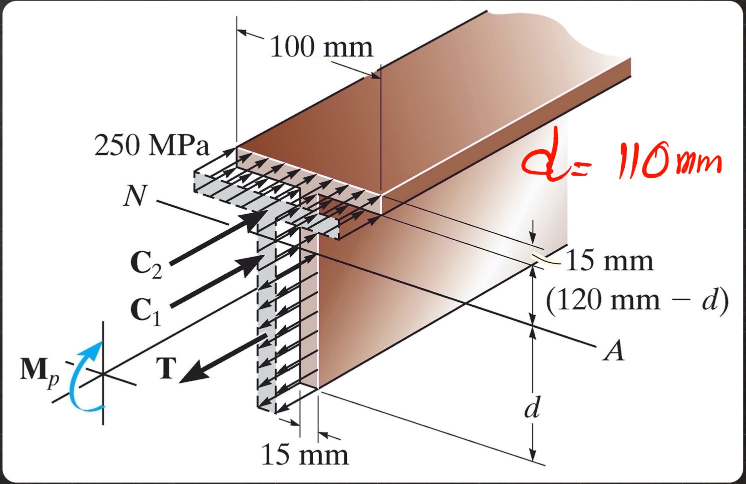
A_t: $0.015 \times d$

A_{C_1}: $0.015 \times (0.12 - d)$

A_{C_2}: 0.1×0.015

$$\Rightarrow d = 110\ \text{mm}$$

< 120 mm



$$T = 250 \times 10^6 \times 0.015 \times 0.11$$

$$\Rightarrow T = 412.5 \text{ kN}$$

$$C_1 = 250 \times 10^6 \times 0.015 \times 0.01$$

$$\Rightarrow C_1 = 37.5 \text{ kN}$$

$$C_2 = 250 \times 10^6 \times 0.1 \times 0.015$$

$$\Rightarrow C_2 = 375 \text{ kN}$$

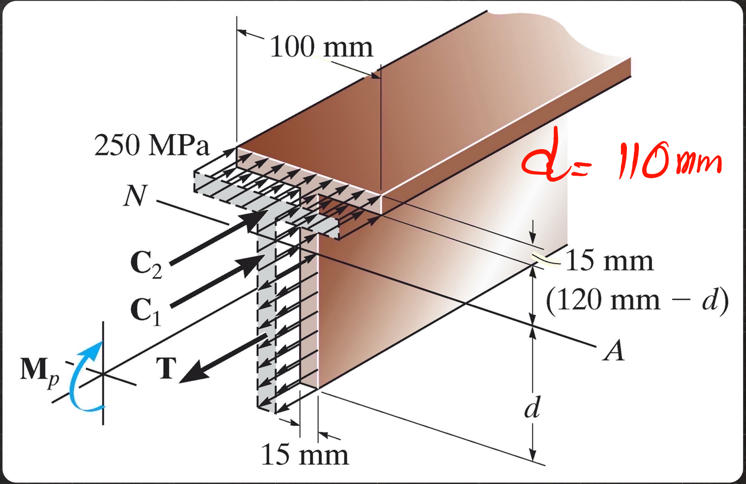
EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma \, dA = \underbrace{T}_{412.5} - \underbrace{C_1}_{37.5} - \underbrace{C_2}_{375} = 0$$

$$M_{pl.} = T \times \frac{d}{2} + C_1 \times \frac{(0.12 - d)}{2} + C_2 \times \left(0.12 - d + \frac{0.015}{2}\right)$$

$d = 110$ mm



$$T = 250 \times 10^6 \times 0.015 \times 0.11$$

$$\Rightarrow T = 412.5 \text{ kN}$$

$$C_1 = 250 \times 10^6 \times 0.015 \times 0.01$$

$$\Rightarrow C_1 = 37.5 \text{ kN}$$

$$C_2 = 250 \times 10^6 \times 0.1 \times 0.015$$

$$\Rightarrow C_2 = 375 \text{ kN}$$

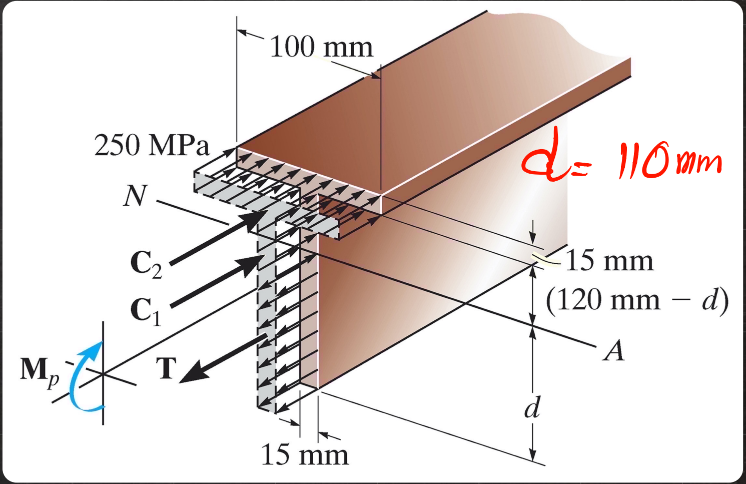
EQUILIBRIUM OF FORCES

$\Downarrow$

$$\int_A \sigma \, dA = \underbrace{T}_{412.5} - \underbrace{C_1}_{37.5} - \underbrace{C_2}_{375} = 0 \checkmark$$

$$M_{pl.} = T \times \frac{d}{2} + C_1 \times \frac{(0.12-d)}{2} + C_2 \times \left(0.12 - d + \frac{0.015}{2}\right)$$

$$\Rightarrow M_{pl.} = 29.4 \text{ kN.m}$$



d = 110 mm

MECHANICS AND MATERIALS I

Bending iv

Sections ... 6.5 ... 6.9 – 6.10

Chap. 6

[Hibbeler 9th edition]

MECHANICS AND MATERIALS I

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