MECHANICS AND MATERIALS I

2

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MECHANICS AND MATERIALS I

Introduction to Statics ii

Rigid Bodies: Equivalent Systems of Forces

Chap. 3

[Beer Johnston et al. 9th edition]

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LAST TIME $\rightsquigarrow$ STATICS OF "PARTICLES" IN 2D, 3D

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$\hookrightarrow$ $\sum \mathbb{F} = \emptyset$

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$\hookrightarrow$ ALL FORCES APPLIED AT THE SAME POINT!

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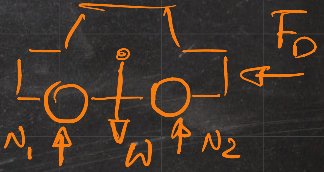
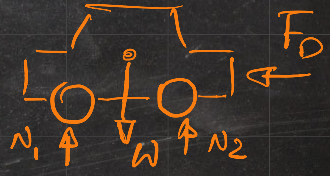


LAST TIME ⇝ STATICS OF "PARTICLES" IN 2D, 3D

THIS TIME ⇝ DEALING WITH "RIGID BODIES" IN 2D, 3D

↳ REPLACE A GIVEN SYSTEM OF FORCES AND MOMENTS ACTING ON A BODY BY AN EQUIVALENT SYSTEM!



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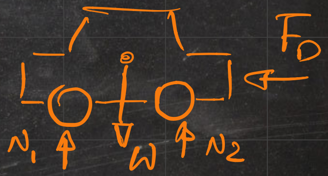
$\Rightarrow$ EQUIVALENT SYSTEM OF FORCES



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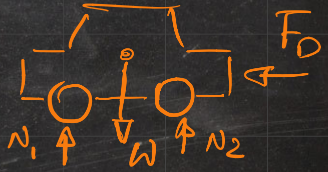

⇒ SOLVE EQUILIBRIUM NEXT TIME!

PRINCIPLE OF TRANSMISSIBILITY $\rightsquigarrow$ EQUIVALENT FORCES

PRINCIPLE OF TRANSMISSIBILITY ⟶ EQUIVALENT FORCES

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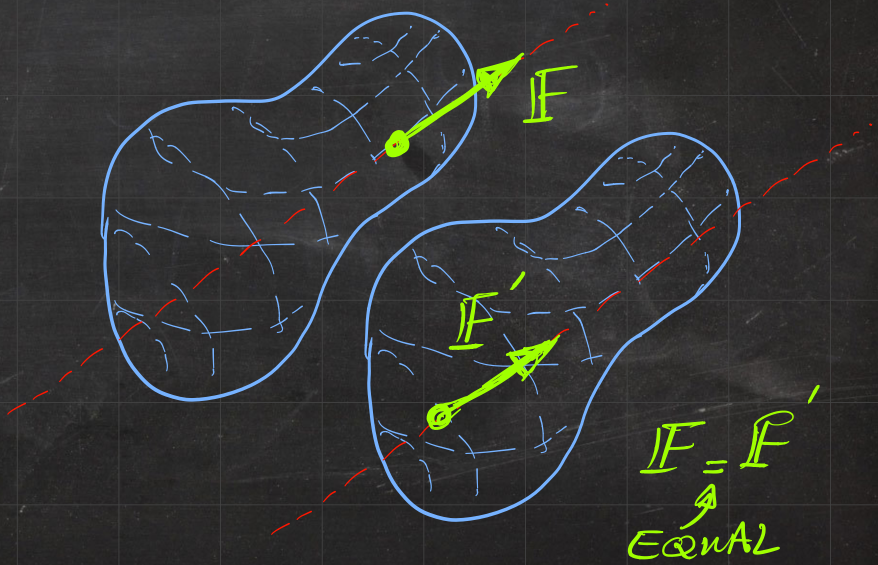
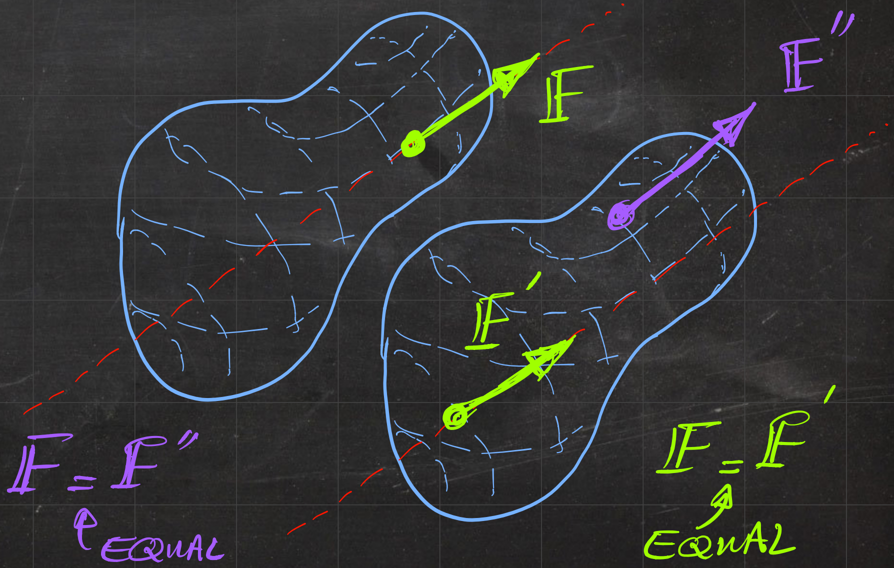


PRINCIPLE OF TRANSMISSIBILITY ⟶ EQUIVALENT FORCES

The effect of a force on a rigid body will not change if it is moved along its line of action!

$\mathbb{F}'$ IS EQUIVALENT TO $\mathbb{F}$

$\mathbb{F}''$ IS NOT EQUIVALENT TO $\mathbb{F}$



The effect of a force on a rigid body will not change if it is moved along its line of action!

TWO FORCE VECTORS ARE

EQUAL

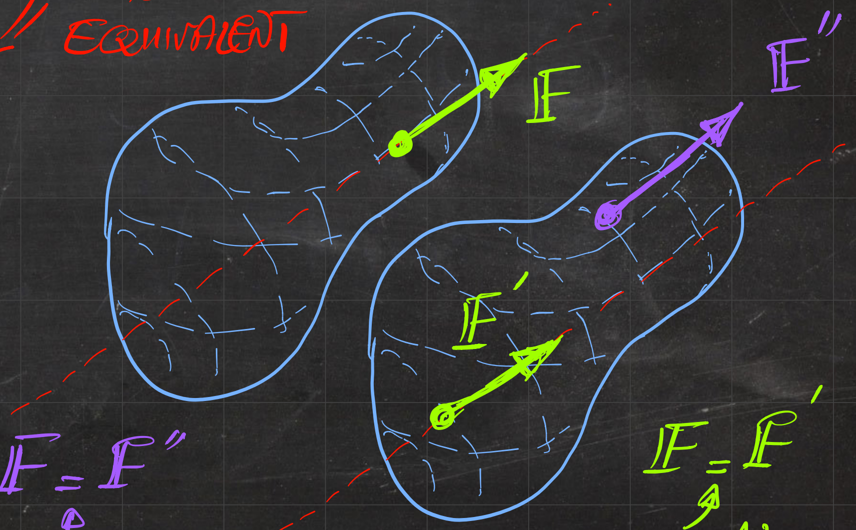
IF THEY HAVE

THE SAME MAGNITUDE

AND

THE SAME DIRECTION

NOT ENOUGH TO BE EQUIVALENT



$\mathbb{F} = \mathbb{F}''$

EQUAL

$\mathbb{F} = \mathbb{F}'$

EQUAL

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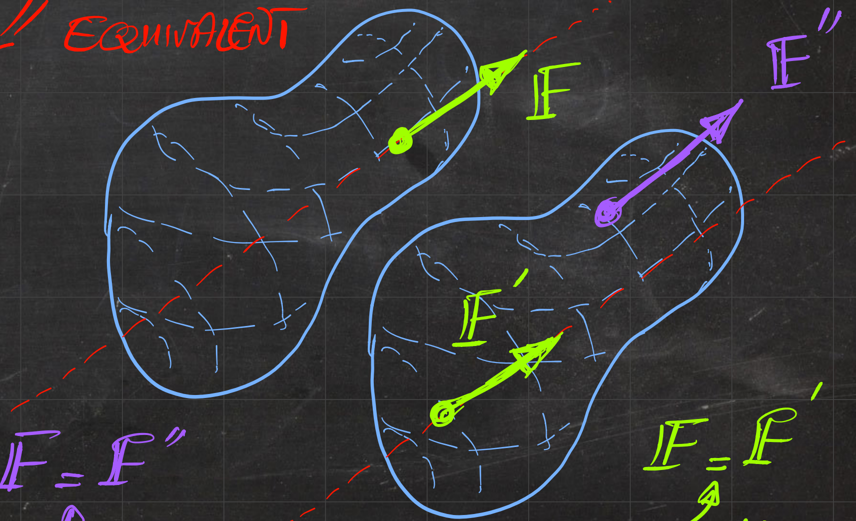

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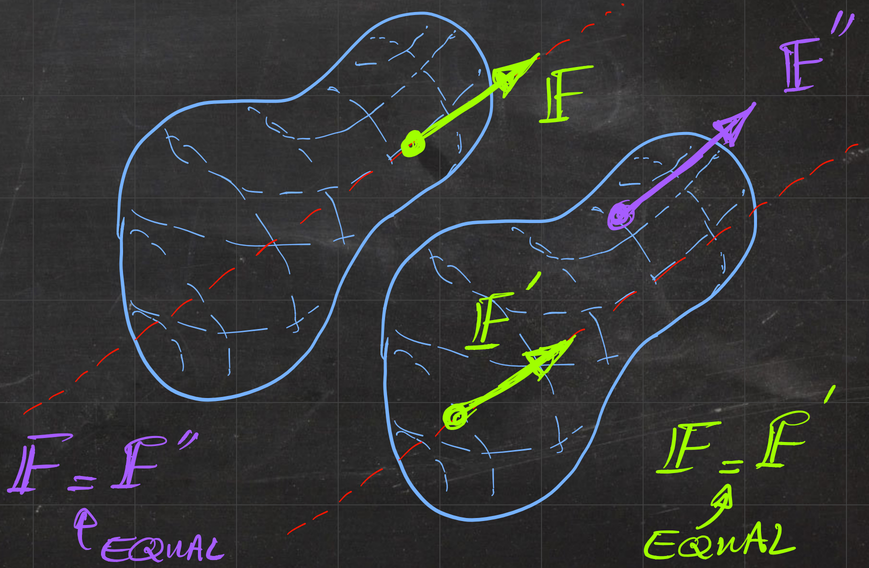
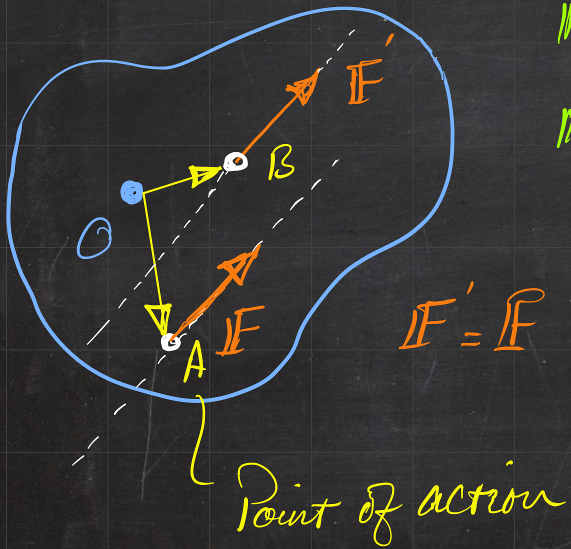


MOMENT DUE TO $\mathbb{F}$

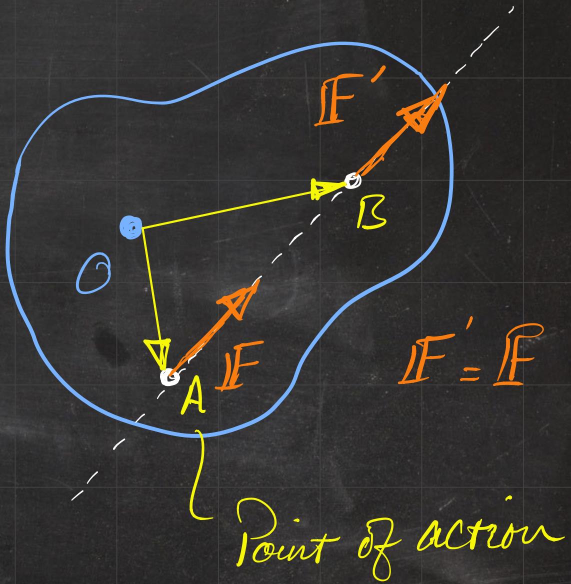
$$\mathbb{M}_O = \vec{OA} \times \mathbb{F}$$

$$\mathbb{M}'_O = \vec{OB} \times \mathbb{F}'$$

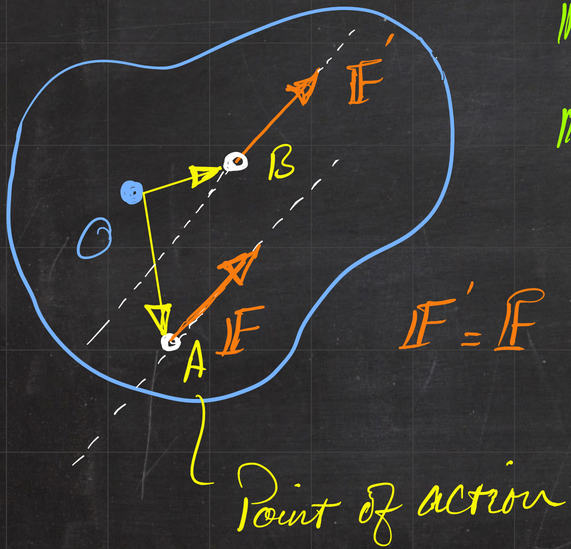
MOMENT DUE TO $\mathbb{F}'$



$$\mathbb{M}'_O \neq \mathbb{M}_O$$



$$\mathbb{M}'_O = \mathbb{M}$$

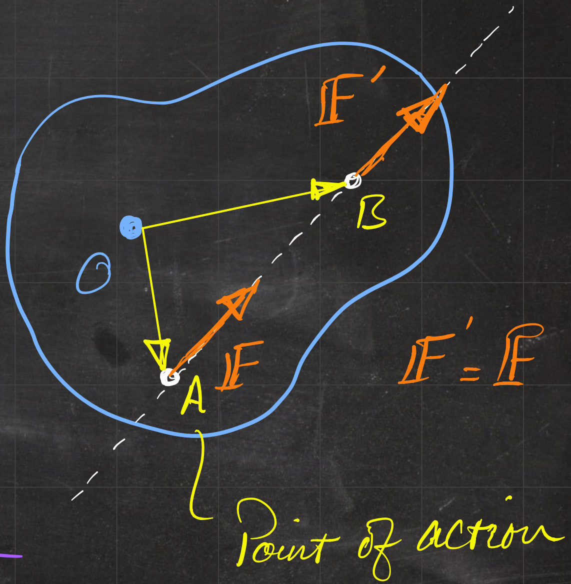


MOMENT DUE TO F

$$\mathbb{M}_O = \overrightarrow{OA} \times \mathbb{F}$$

$$\mathbb{M}'_O = \overrightarrow{OB} \times \mathbb{F}'$$

MOMENT DUE TO F'



EQUAL

$\Uparrow \quad \not\Downarrow$

EQUIVALENT

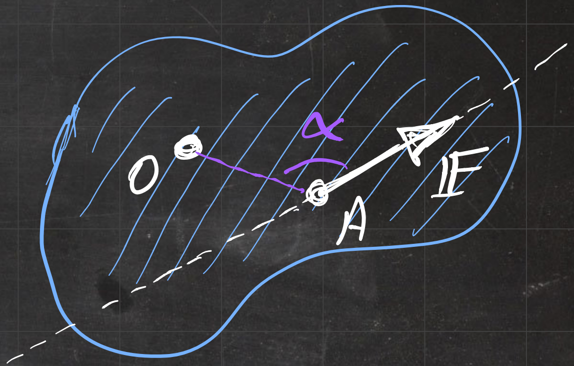
$$\mathbb{M}'_O \neq \mathbb{M}_O$$

$$\mathbb{M}'_O = \mathbb{M}$$

MOMENT OF A FORCE ABOUT A POINT

MOMENT OF A FORCE ABOUT A POINT

$$\mathbb{M}_O = \vec{OA} \times \mathbb{F}$$



$$|\mathbb{M}_O| = |\vec{OA}|\,|\mathbb{F}|\,\sin\alpha$$

$$0 \leq \alpha \leq 180 \Rightarrow \sin\alpha \geq 0$$

RECALL: VECTOR PRODUCT OF TWO VECTORS

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↳ OUTER PRODUCT or CROSS PRODUCT $\times$

COMPARE WITH

SCALAR PRODUCT OF TWO VECTORS

↳ INNER PRODUCT or DOT PRODUCT $\cdot$

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COMPARE WITH

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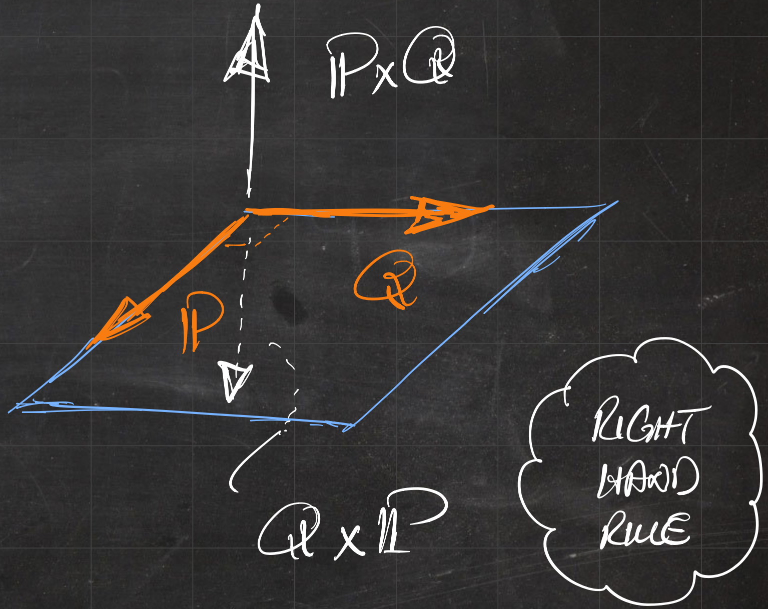
$$\underbrace{\mathbb{P} \times \mathbb{Q}}_{\text{VECTOR}} = - \underbrace{\mathbb{Q} \times \mathbb{P}}_{\text{VECTOR}} \qquad \underbrace{\mathbb{P} \cdot \mathbb{Q}}_{\text{SCALAR}} = \underbrace{\mathbb{Q} \cdot \mathbb{P}}_{\text{SCALAR}}$$

↳ $PQ \cos\theta$

RECALL: VECTOR PRODUCT OF TWO VECTORS



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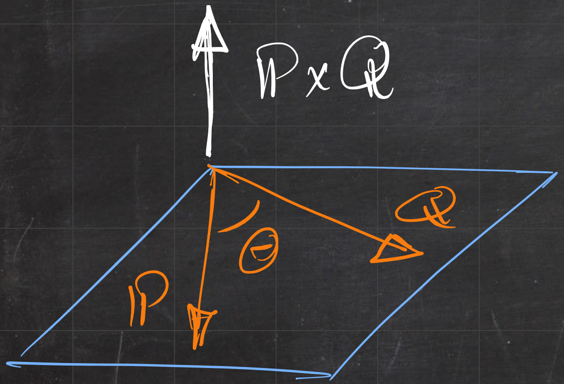

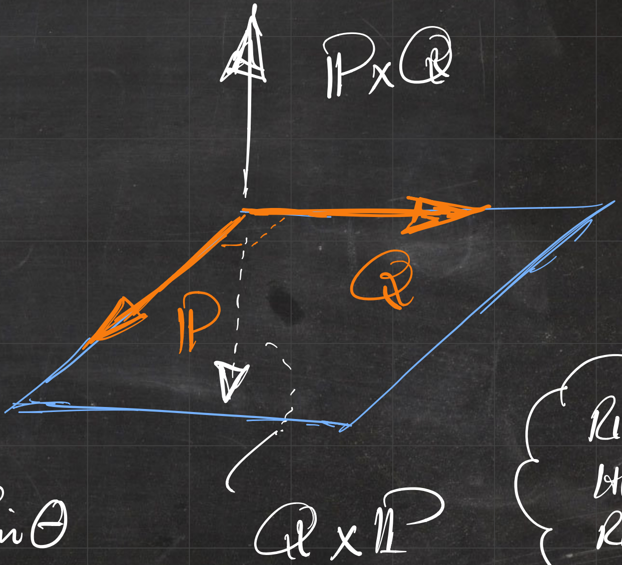

RIGHT HAND RULE

$$|\mathbb{P} \times \mathbb{Q}| = |\mathbb{P}|\,|\mathbb{Q}|\,\sin\theta$$

RECALL: VECTOR PRODUCT OF TWO VECTORS

$$\mathbb{V} = \mathbb{P} \times \mathbb{Q}$$

$$\Downarrow$$

$$\mathbb{V} = [P_y Q_z - P_z Q_y]\,\mathbb{i} + [P_z Q_x - P_x Q_z]\,\mathbb{j} + [P_x Q_y - P_y Q_x]\,\mathbb{k}$$

RECALL: VECTOR PRODUCT OF TWO VECTORS

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$$\Downarrow$$

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$$\mathbb{V} = \begin{vmatrix} \mathbb{i} & \mathbb{j} & \mathbb{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

RECALL: VECTOR PRODUCT OF TWO VECTORS

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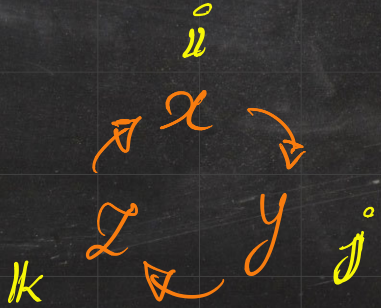
$$\mathbb{V} = [P_y Q_z - P_z Q_y]\,\mathbf{i} + [P_z Q_x - P_x Q_z]\,\mathbf{j} + [P_x Q_y - P_y Q_x]\,\mathbb{k}$$



MOMENT OF A FORCE ABOUT A GIVEN AXIS

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

VECTOR

$\vec{OA}$

$M_{OL} = ?$ → PROJECTION OF $\mathbf{M}_O$ ONTO THE AXIS $\vec{OL}$

SCALAR

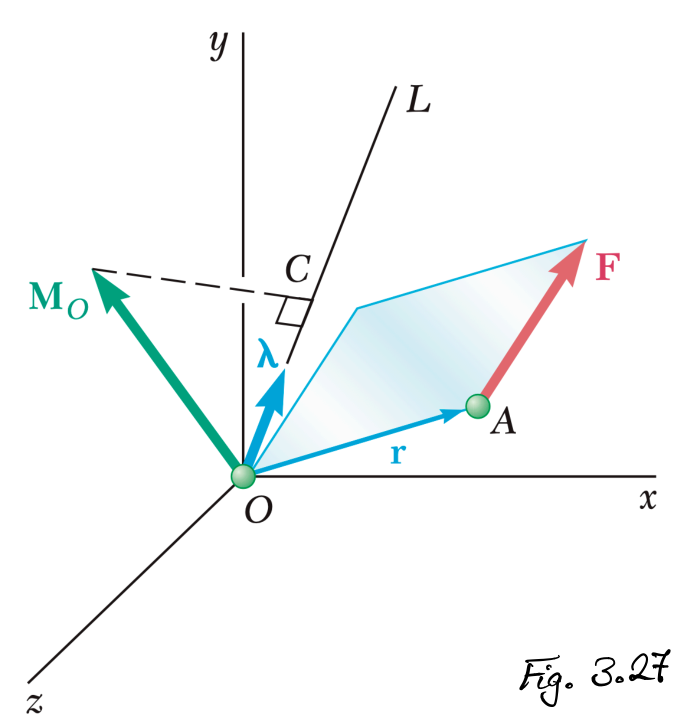


Fig. 3.27

MOMENT OF A FORCE ABOUT A GIVEN AXIS

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VECTOR

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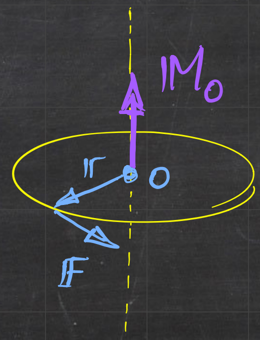

$M_{OL} = ?$ PROJECTION OF $\mathbf{M}_O$ ONTO THE AXIS $\overrightarrow{OL}$

SCALAR

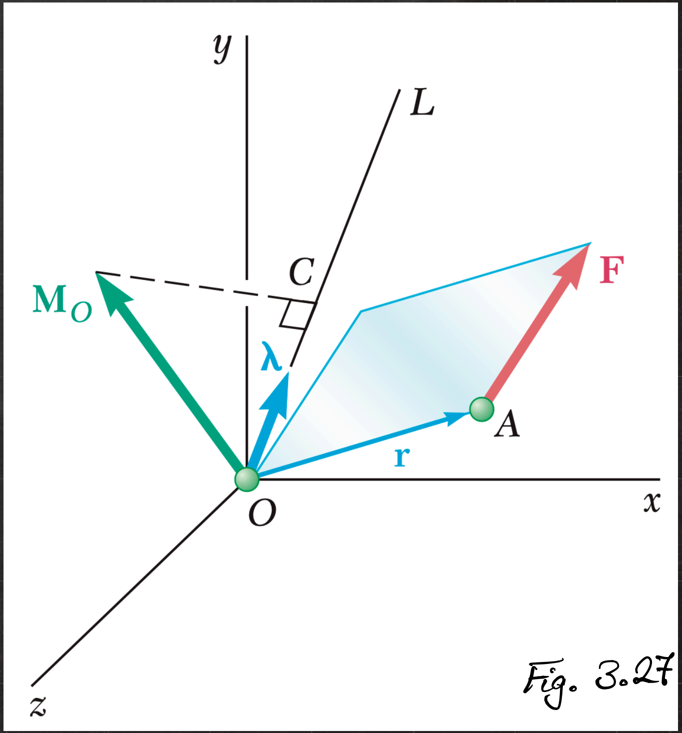

Fig. 3.27

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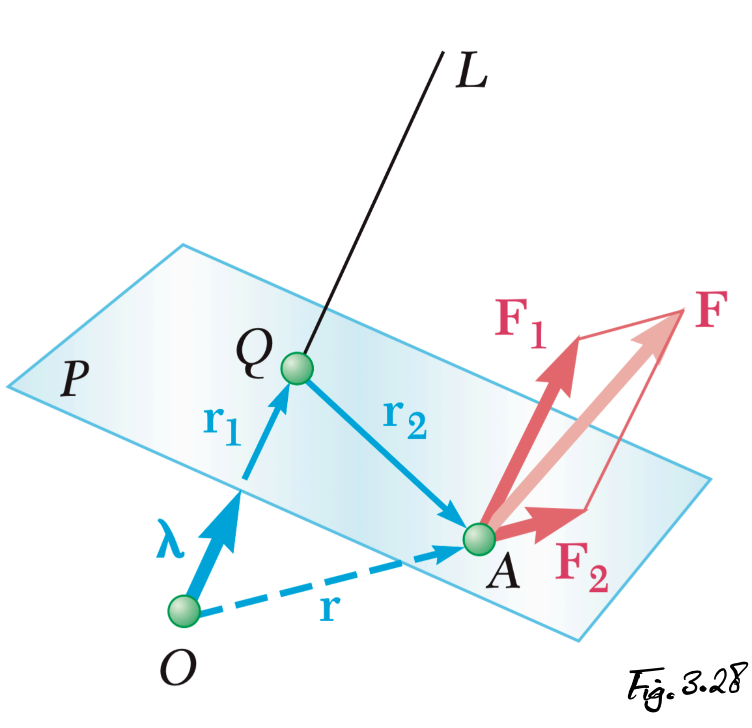
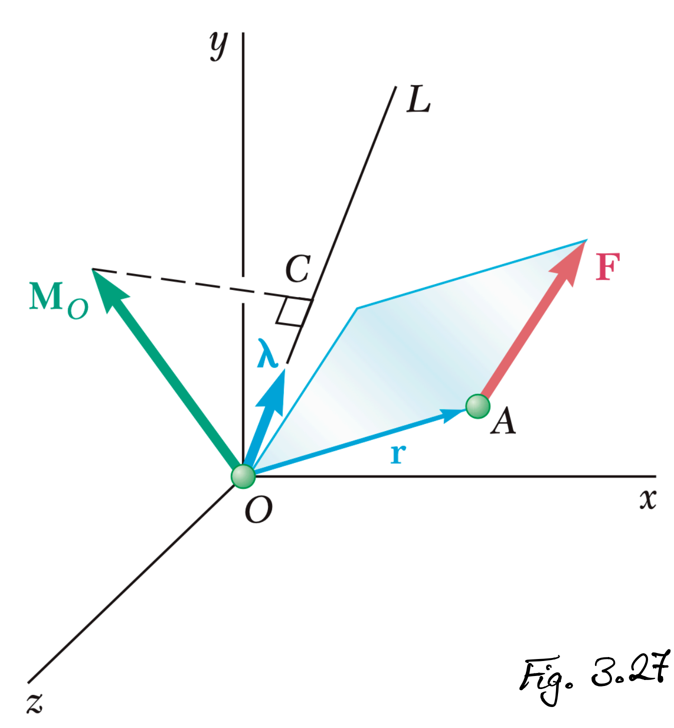


Fig. 3.28



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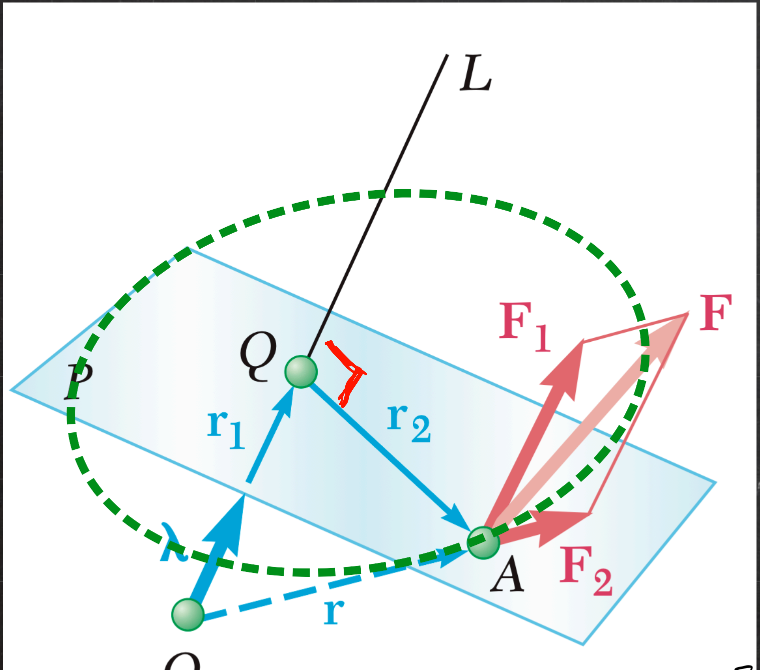

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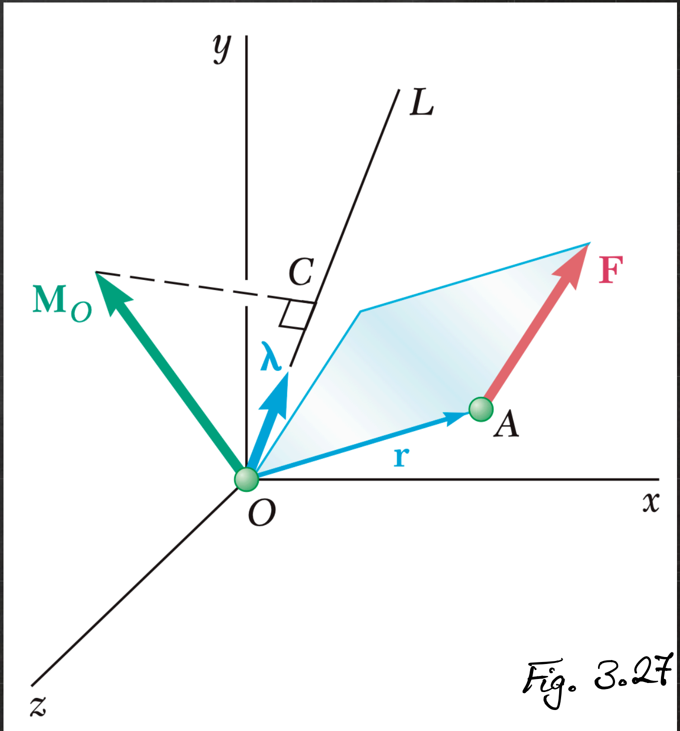

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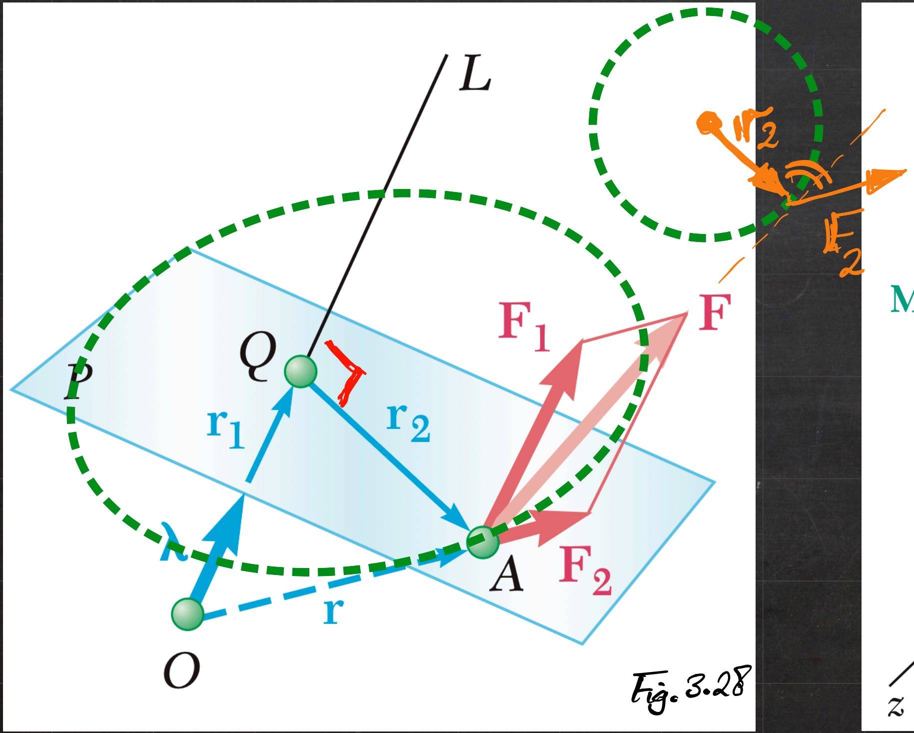

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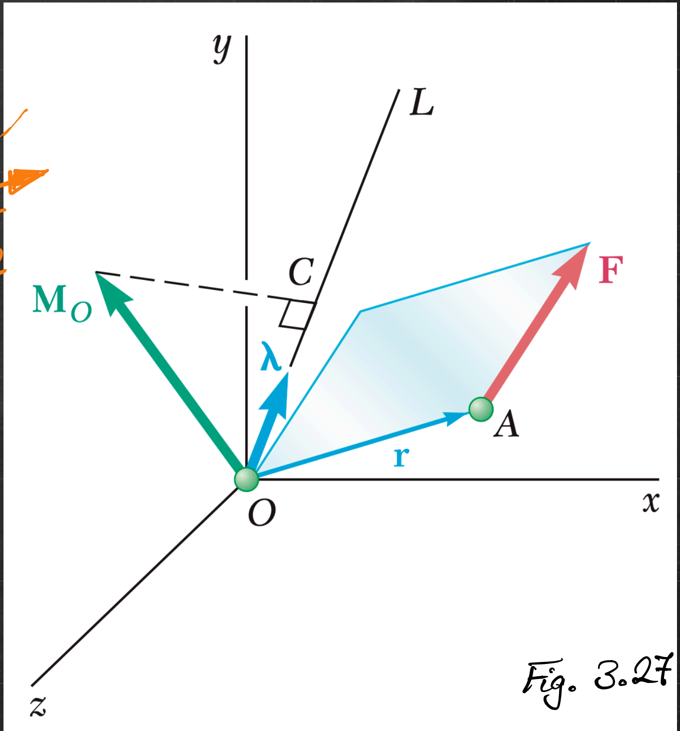

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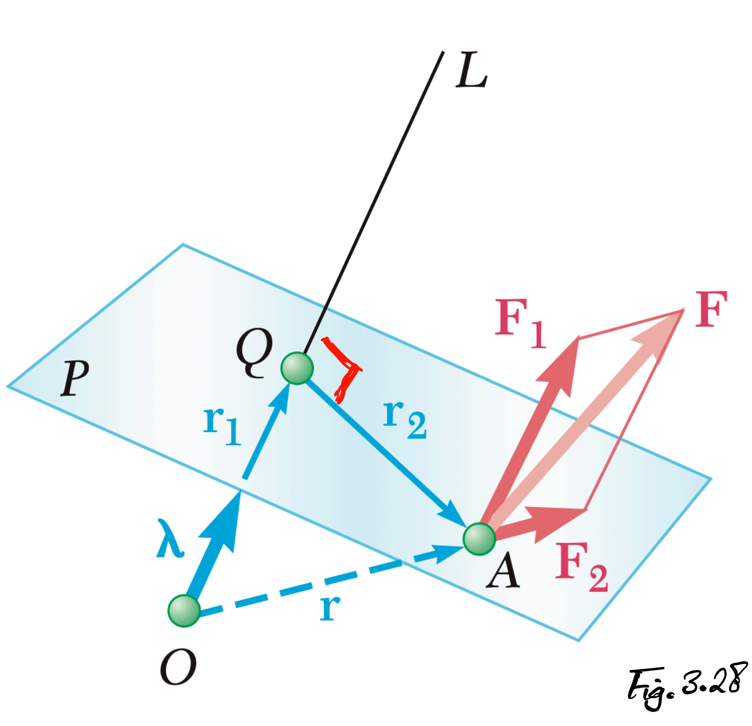


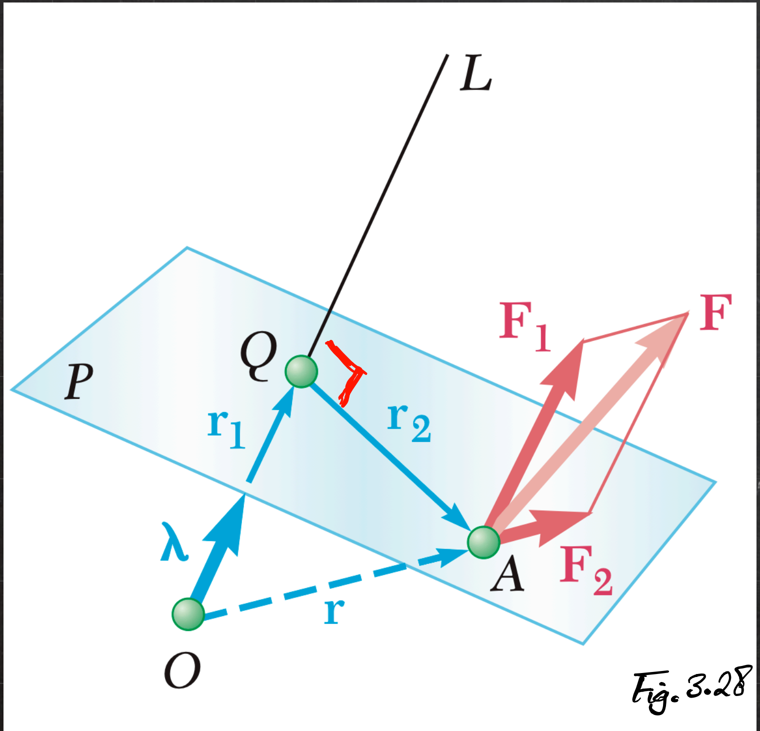
Fig. 3.28

$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O$$

$$= \boldsymbol{\lambda} \cdot [\mathbf{r} \times \mathbf{F}]$$

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE ABOUT A GIVEN AXIS



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$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O$$

$$= \boldsymbol{\lambda} \cdot [\mathbf{r} \times \mathbf{F}]$$

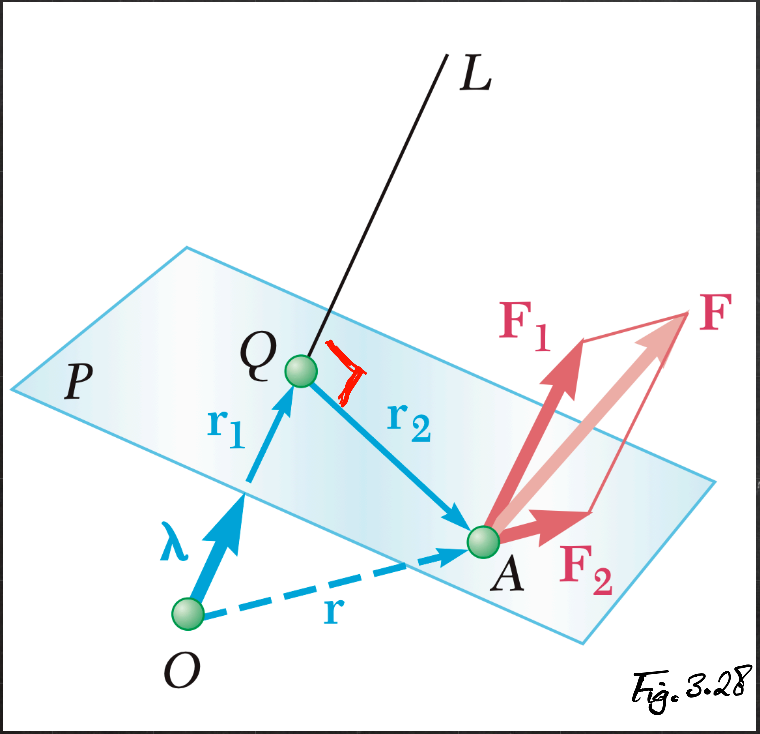
$$\Downarrow$$

$$M_{OL} = \boldsymbol{\lambda} \cdot \big[[\mathbf{r}_1 + \mathbf{r}_2] \times [\mathbf{F}_1 + \mathbf{F}_2] \big]$$

$$\overset{!}{=} \boldsymbol{\lambda} \cdot [\mathbf{r}_1 \times \mathbf{F}_1] + \boldsymbol{\lambda} \cdot [\mathbf{r}_1 \times \mathbf{F}_2]$$

$$+ \boldsymbol{\lambda} \cdot [\mathbf{r}_2 \times \mathbf{F}_1] + \boldsymbol{\lambda} \cdot [\mathbf{r}_2 \times \mathbf{F}_2]$$

MOMENT OF A FORCE ABOUT A GIVEN AXIS



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$$= \underbrace{\boldsymbol{\lambda} \cdot [\mathbf{r}_1 \times \mathbf{F}_1]}_{0} + \underbrace{\boldsymbol{\lambda} \cdot [\mathbf{r}_1 \times \mathbf{F}_2]}_{0}$$

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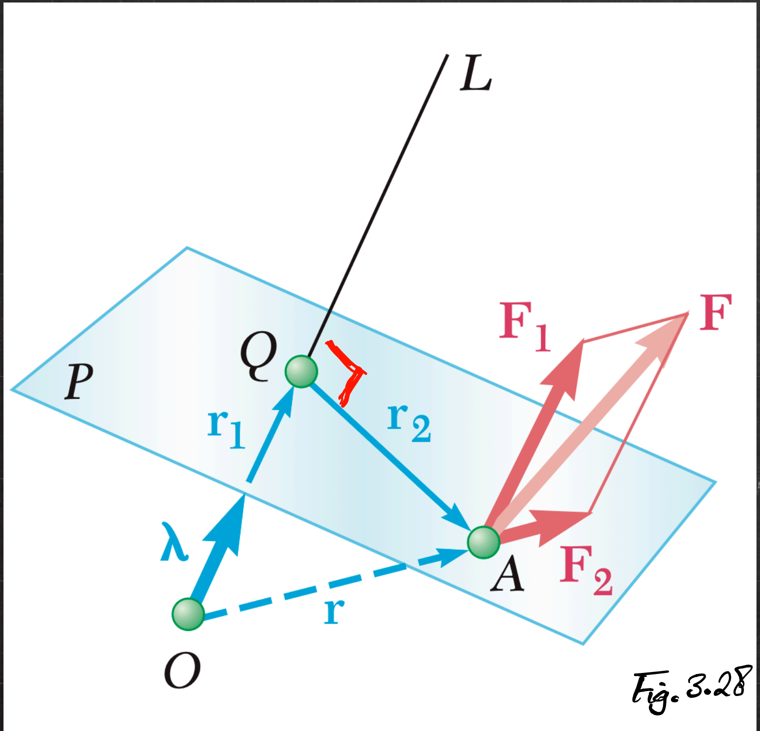

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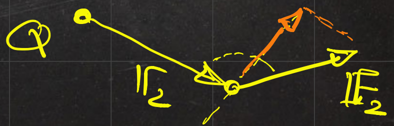
$$\Downarrow$$

$$M_{OL} = \boldsymbol{\lambda} \cdot [\mathbf{r}_2 \times \mathbf{F}_2]$$

$$\Rightarrow M_{OL} = r_2 \, F_2^{\perp}$$

TOP VIEW



MOMENT OF A FORCE ABOUT A GIVEN AXIS

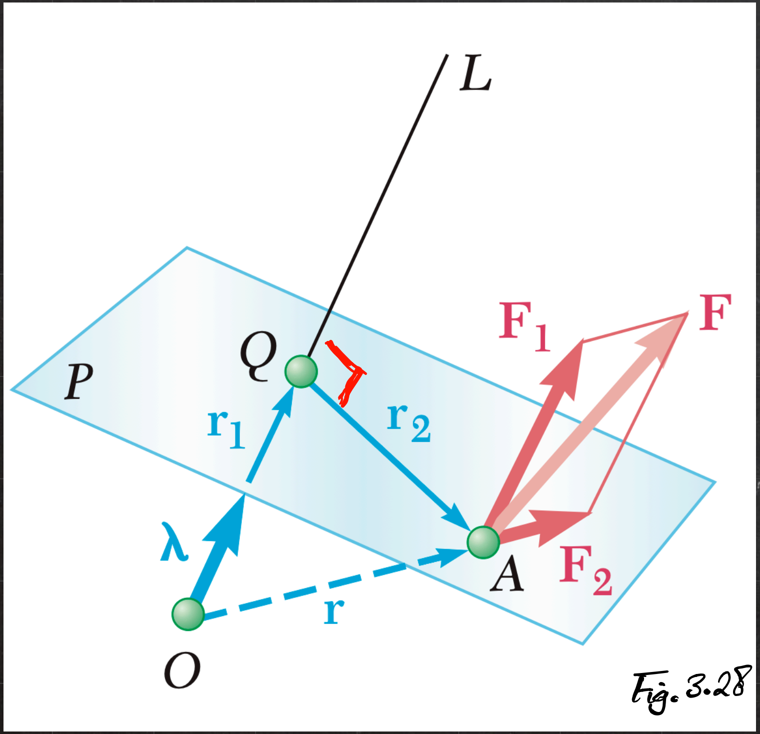


Fig. 3.28

$$M_{OL} = \lambda \cdot M_O$$
$$= \lambda \cdot [r \times F]$$

SCALAR

PARALLEL TO λ

$$- \quad M_{OL} = \lambda \cdot [r_2 \times F_2]$$

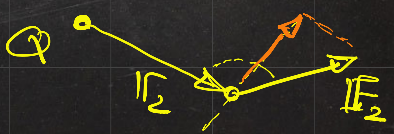
INDICATES DIRECTION

$+$

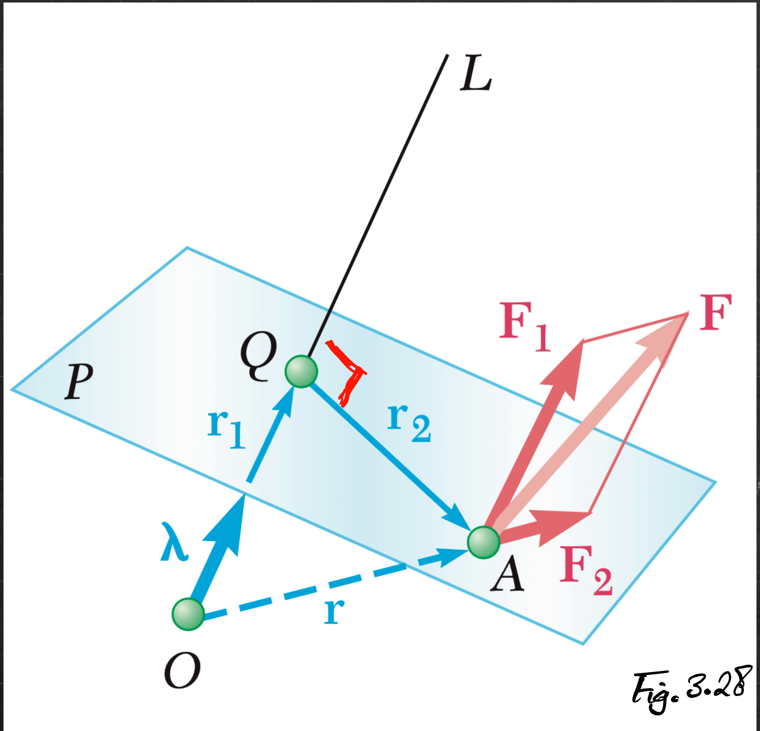
$$\Rightarrow M_{OL} = r_2 \, F_2^{\perp}$$

TOP VIEW

$F_2^{\perp}$



MOMENT OF A FORCE ABOUT A GIVEN AXIS



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$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O$$

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SCALAR ⇓

PARALLEL TO $\boldsymbol{\lambda}$

$$M_{OL} = \boldsymbol{\lambda} \cdot [\mathbf{r}_2 \times \mathbf{F}_2]$$

−

INDICATES DIRECTION

\+

$$\Rightarrow |M_{OL}| = |\mathbf{r}_2 \times \mathbf{F}_2|$$

Exercise 1 . [similar to … P. 100 … 3.5]

FORCE $\mathbb{P}$ ACTS ON A CUBE OF SIDE a.

- DETERMINE THE MOMENT OF $\mathbb{P}$

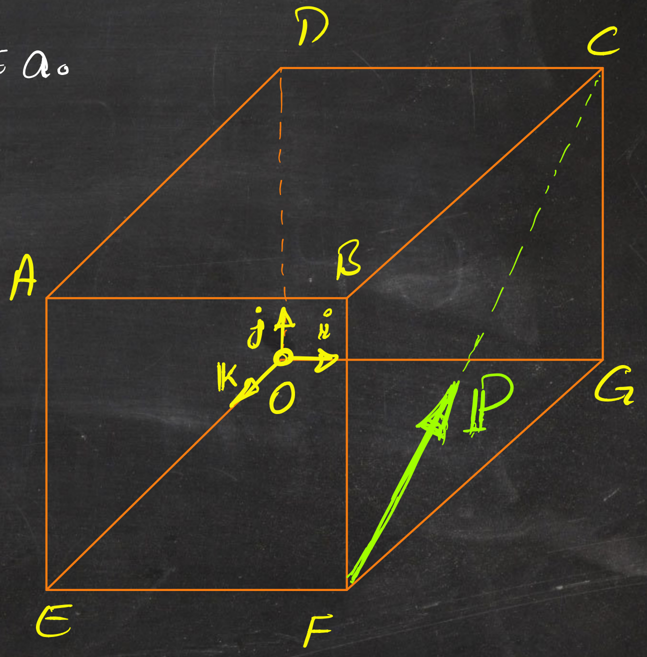
  a) ABOUT A

  b) ABOUT AB

  c) ABOUT AG

- USING THE RESULT OF (C),

  DETERMINE THE PERPENDICULAR DISTANCE BETWEEN AG AND FC.



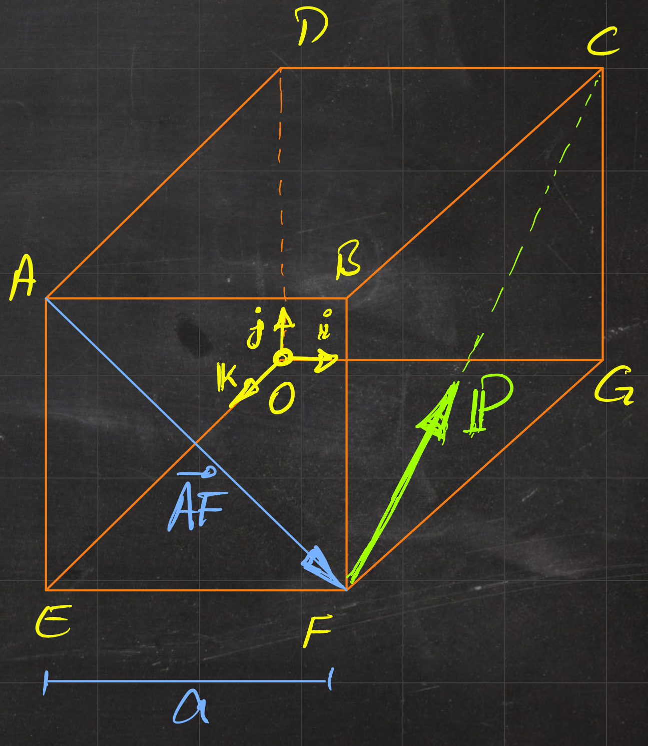
a) $\mathbb{M}_A = ?$

$$\mathbb{M}_A = \frac{aP}{\sqrt{2}}\left[\mathbf{i}+\mathbf{j}+\mathbf{k}\right]$$

$$\mathbb{M}_A = \vec{AF} \times \mathbb{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & -a & 0 \\ 0 & \frac{P}{\sqrt{2}} & -\frac{P}{\sqrt{2}} \end{vmatrix} = \begin{bmatrix} aP/\sqrt{2} \\ aP/\sqrt{2} \\ aP/\sqrt{2} \end{bmatrix}$$

$a\,\mathbf{i} - a\,\mathbf{j}$

$\frac{P}{\sqrt{2}}\,\mathbf{j} - \frac{P}{\sqrt{2}}\,\mathbf{k}$



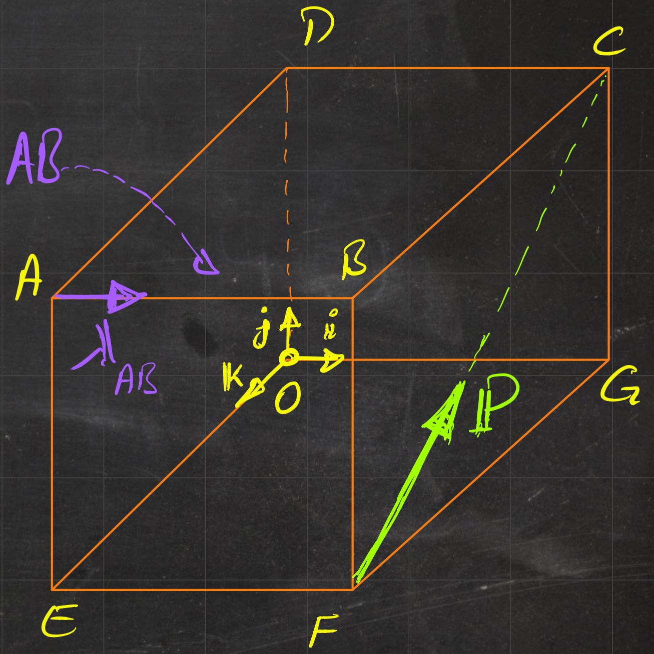
b) $M_{AB} = ?$

$$M_{AB} = \lambda_{AB} \cdot \mathbb{M}_A = \frac{aP}{\sqrt{2}}$$

$\mathbb{i}$

from (a)

$$\frac{aP}{\sqrt{2}}\left[\mathbb{i} + \mathbb{j} + \mathbb{k}\right]$$

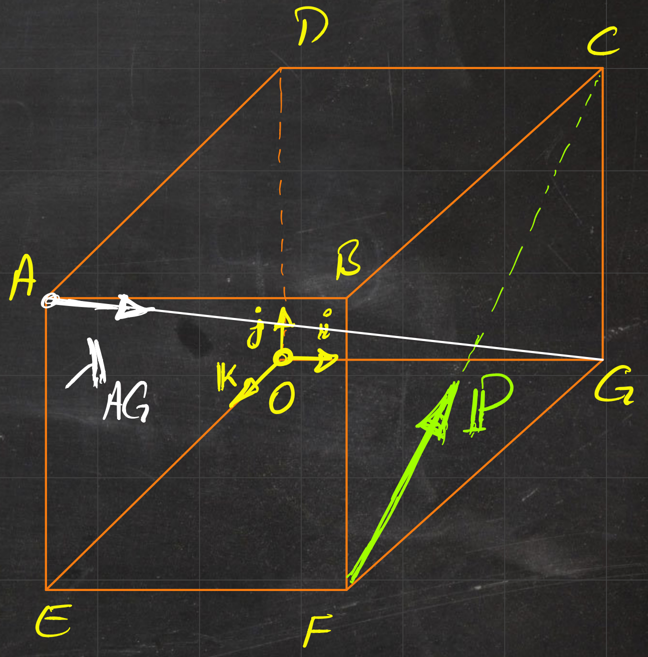


c) $M_{AG} = ?$

$$M_{AG} = \boldsymbol{\lambda}_{AG} \cdot \mathbf{M}_A$$

$$\boldsymbol{\lambda}_{AG} = \frac{\vec{AG}}{|\vec{AG}|}$$

$$\vec{AG} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$



c) $M_{AG} = ?$

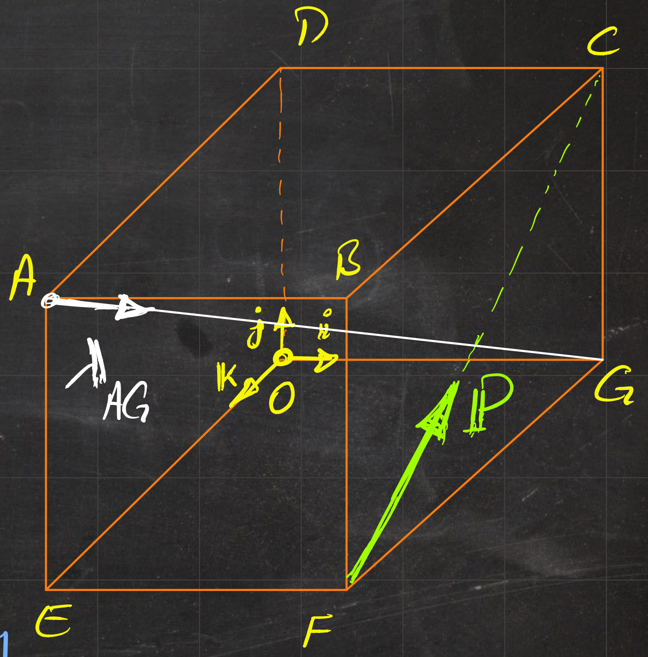
$$M_{AG} = \lambda_{AG} \cdot \mathbf{M}_A$$

$$\lambda_{AG} = \frac{\vec{AG}}{|\vec{AG}|}$$

$$|\vec{AG}| = \sqrt{3}$$

$$\Uparrow$$

$$\vec{AG} = \overset{G}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} - \overset{A}{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}} = [\hat{i} - \hat{j} - \mathbb{k}]$$

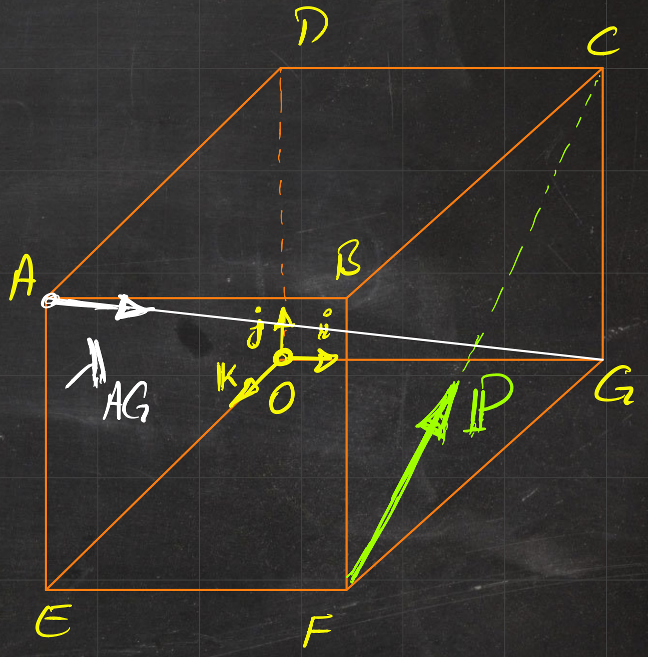


c) $M_{AG} = ?$

$$M_{AG} = \lambda_{AG} \cdot \mathbb{M}_A$$

$$\lambda_{AG} = \frac{\vec{AG}}{|\vec{AG}|} \qquad \frac{aP}{\sqrt{2}}\left[\hat{i}+\hat{j}+\mathbb{k}\right]$$

$$= \frac{1}{\sqrt{3}}\left[\hat{i}-\hat{j}-\mathbb{k}\right]$$



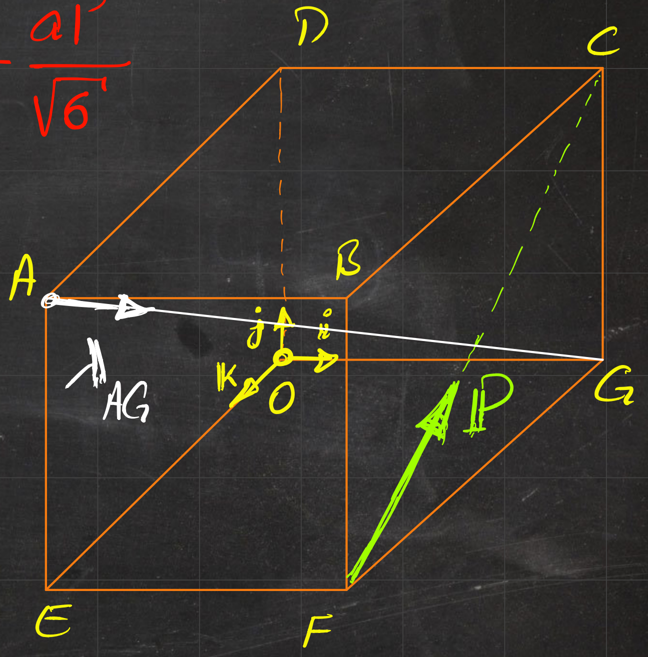
c) $M_{AG} = ?$

$$M_{AG} = -\frac{aP}{\sqrt{6}}$$

$$M_{AG} = \boldsymbol{\lambda}_{AG} \cdot \mathbf{M}_A$$

$$\boldsymbol{\lambda}_{AG} = \frac{\vec{AG}}{|\vec{AG}|} = \frac{1}{\sqrt{3}}[\hat{\mathbf{i}} - \hat{\mathbf{j}} - \mathbf{k}]$$

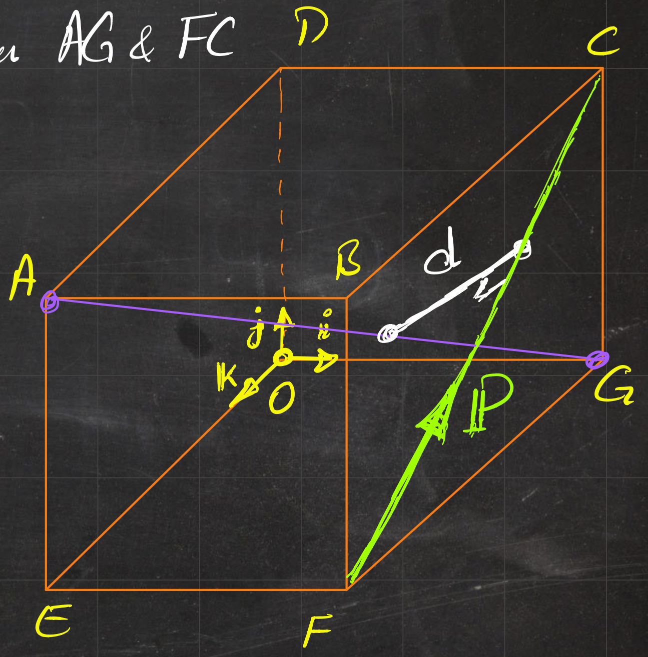
$$\mathbf{M}_A = \frac{aP}{\sqrt{2}}[\hat{\mathbf{i}} + \hat{\mathbf{j}} + \mathbf{k}]$$



d) Perpendicular distance between AG & FC

$$\mathbb{P} \cdot \lambda_{AG} = 0$$

$$\frac{P}{\sqrt{2}}\left[\mathbf{j} - \mathbf{k}\right] \qquad \frac{1}{\sqrt{3}}\left[\mathbf{i} - \mathbf{j} - \mathbf{k}\right]$$



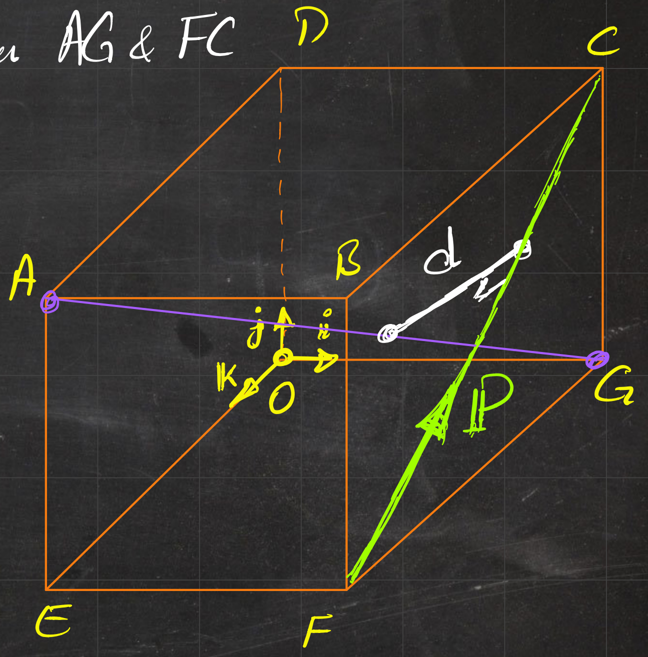
d) Perpendicular distance between AG & FC

↳ $\mathbb{P} \cdot \mathbb{1}_{AG} = 0$

↳ LINE OF ACTION OF $\mathbb{P}$ IS FC

$\Downarrow$

AG $\perp$ FC



d) Perpendicular distance between AG & FC

c) $|M_{AG}| = d\,P$

Perpendicular Distance

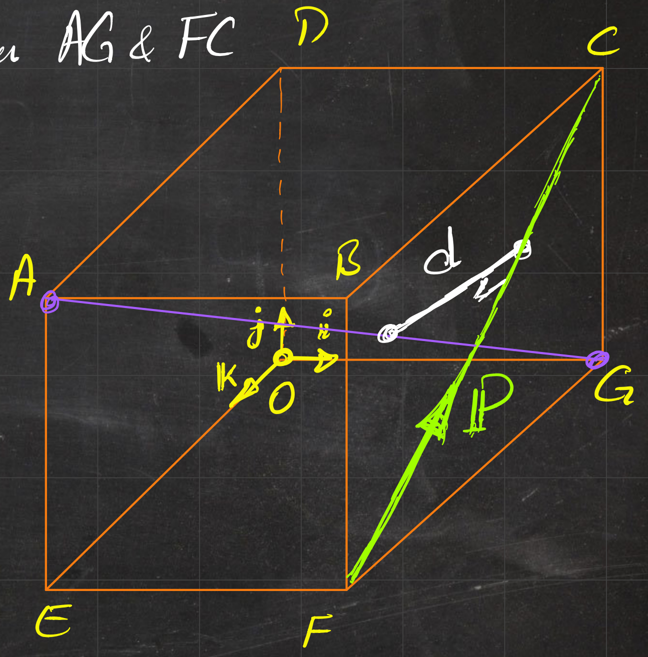
$$|M_{OL}| = |\mathbf{r}_2 \times \mathbf{F}_2|$$

RECALL

$$\Downarrow$$

$$\frac{aP}{\sqrt{6}} = d\,P$$

$$\Rightarrow d = a/\sqrt{6}$$



IDEAL MOMENT

IDEAL MOMENT

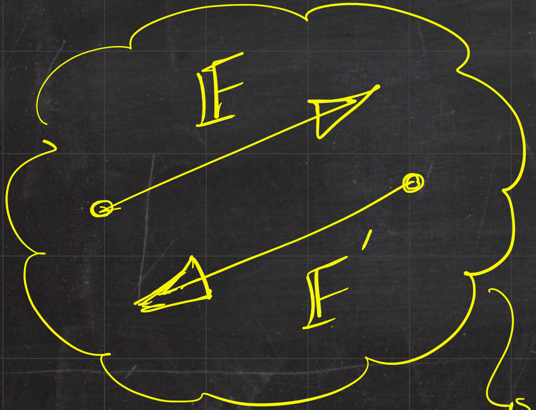
TWO FORCES ARE A COUPLE IF THEY HAVE
- SAME MAGNITUDE
- OPPOSITE DIRECTIONS

IDEAL MOMENT

TWO FORCES ARE A COUPLE IF THEY HAVE

SAME MAGNITUDE

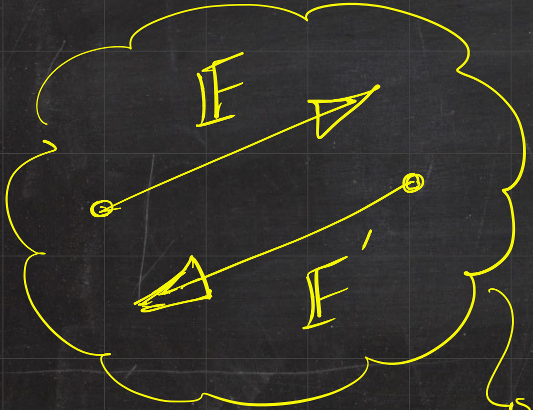
OPPOSITE DIRECTIONS



A Couple

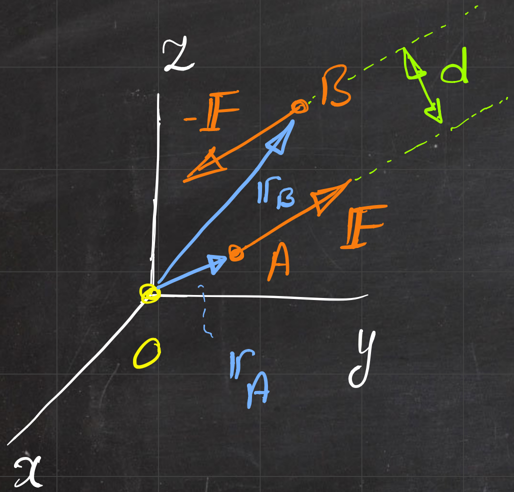
IDEAL MOMENT

Two forces are a couple if they have
- same magnitude
- opposite directions

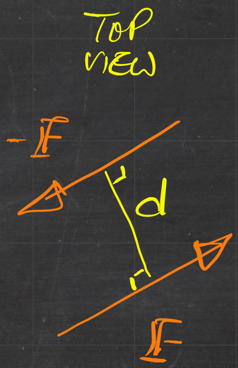


A Couple

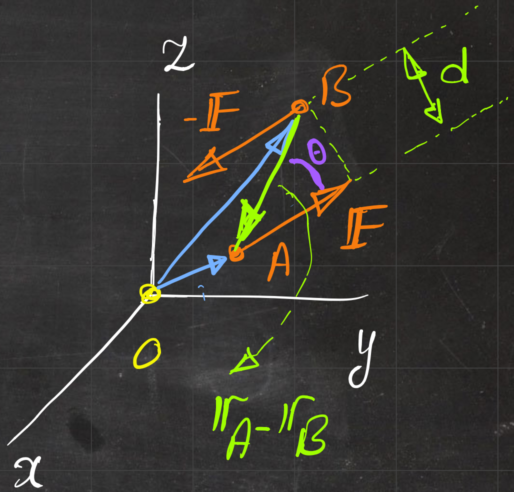
THE MOMENT CAUSED BY A COUPLE IS IDEAL MOMENT!



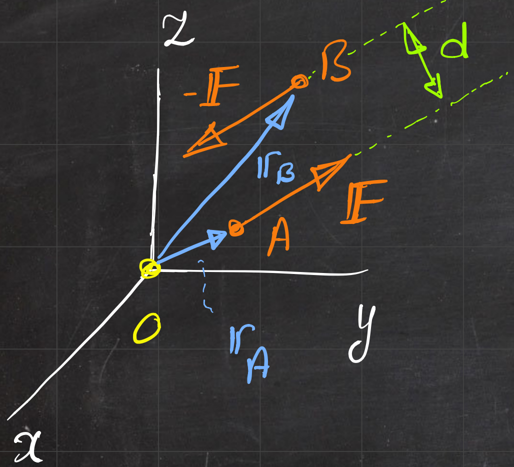
z
-F
B
d
r_B
F
A
O
r_A
y
x

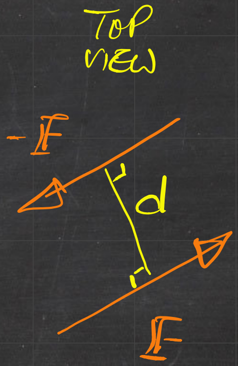


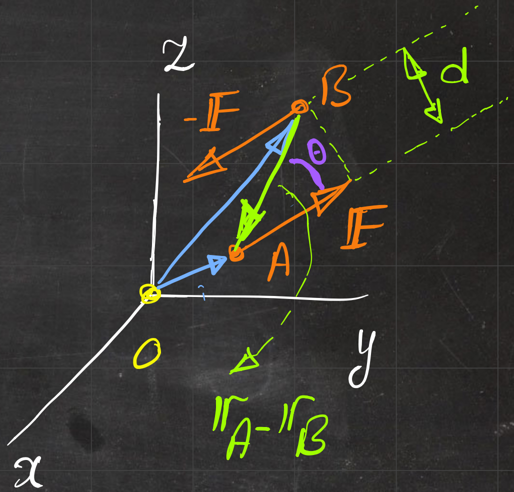
TOP VIEW
-F
d
F

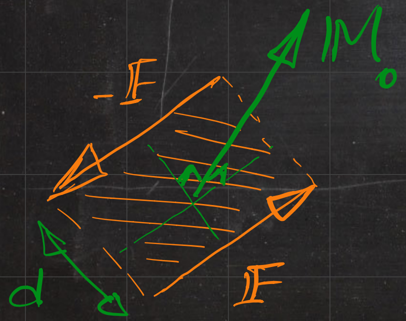


z
-F
B
d
θ
F
A
O
y
r_A - r_B
x





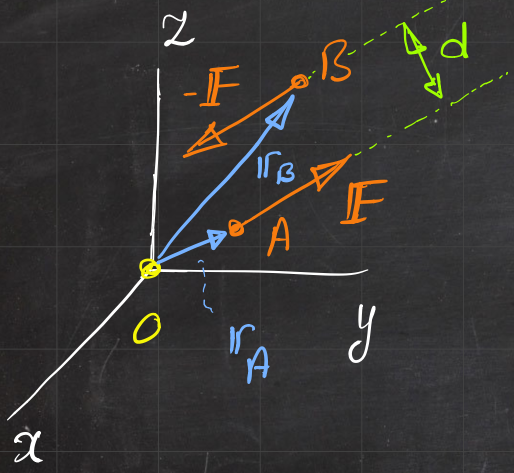


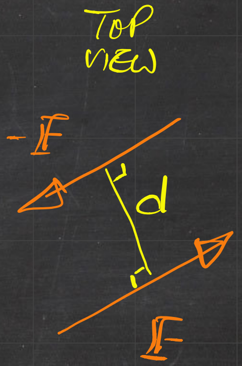


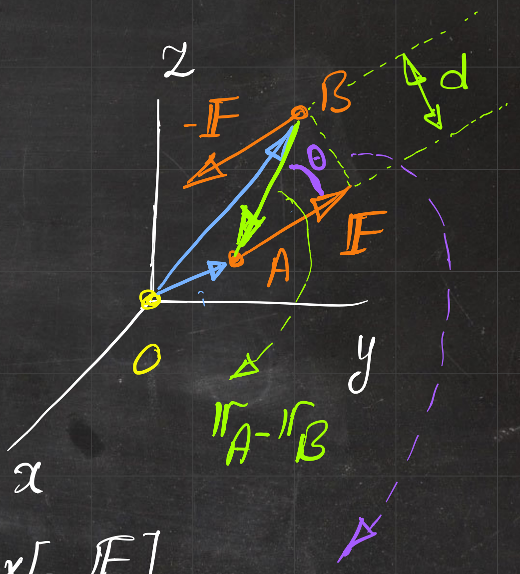
$$\mathbb{M}_o = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times [-\mathbf{F}]$$

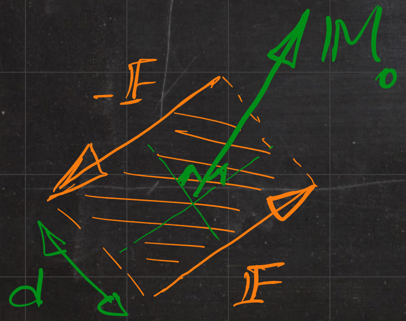
$$\stackrel{!}{=} [\mathbf{r}_A - \mathbf{r}_B] \times \mathbf{F}$$

$$\Rightarrow M_o = |\mathbf{r}_A - \mathbf{r}_B| \, F \sin\theta$$









$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times [-\mathbf{F}]$$

$$\stackrel{!}{=} [\mathbf{r}_A - \mathbf{r}_B] \times \mathbf{F}$$

$$\Rightarrow M_O = |\mathbf{r}_A - \mathbf{r}_B| \; F \sin\theta$$

$$d = |\mathbf{r}_A - \mathbf{r}_B| \sin\theta$$

$$\Rightarrow M_O = Fd$$

$$M = Fd$$

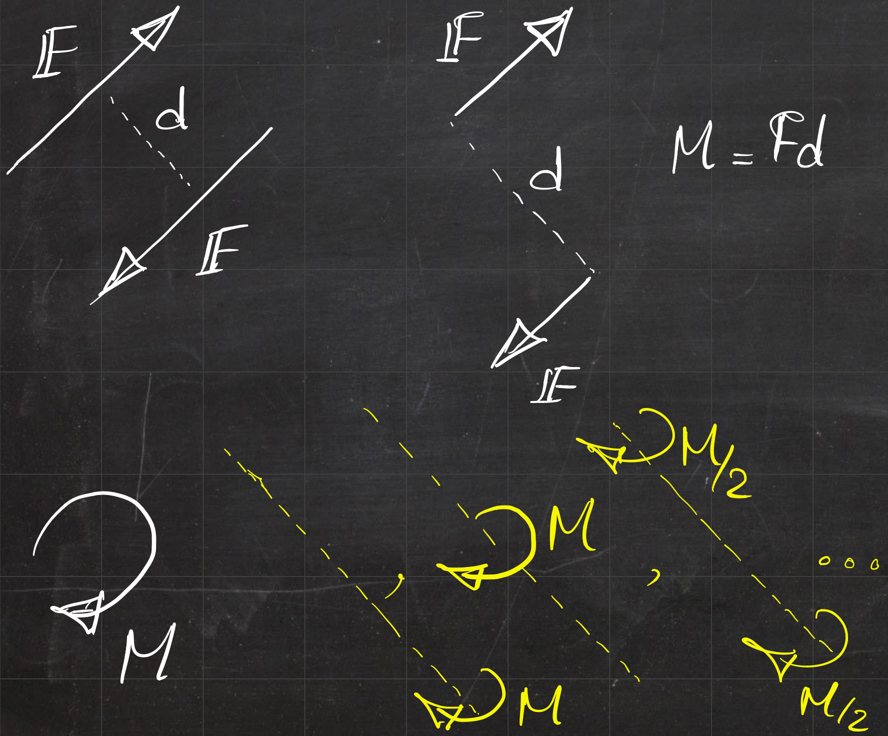
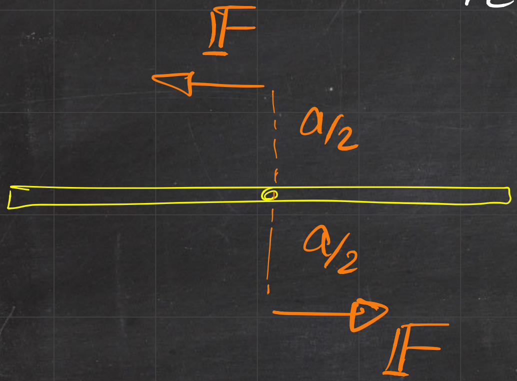
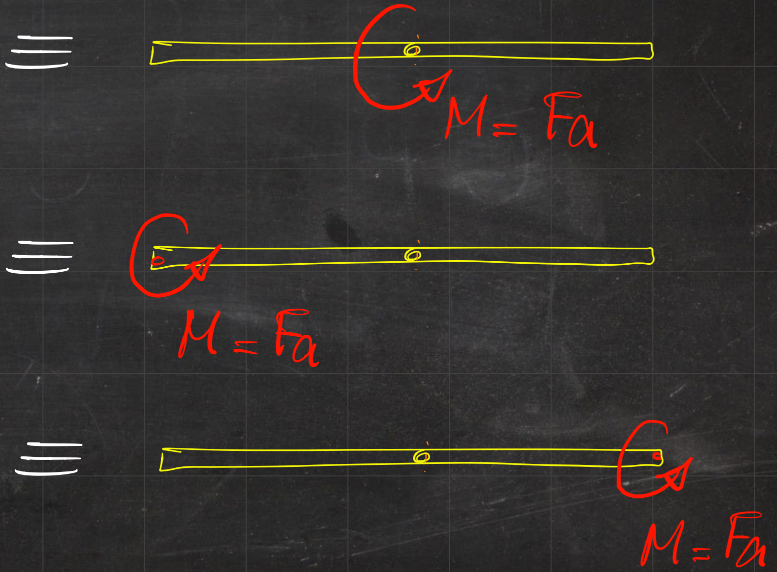


Photo 3.1 The parallel upward and downward forces of equal magnitude exerted on the arms of the lug nut wrench are an example of a couple.

RESULTANT FORCE

↳ VECTORIAL SUM OF FORCES



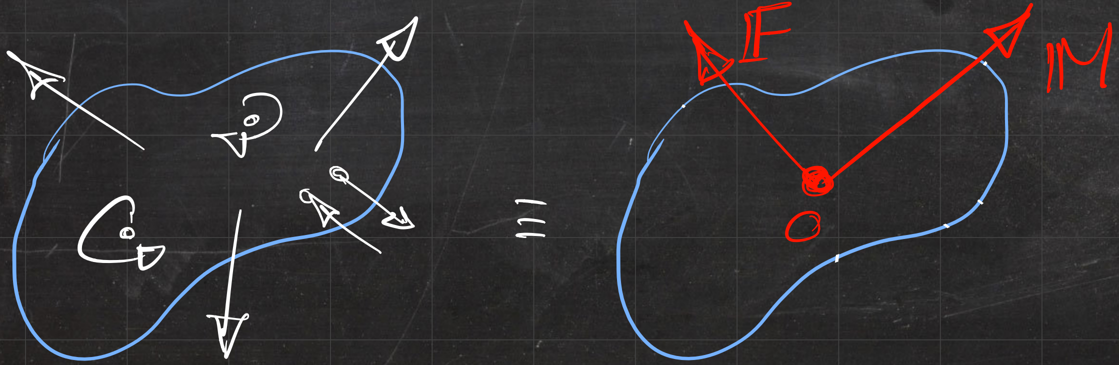


RESULTANT MOMENT

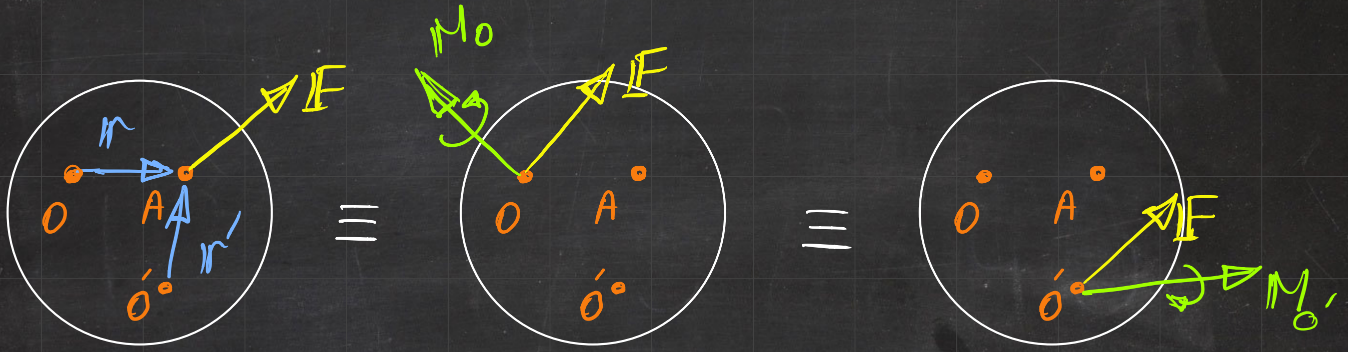
↳ VECTORIAL SUM OF MOMENTS

ANY SYSTEM OF FORCES AND MOMENTS CAN BE REDUCED TO

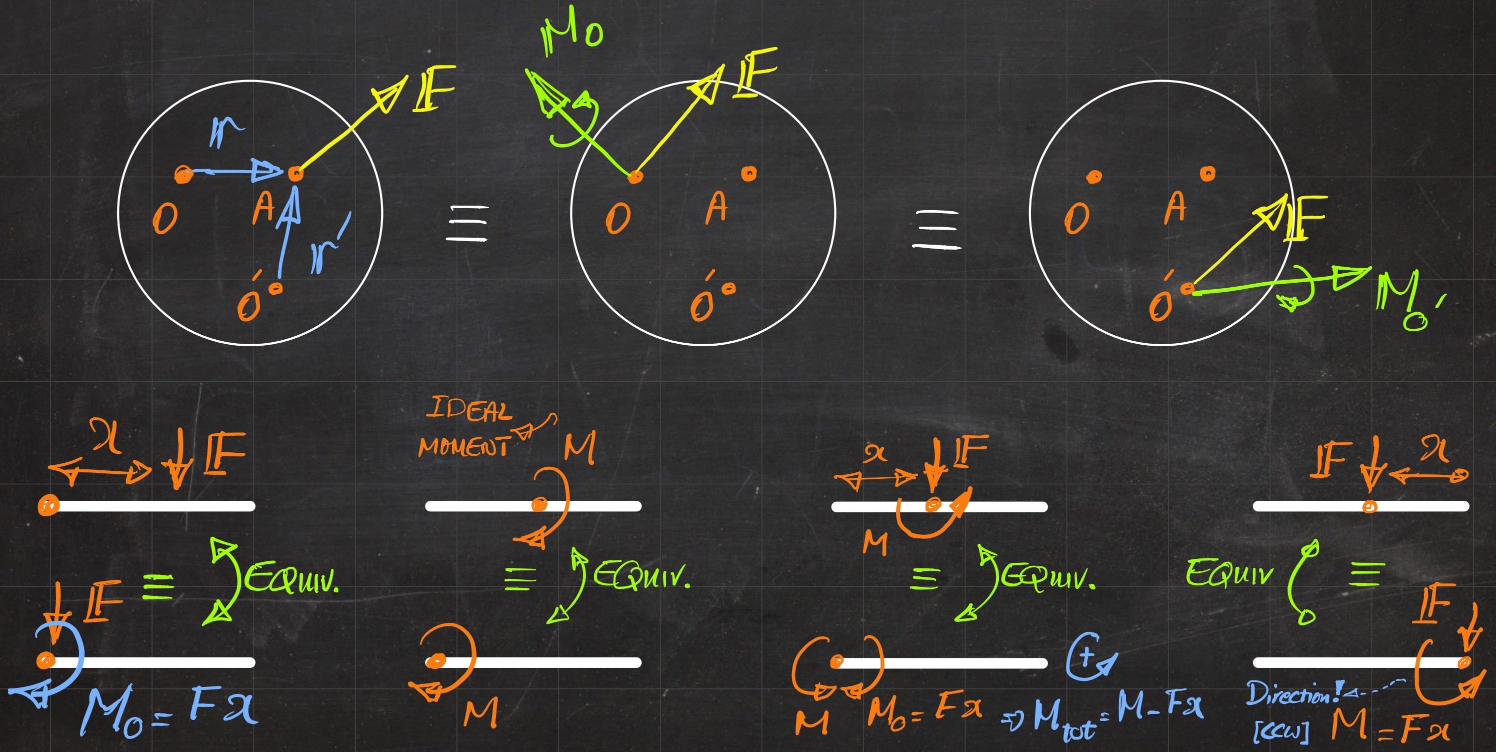
A RESULTANT FORCE + A RESULTANT MOMENT



STATICALLY EQUIVALENT SYSTEM OF FORCES



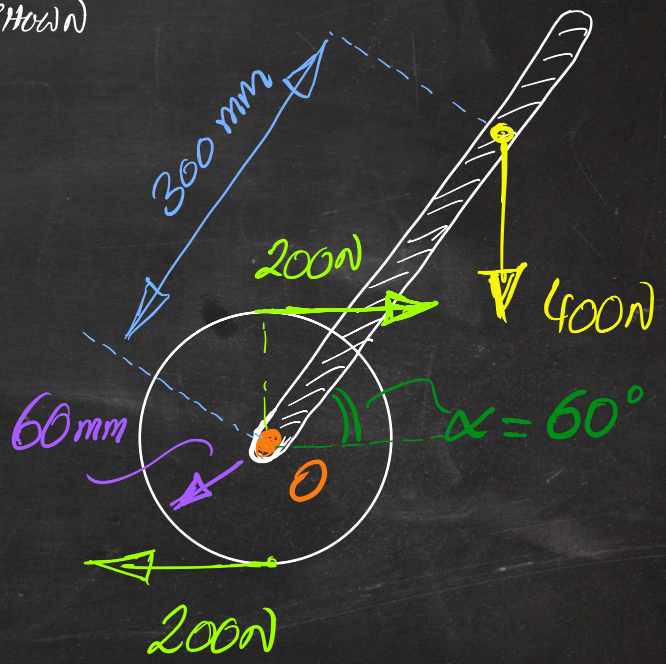
STATICALLY EQUIVALENT SYSTEM OF FORCES



Exercise 2 . [similar to … P. 115 … 3.7]

REPLACE THE COUPLE AND FORCE SHOWN BY AN EQUIVALENT SINGLE FORCE APPLIED TO THE LEVER.

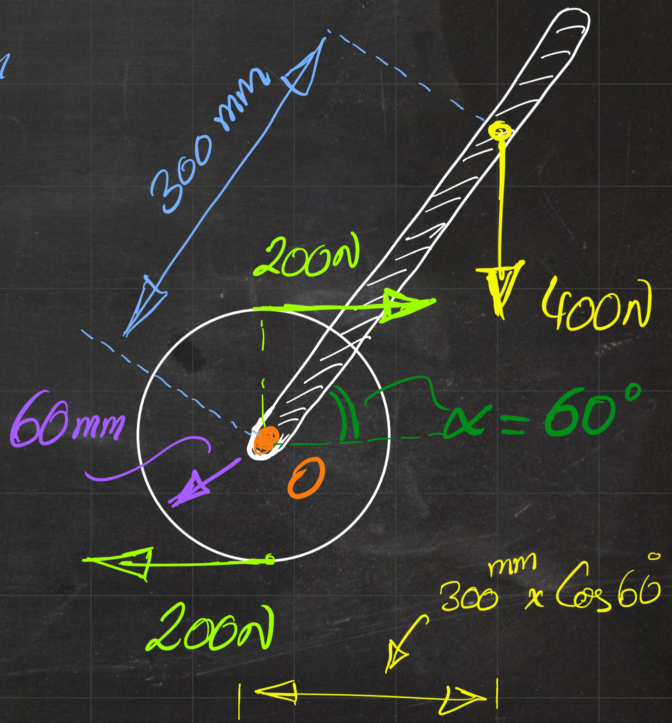
DETERMINE THE DISTANCE FROM THE SHAFT TO THE POINT OF APPLICATION OF THE EQUIVALENT FORCE.



- Step 1: TRANSFER EVERYTHING TO O ↲ Shaft

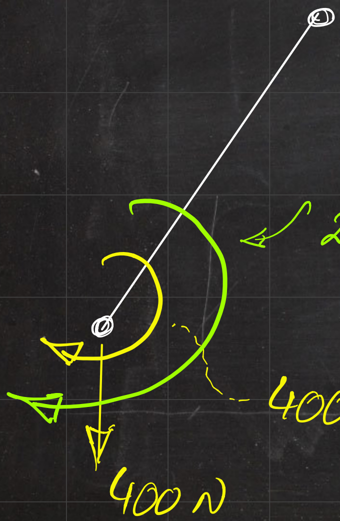
- Step 2: FIND EQUIVALENT SYSTEM

$400 \times 0.3 \times \cos 60^\circ = 60 \text{ N.m}$



- Step 1: TRANSFER EVERYTHING TO O ↶ Shaft

- Step 2: FIND EQUIVALENT SYSTEM

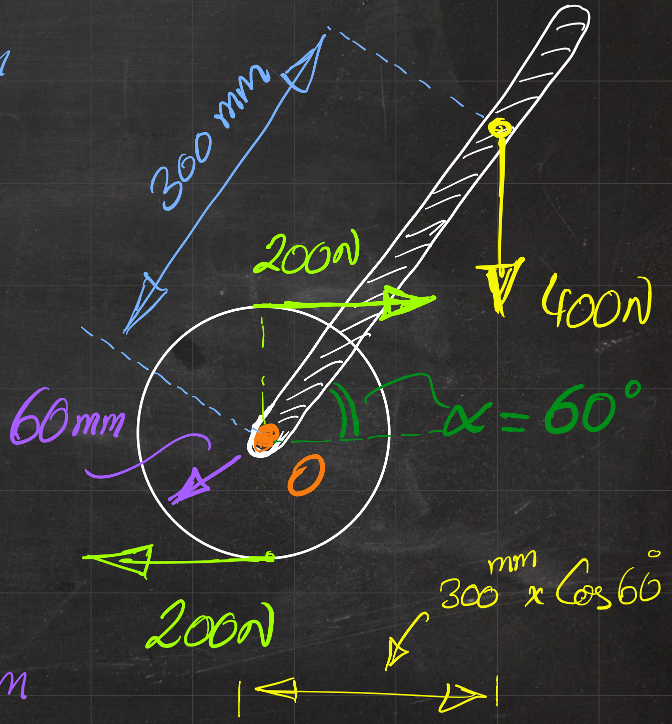


$2 \times 200 \times 0.6 = 24\ \mathrm{N.m}$

$+$

$400 \times 0.3 \times \cos 60^\circ = 60\ \mathrm{N.m}$

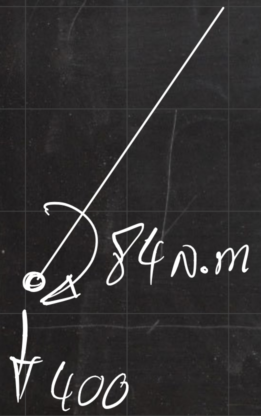
$= 84\ \mathrm{N.m}$



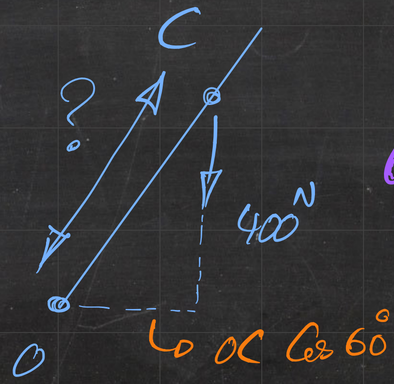
- Step 1: TRANSFER EVERYTHING TO O ↙ Shaft

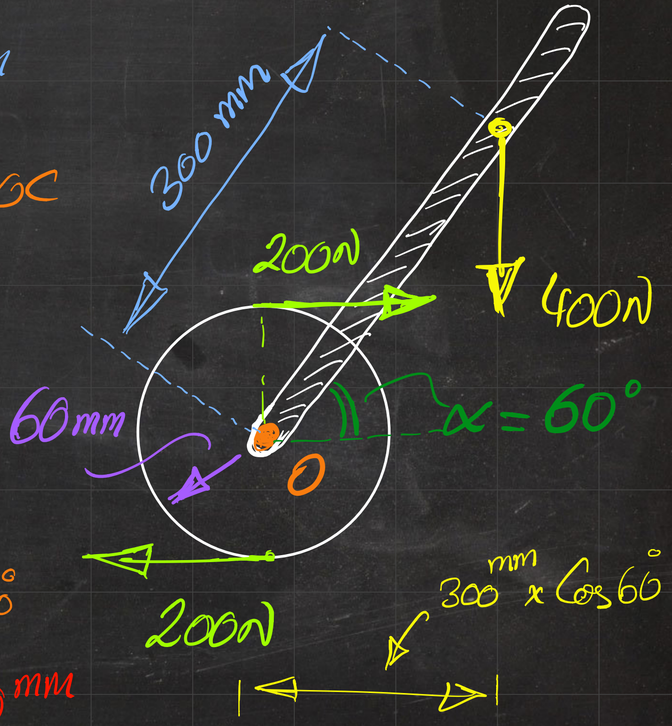
- Step 2: FIND EQUIVALENT SYSTEM

$$84 = 400 \times \tfrac{1}{2} \times OC$$



=

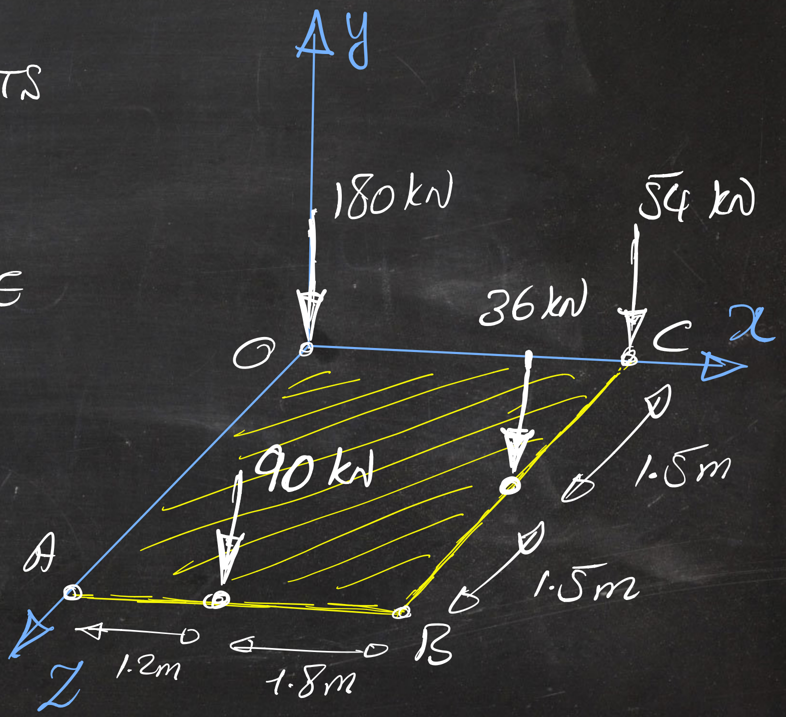


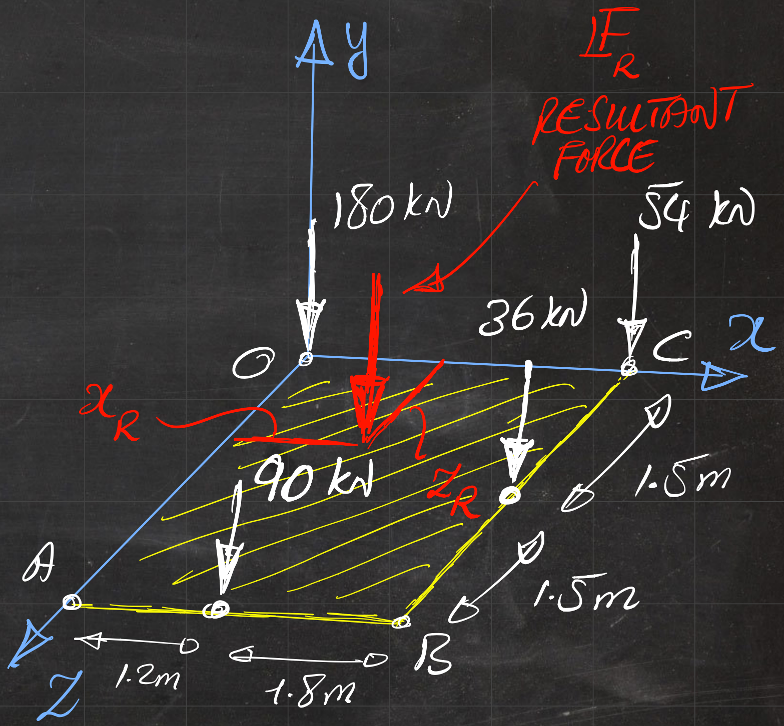


$$\Rightarrow OC = 420 \text{ mm}$$

Exercise 3 . [similar to … P. 133 … 3.11]

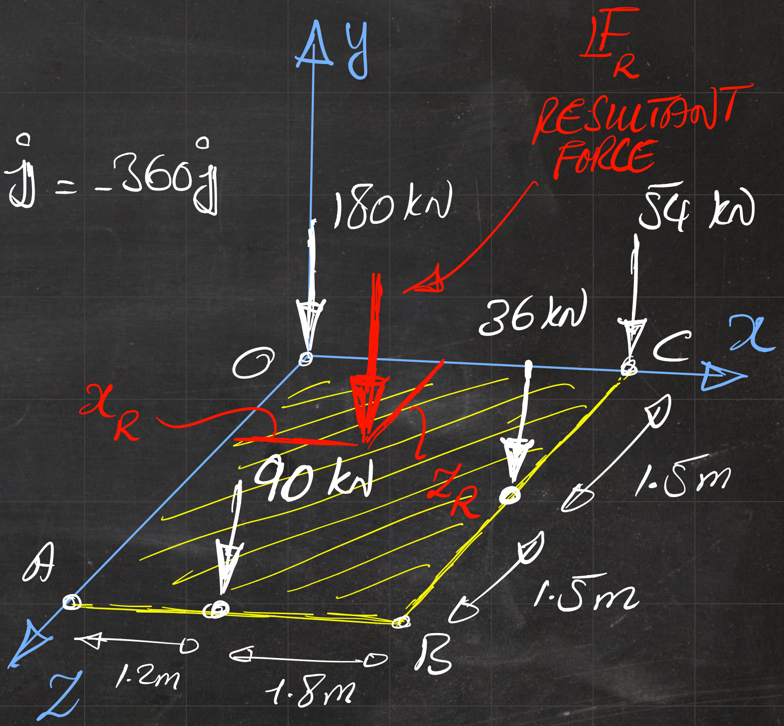
A SQUARE FOUNDATION SUPPORTS THE FOUR FORCES SHOWN. DETERMINE THE MAGNITUDE AND POINT OF APPLICATION OF THE RESULTANT FORCE.





y
F_R
RESULTANT FORCE
180 kN
54 kN
36 kN
C
x
O
x_R
90 kN
z_R
1.5 m
1.5 m
A
B
1.2 m
1.8 m
z

$$\mathbb{F}_R = \sum \mathbb{F} = [-180 - 54 - 36 - 90]\,\mathbf{j} = -360\,\mathbf{j}$$



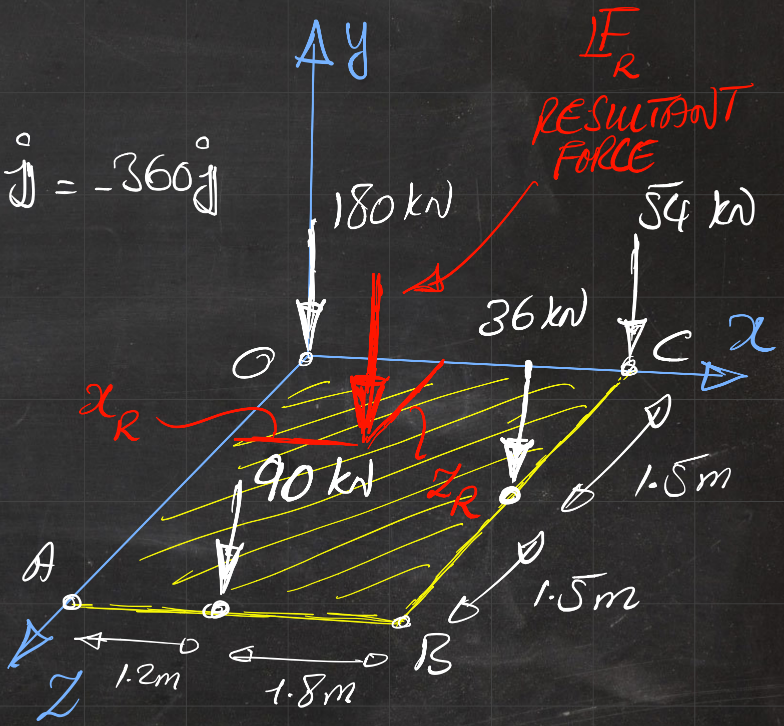
$$\mathbb{F}_R = \sum \mathbb{F} = [-180-54-36-90]\,\mathbf{j} = -360\,\mathbf{j}$$

$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

$$= [0+0+36\times1.5+90\times3]\,\mathbf{i}$$

$$+[0-54\times3-36\times3-90\times1.2]\,\mathbf{k}$$

$$= 324\,\mathbf{i} - 378\,\mathbf{k}$$



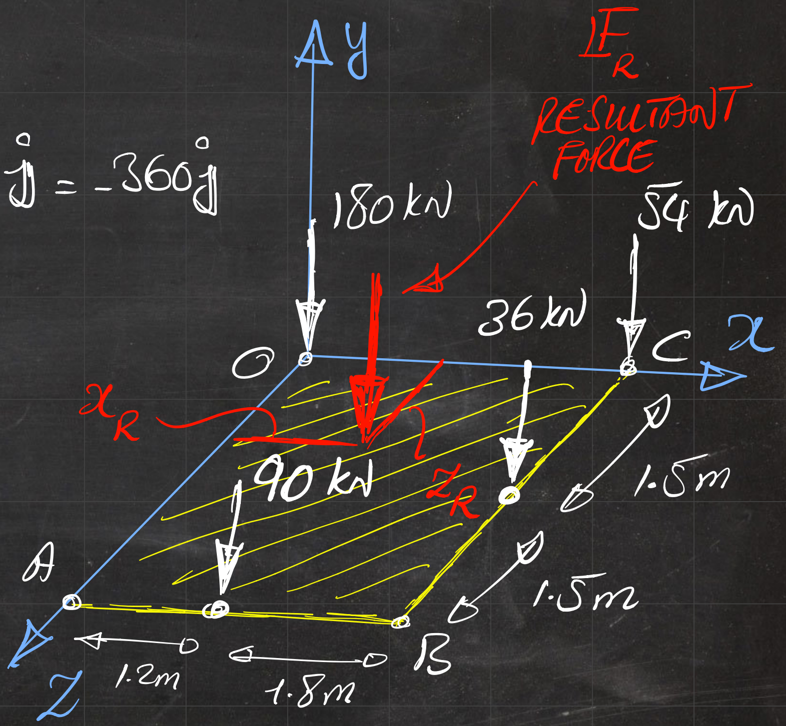
$$\mathbb{F}_R = \sum \mathbb{F} = [-180-54-36-90]\,\mathbf{j} = -360\,\mathbf{j}$$

$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

$$= [0+0+36\times1.5+90\times3]\,\mathbf{i}$$

$$+[0-54\times3-36\times3-90\times1.2]\,\mathbf{k}$$

$$= 324\,\mathbf{i} - 378\,\mathbf{k}$$



$$360\times x_R = 378\,,\quad 360\times z_R = 324$$

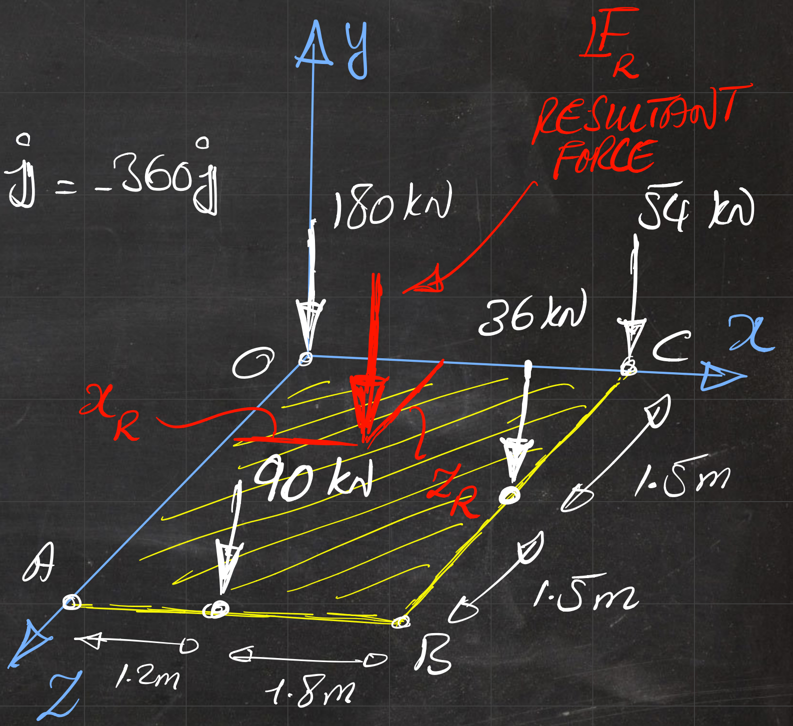
$$\mathbb{F}_R = \sum \mathbb{F} = [-180 - 54 - 36 - 90]\,\mathbf{j} = -360\,\mathbf{j}$$

$$\mathbb{M}_{R_O} = \sum \mathbb{M}_O$$

$$= [0 + 0 + 36 \times 1.5 + 90 \times 3]\,\mathbf{i}$$

$$+ [0 - 54 \times 3 - 36 \times 3 - 90 \times 1.2]\,\mathbf{k}$$

$$= 324\,\mathbf{i} - 378\,\mathbf{k}$$



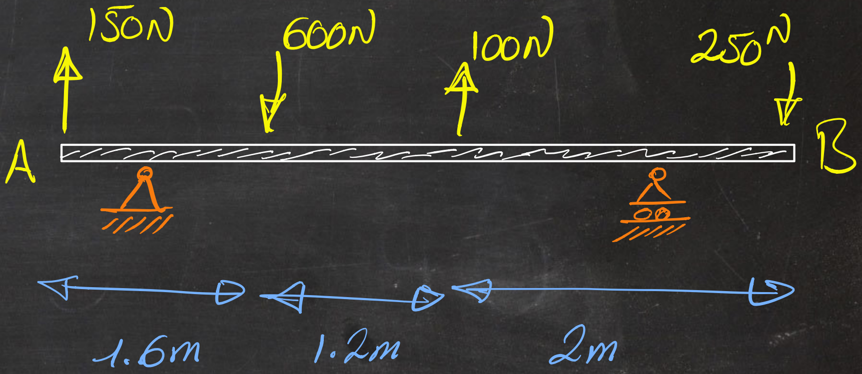
$$360 \times x_R = 378\,,\quad 360 \times z_R = 324$$

$$\Rightarrow x_R = 1.05\,m\,,\quad z_R = 0.9\,m$$

Exercise 4 . [similar to … P. 130 … 3.8]

THE BEAM IS SUBJECTED TO THE FORCES SHOWN.

REDUCE THE GIVEN SYSTEM



a) TO AN EQUIVALENT FORCE-COUPLE SYSTEM AT A.

b) TO AN EQUIVALENT FORCE-COUPLE SYSTEM AT B.

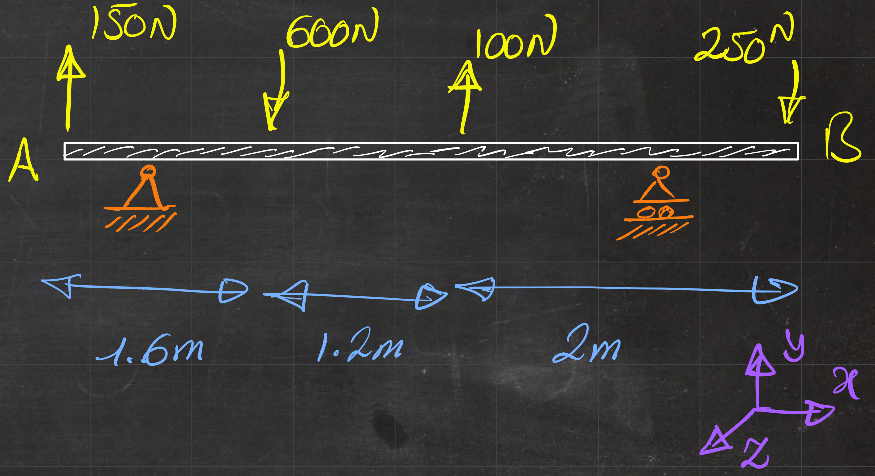
c) TO A SINGLE FORCE.

a)

$$\mathbb{R} = \sum \mathbb{F}$$

$$= [150 - 600 + 100 - 250]\,\mathbf{j}$$

$$= -600^{N}\,\mathbf{j}$$

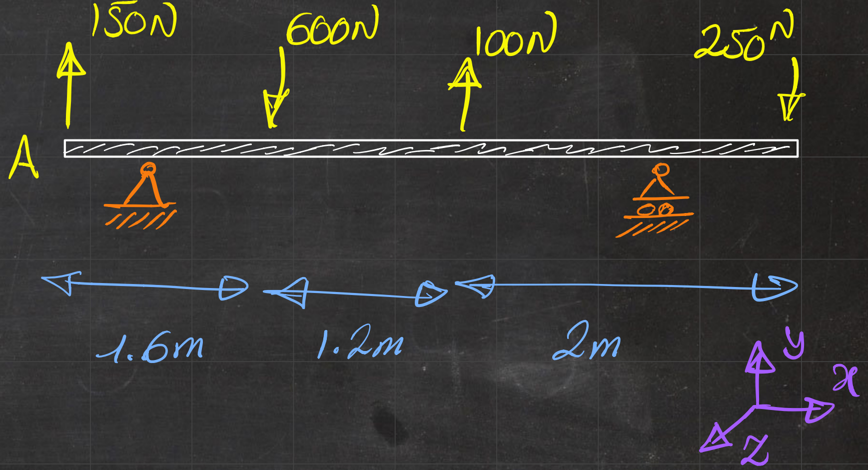


a)

$$\mathbb{R} = \sum \mathbb{F}$$

$$= [150 - 600 + 100 - 250]\,\hat{j}$$

$$= -600^{N}\,\hat{j}$$



$$\mathbb{M}^{\mathbb{R}}_{A} = \sum \mathbb{M}_A = [1.6\,\hat{i}] \times [-600\,\hat{j}]$$

$$+ [2.8\,\hat{i}] \times [100\,\hat{j}]$$

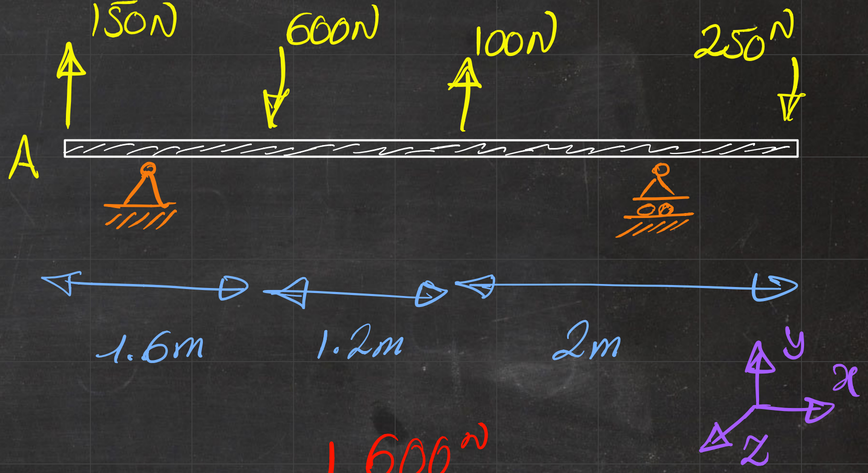
$$+ [4.8\,\hat{i}] \times [-250\,\hat{j}] = -1880^{N.m}\,\mathbb{k}$$

a)

$$\mathbb{R} = \sum \mathbb{F}$$

$$= [150 - 600 + 100 - 250]\,\hat{j}$$

$$= -600^{N}\,\hat{j}$$

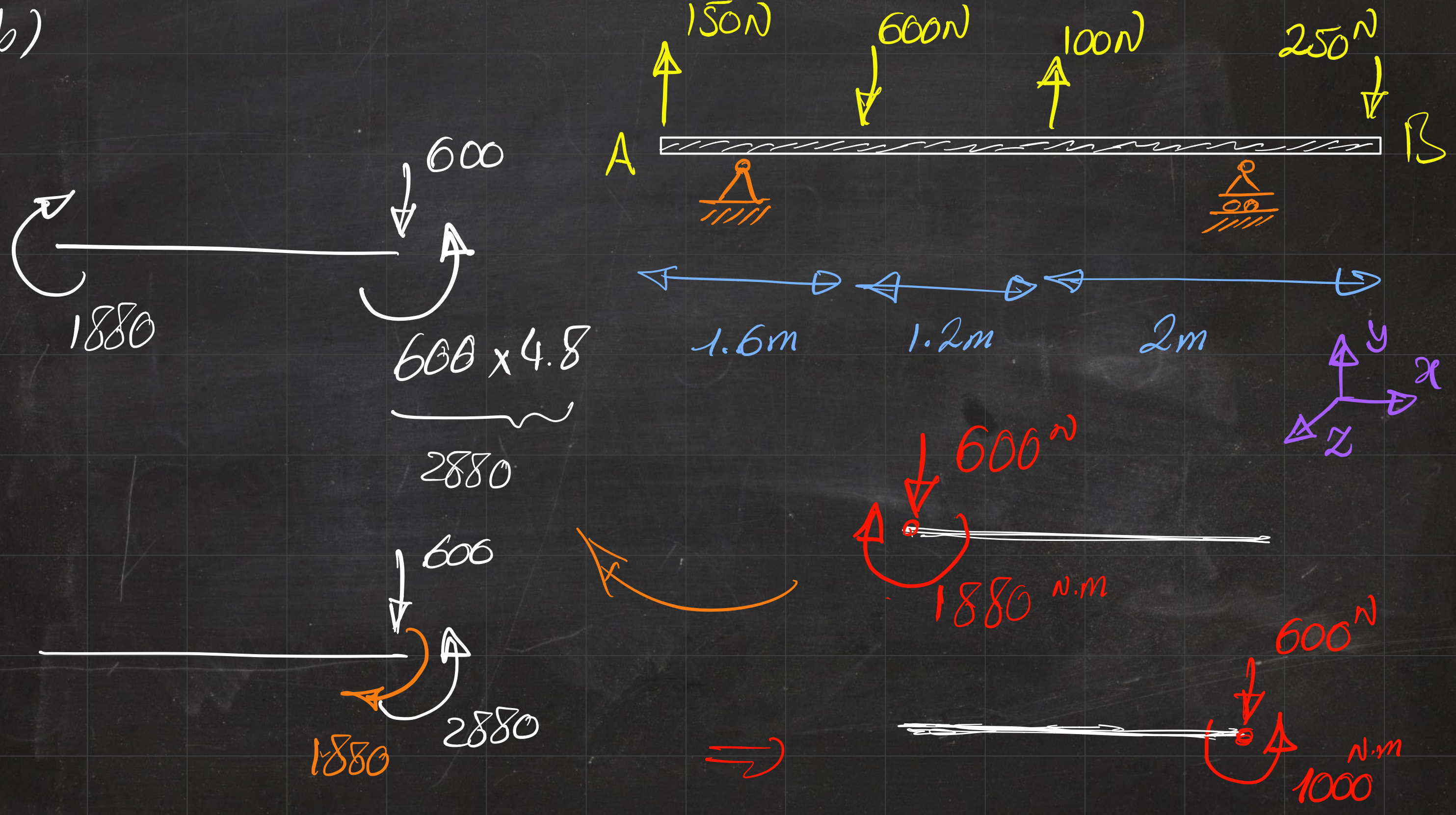


$$\mathbb{M}_A^{\mathbb{R}} = \sum \mathbb{M}_A = [1.6\,\hat{i}] \times [-600\,\hat{j}]$$

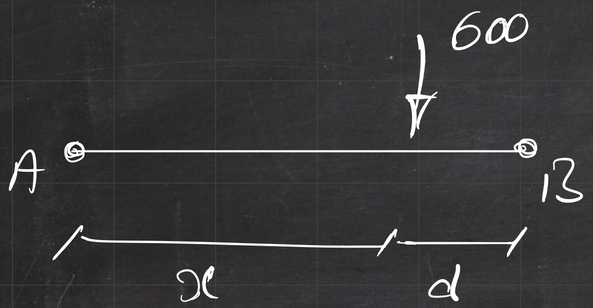
$$+ [2.8\,\hat{i}] \times [100\,\hat{j}]$$

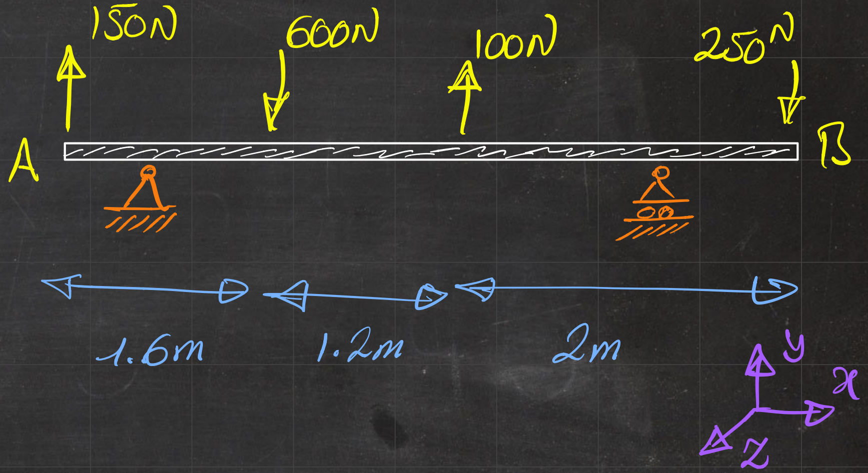
$$+ [4.8\,\hat{i}] \times [-250\,\hat{j}] = -1880^{N.m}\,\mathbb{K}$$

b)

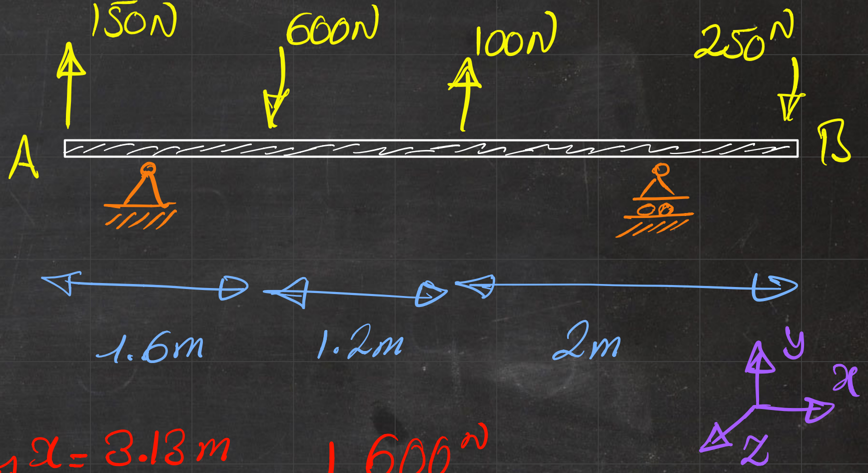

150N
600N
100N
250N
A
B
1.6m
1.2m
2m
y
x
z
600
1880
600 x 4.8
2880
600
2880
1880
600N
1880 N.m
600N
N.m
1000

c)





c)



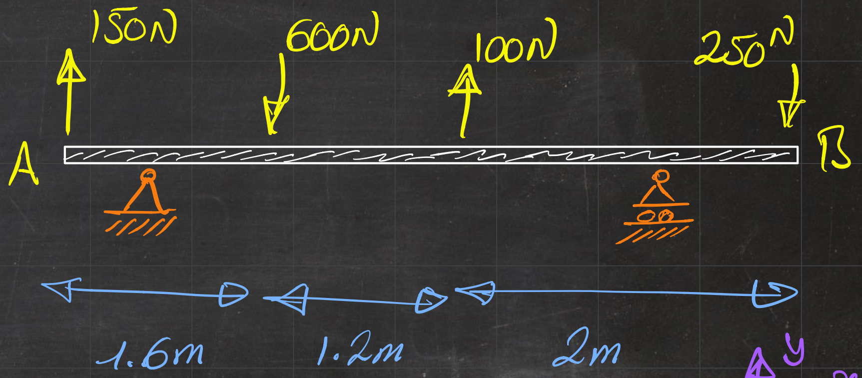
w.r.t. A $\Rightarrow$ $600x = 1880$

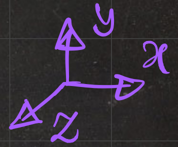
$x = 3.13\ m$

$\Downarrow$

$x + d = 4.8\ m$ ✓

c)





w.r.t. A $\Rightarrow$ $600\,x = 1880$ $\Rightarrow$ $x = 3.13\text{ m}$

w.r.t. B $\Rightarrow$ $600\,d = 1000$ $\Rightarrow$ $d = 1.67\text{ m}$

$x + d = 4.8\text{ m}$ ✓

ALTERNATIVELY

MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

Introduction to Statics ii

Rigid Bodies: Equivalent Systems of Forces

Chap. 3

[Beer Johnston et al. 9th edition]

MECHANICS AND MATERIALS I

MECHANICS AND MATERIALS I

2

MECHANICS AND MATERIALS I

2

Appendix I

MOMENT OF A FORCE ABOUT A GIVEN AXIS

$\mathbb{M}_O = \mathbb{r} \times \mathbb{F}$

VECTOR

$\overrightarrow{OA}$

$M_{OL} = ?$ PROJECTION OF $\mathbb{M}_O$ ONTO THE AXIS $\overrightarrow{OL}$

SCALAR

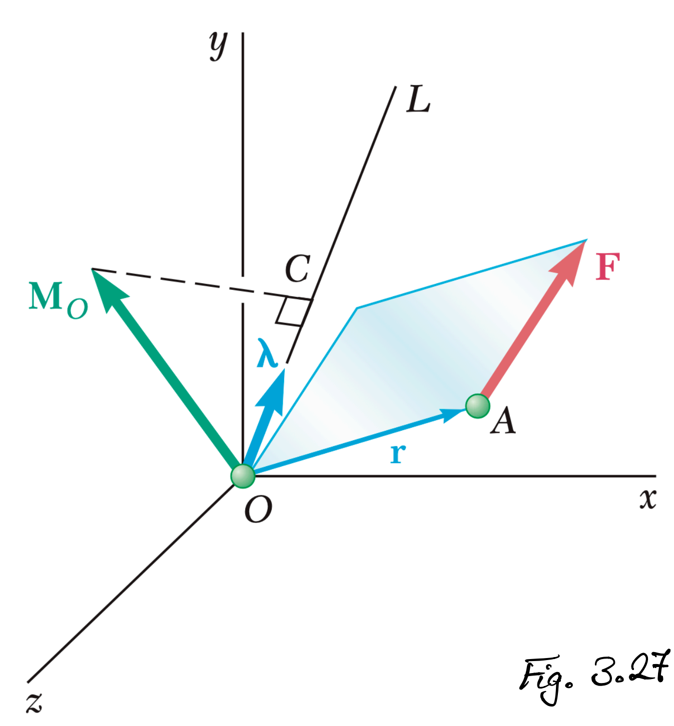
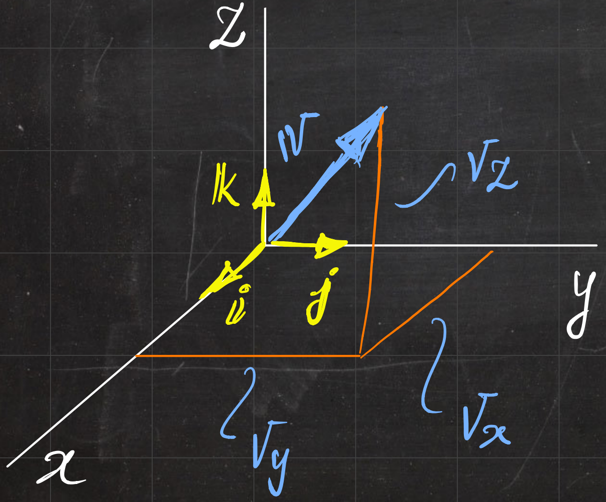


Fig. 3.27

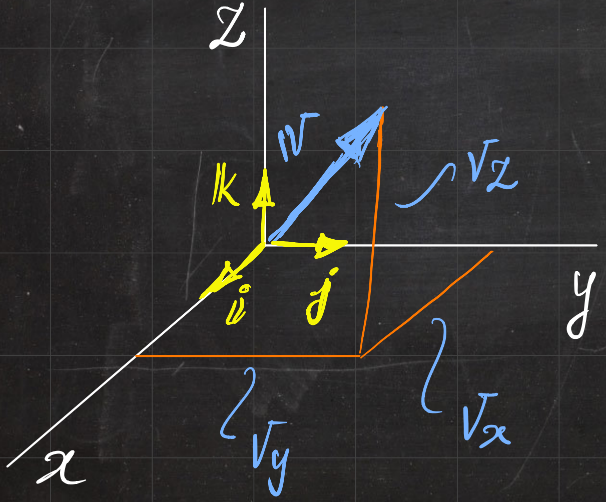
Components of a vector

$$\mathbb{V} = V_x\,\mathbb{i} + V_y\,\mathbb{j} + V_z\,\mathbb{k}$$



Components of a vector

$$\mathbb{V} = V_x\, \mathbb{i} + V_y\, \mathbb{j} + V_z\, \mathbb{k}$$



$$\Downarrow$$

$$V_x = \mathbb{V} \cdot \mathbb{i}$$

$$V_y = \mathbb{V} \cdot \mathbb{j}$$

$$V_z = \mathbb{V} \cdot \mathbb{k}$$

Components of a vector

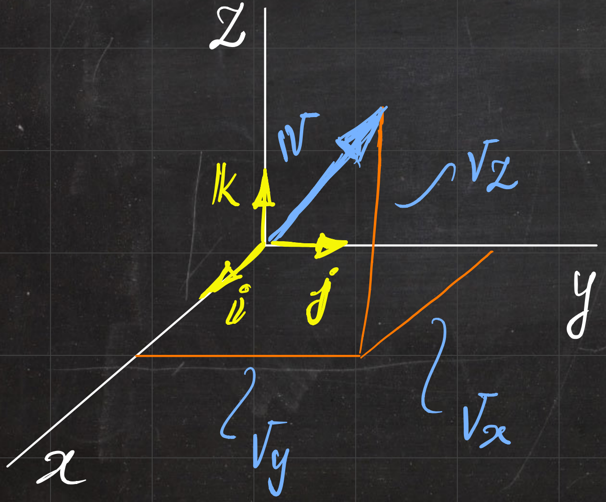
$$\mathbb{V} = V_x \, \mathbb{i} + V_y \, \mathbb{j} + V_z \, \mathbb{k}$$

$$\Downarrow$$

$$V_x = \mathbb{V} \cdot \mathbb{i}$$

$$V_y = \mathbb{V} \cdot \mathbb{j}$$

$$V_z = \mathbb{V} \cdot \mathbb{k}$$



x-Component is obtained via projection on x-axis!

Components of a vector

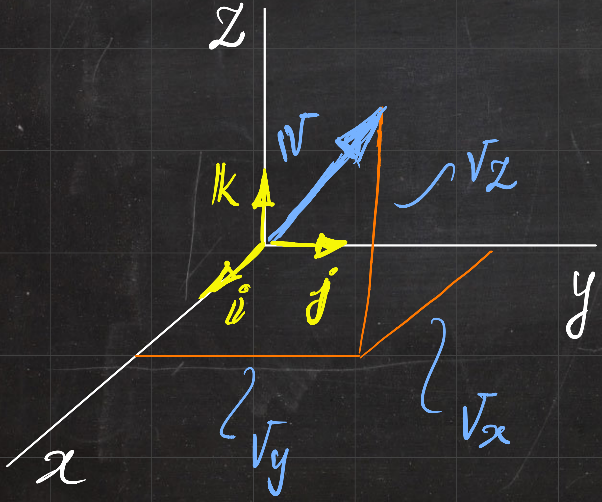
$$\mathbf{V} = V_x\,\mathbf{i} + V_y\,\mathbf{j} + V_z\,\mathbf{k}$$

$$\Downarrow$$

$$V_x = \mathbf{V}\cdot\mathbf{i}$$

$$V_y = \mathbf{V}\cdot\mathbf{j}$$

$$V_z = \mathbf{V}\cdot\mathbf{k}$$



x-Component is obtained via projection on x-axis!

dot product with the unit vector of x-direction

Components of a vector

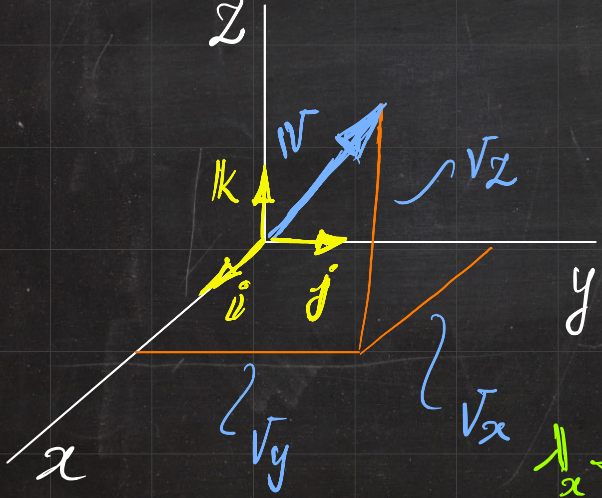
$$\mathbb{V} = V_x\,\mathbb{i} + V_y\,\mathbb{j} + V_z\,\mathbb{k}$$

$$\Downarrow$$

$$V_x = \mathbb{V} \cdot \mathbb{i}$$

$$V_y = \mathbb{V} \cdot \mathbb{j}$$

$$V_z = \mathbb{V} \cdot \mathbb{k}$$



$\lambda_x \leftarrow \mathbb{i} \Leftarrow$ { Director of x-direction

x-Component is obtained via projection on x-axis!

dot product with the unit vector of x-direction

Components of a vector

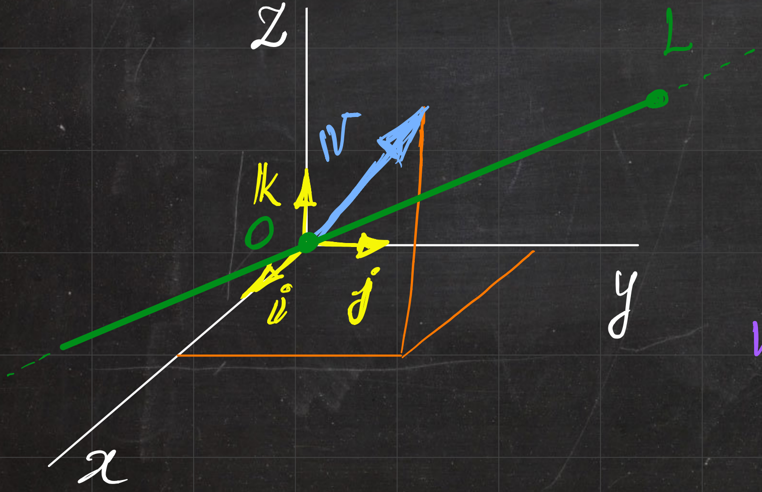
$$\mathbb{V} = V_x\, \mathbb{i} + V_y\, \mathbb{j} + V_z\, \mathbb{k}$$

$\Downarrow$

$$V_x = \mathbb{V} \cdot \mathbb{i}$$

$$V_y = \mathbb{V} \cdot \mathbb{j}$$

$$V_z = \mathbb{V} \cdot \mathbb{k}$$



What is the component of $\mathbb{V}$ along OL-line?

Components of a vector

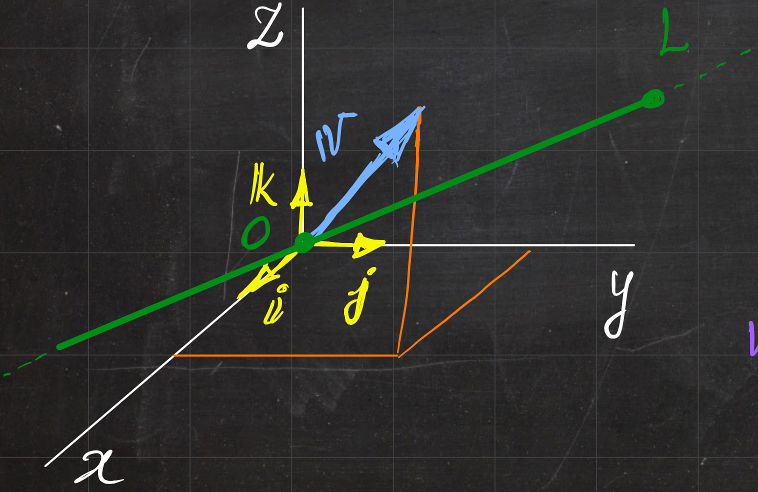
$$\mathbb{V} = V_x\,\mathbb{i} + V_y\,\mathbb{j} + V_z\,\mathbb{k}$$

$$\Downarrow$$

$$V_x = \mathbb{V}\cdot\mathbb{i}$$

$$V_y = \mathbb{V}\cdot\mathbb{j}$$

$$V_z = \mathbb{V}\cdot\mathbb{k}$$

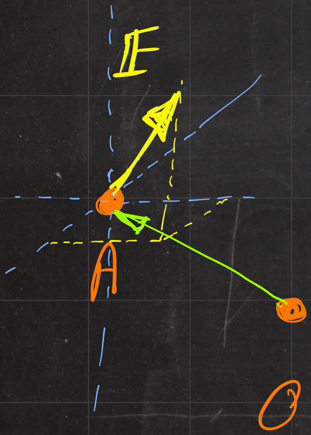


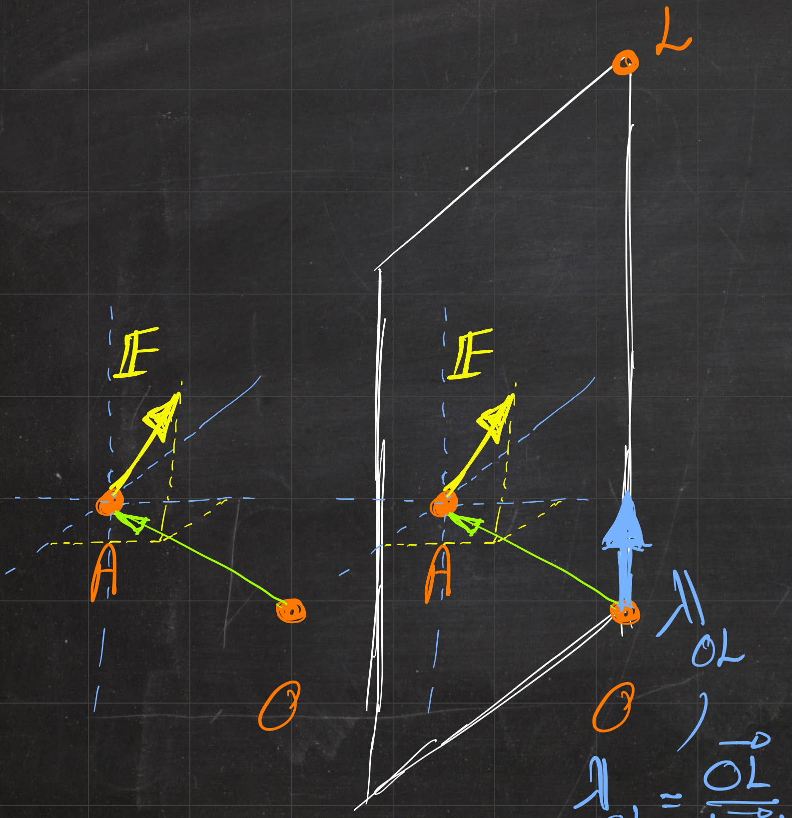
What is the component of $\mathbb{V}$ along OL-line?

$$V_{OL} = \mathbb{V}\cdot\lambda_{OL} = \lambda_{OL}\cdot\mathbb{V}$$

remember

$$\mathbb{M}_O = \overrightarrow{OA} \times \mathbb{F}$$

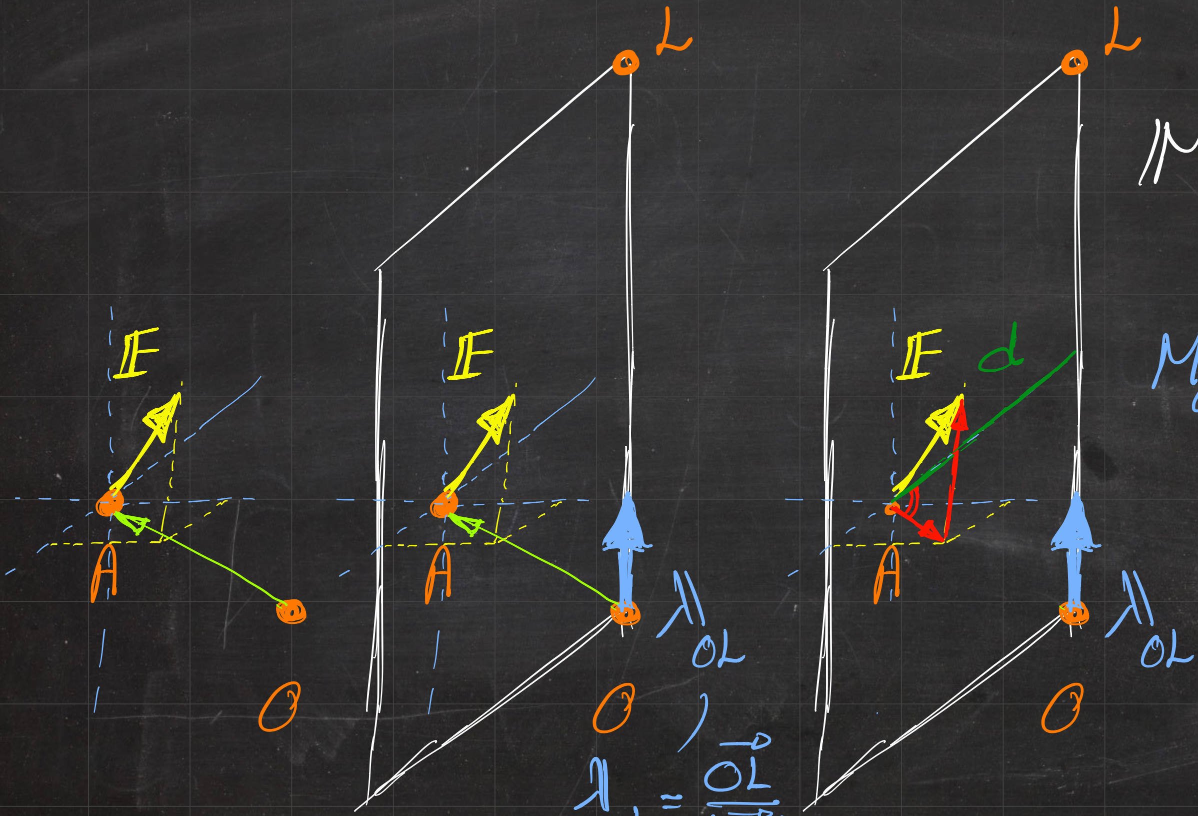




$$\lambda_{OL} = \frac{\vec{OL}}{|\vec{OL}|}$$

$$\mathbb{M}_O = \vec{OA} \times \mathbb{F}$$

$$M_{OL} = \lambda_{OL} \cdot \mathbb{M}_O$$



$$\mathbb{M}_O = \vec{OA} \times \mathbb{F}$$

$$M_{OL} = \lambda_{OL} \cdot \mathbb{M}_O$$

$$\lambda_{OL} = \frac{\vec{OL}}{|\vec{OL}|}$$

MOMENT OF A FORCE ABOUT A GIVEN AXIS

$\mathbb{M}_O = \mathbb{r} \times \mathbb{F}$

VECTOR

$\overrightarrow{OA}$

$M_{OL} = \,?$ → PROJECTION OF $\mathbb{M}_O$ ONTO THE AXIS $\overrightarrow{OL}$

SCALAR

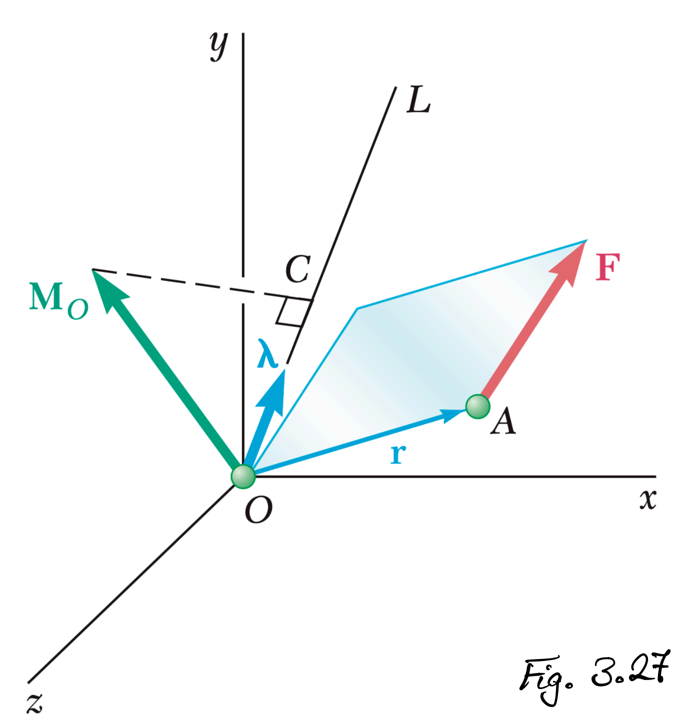


Fig. 3.27

MOMENT OF A FORCE ABOUT A GIVEN AXIS

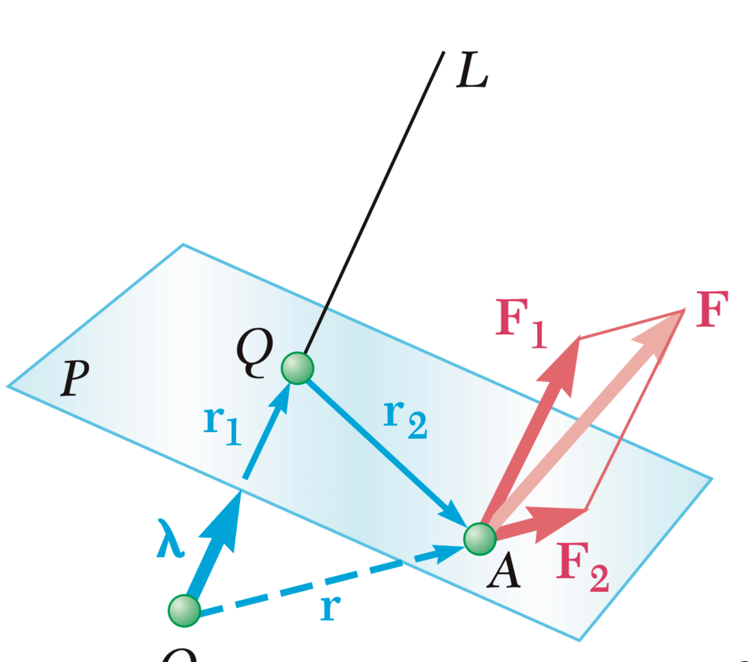


Fig. 3.28

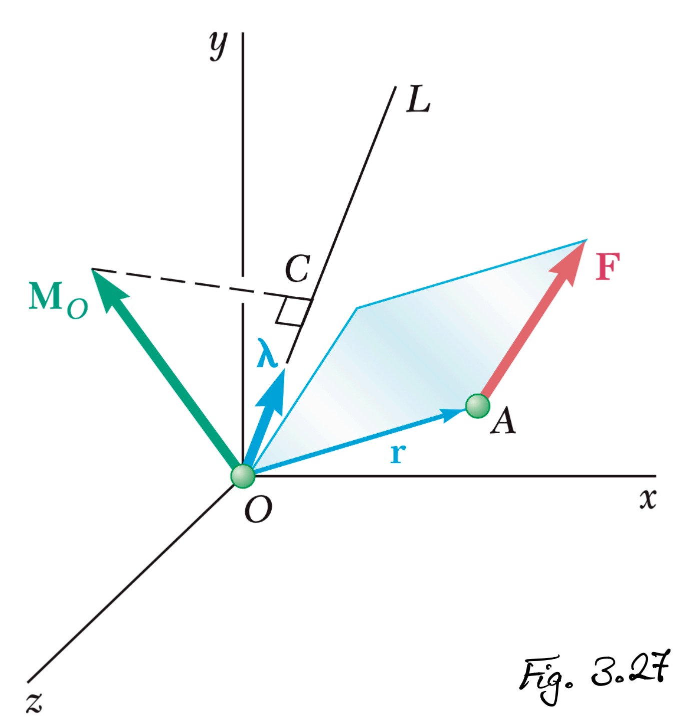


Fig. 3.27

MOMENT OF A FORCE ABOUT A GIVEN AXIS

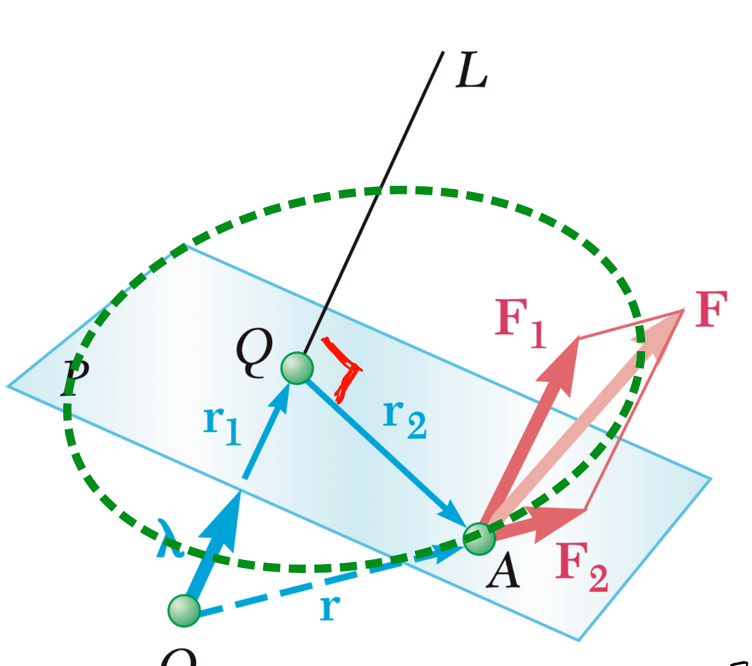


Fig. 3.28

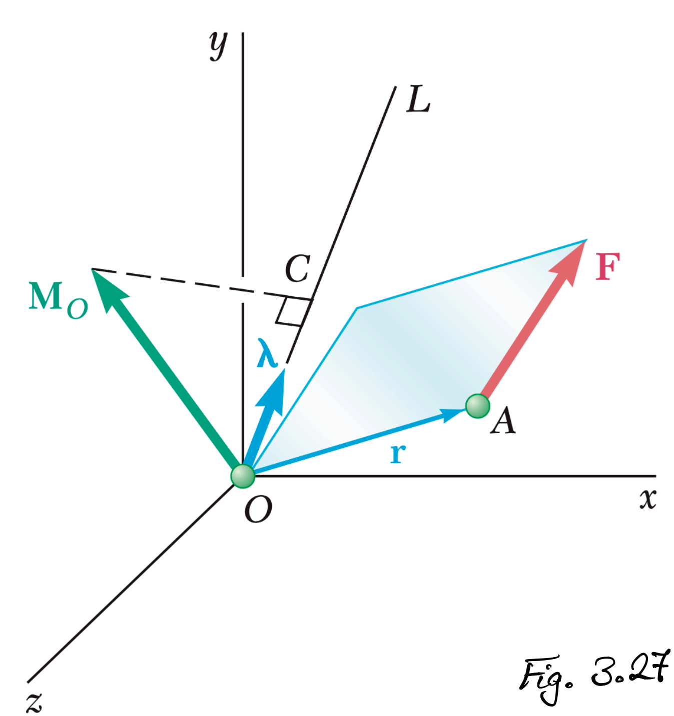


Fig. 3.27

MOMENT OF A FORCE ABOUT A GIVEN AXIS

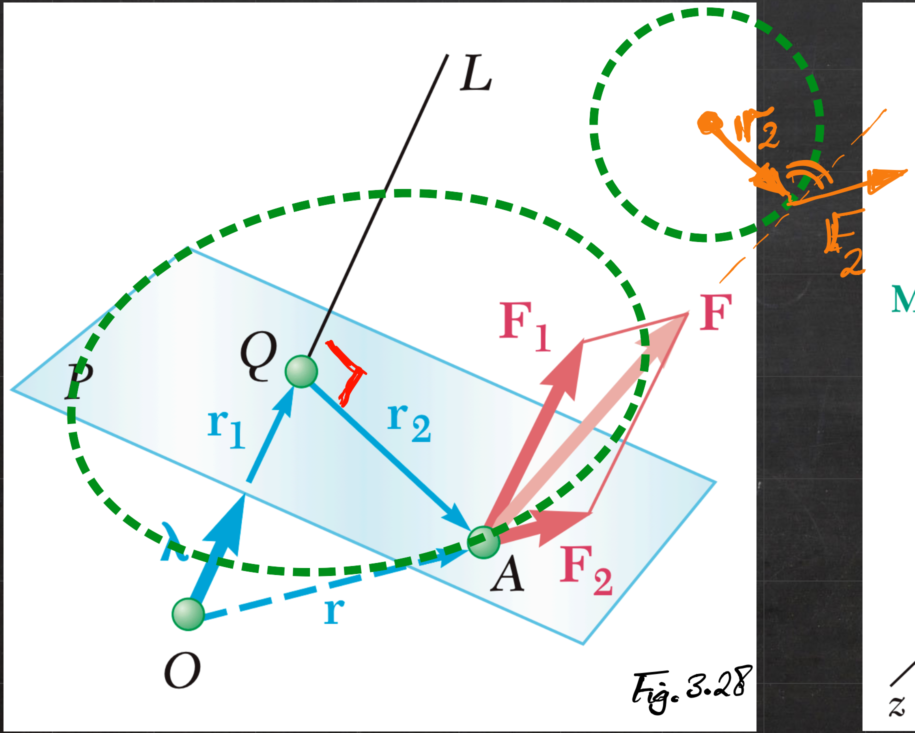


Fig. 3.28

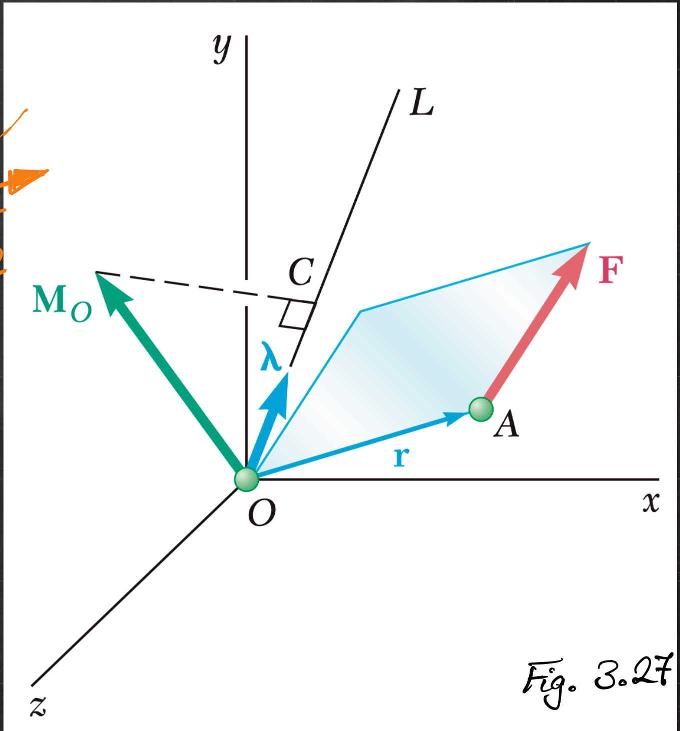


Fig. 3.27

MOMENT OF A FORCE ABOUT A GIVEN AXIS

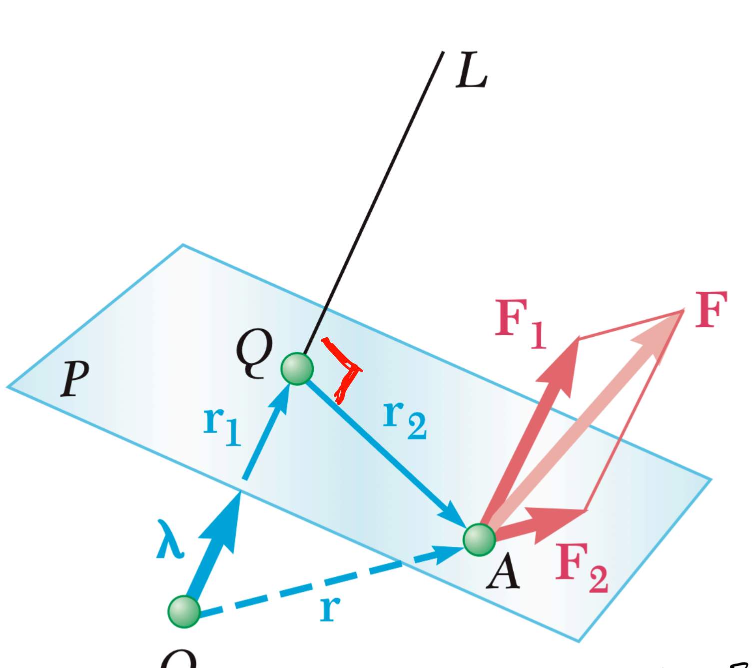


Fig. 3.28

$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O$$

$$= \boldsymbol{\lambda} \cdot [\mathbf{r} \times \mathbf{F}]$$

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

MOMENT OF A FORCE ABOUT A GIVEN AXIS

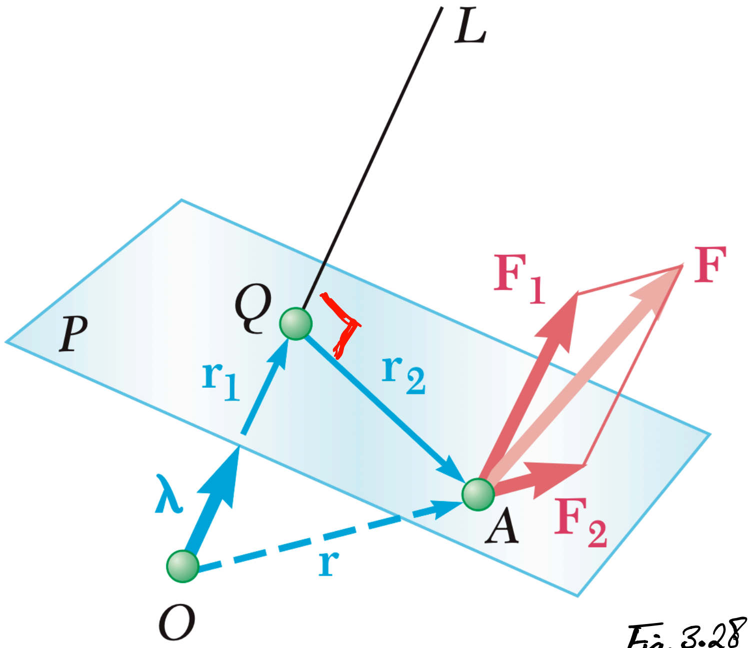


Fig. 3.28

$$M_{OL} = \lambda \cdot M_O$$
$$= \lambda \cdot [\, r \times F \,]$$

$$\Downarrow$$

$$M_{OL} = \lambda \cdot \big[[r_1 + r_2] \times [F_1 + F_2] \big]$$
$$= \lambda \cdot [r_1 \times F_1] + \lambda \cdot [r_1 \times F_2]$$
$$+ \lambda \cdot [r_2 \times F_1] + \lambda \cdot [r_2 \times F_2]$$

MOMENT OF A FORCE ABOUT A GIVEN AXIS

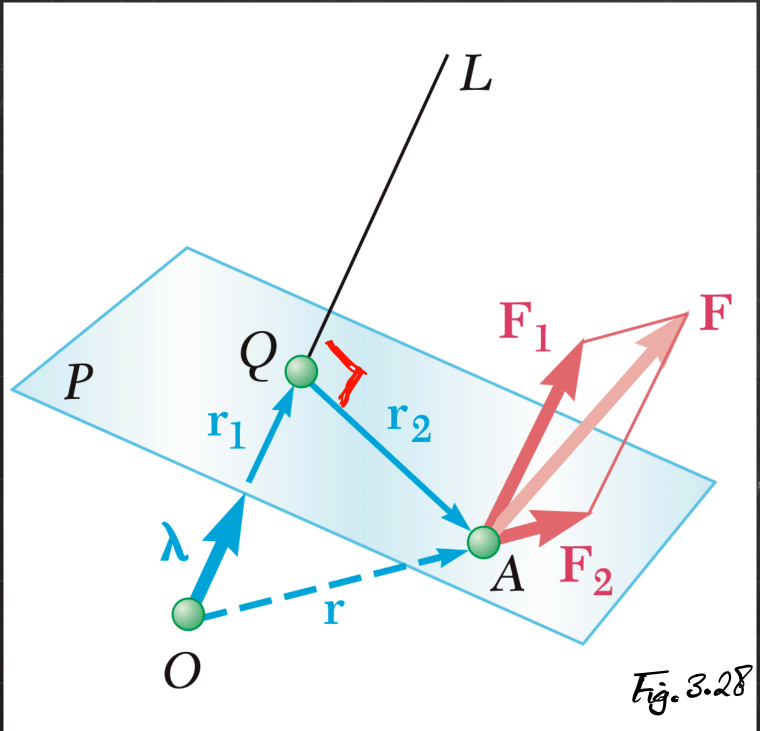


Fig. 3.28

$$M_{OL} = \lambda \cdot M_O$$
$$= \lambda \cdot [r \times F]$$
$$\Downarrow$$
$$M_{OL} = \lambda \cdot \big[[r_1 + r_2] \times [F_1 + F_2] \big]$$
$$= \underbrace{\lambda \cdot [r_1 \times F_1]}_{0} + \underbrace{\lambda \cdot [r_1 \times F_2]}_{0}$$
$$+ \underbrace{\lambda \cdot [r_2 \times F_1]}_{0} + \lambda \cdot [r_2 \times F_2]$$

MOMENT OF A FORCE ABOUT A GIVEN AXIS

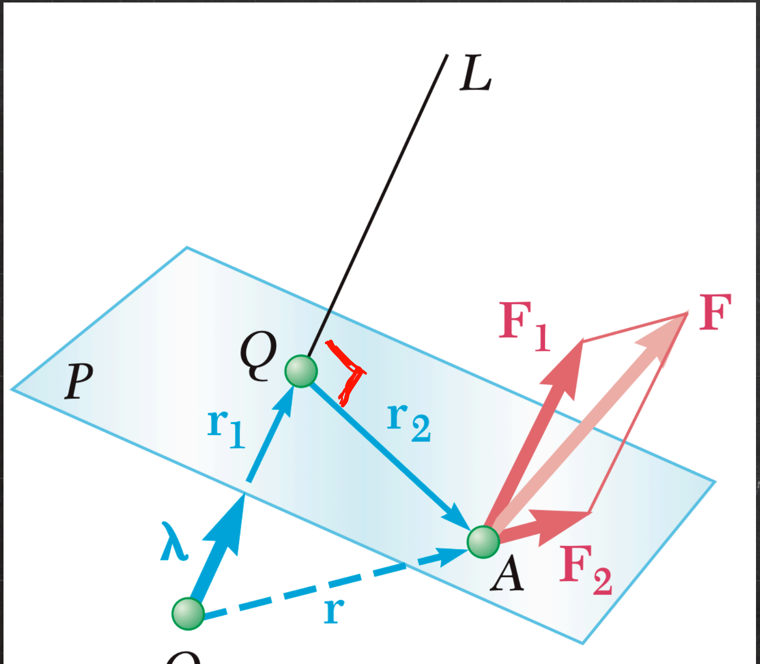


Fig. 3.28

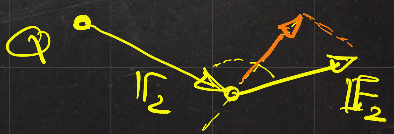
$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O$$

$$= \boldsymbol{\lambda} \cdot [\mathbf{r} \times \mathbf{F}]$$

$$\Downarrow$$

$$M_{OL} = \boldsymbol{\lambda} \cdot [\mathbf{r}_2 \times \mathbf{F}_2]$$

$$\Rightarrow M_{OL} = r_2 \, F_2^{\perp}$$



MOMENT OF A FORCE ABOUT A GIVEN AXIS

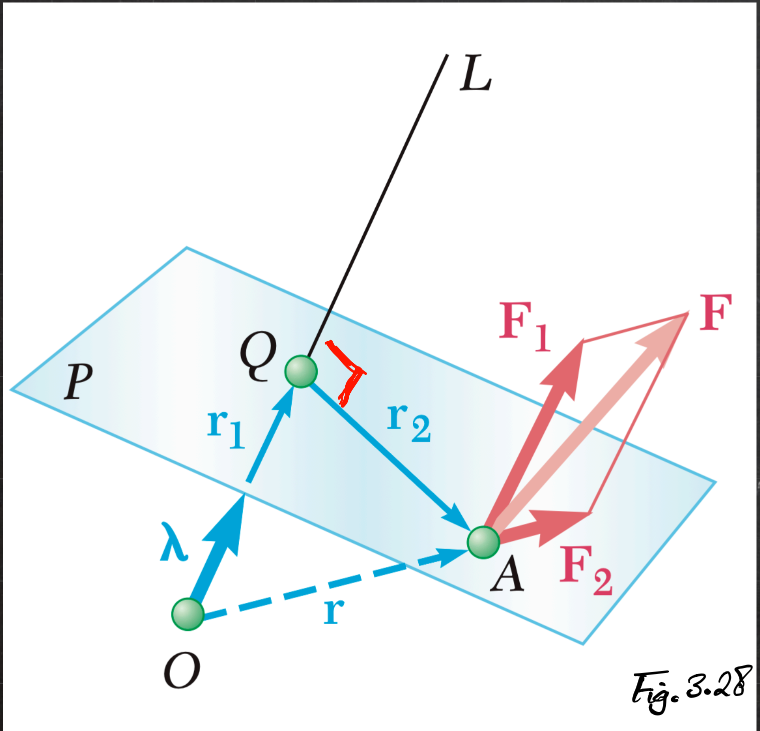


Fig. 3.28

$$M_{OL} = \lambda \cdot M_O$$

$$= \lambda \cdot [r \times F]$$

SCALAR

PARALLEL TO λ

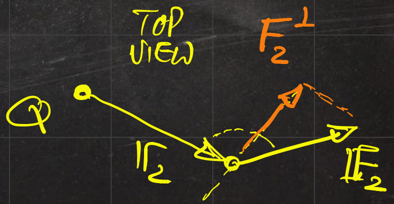
$$M_{OL} = \lambda \cdot [r_2 \times F_2]$$

−

INDICATES DIRECTION

+

$$\Rightarrow M_{OL} = r_2 \, F_2^{\perp}$$



MOMENT OF A FORCE ABOUT A GIVEN AXIS

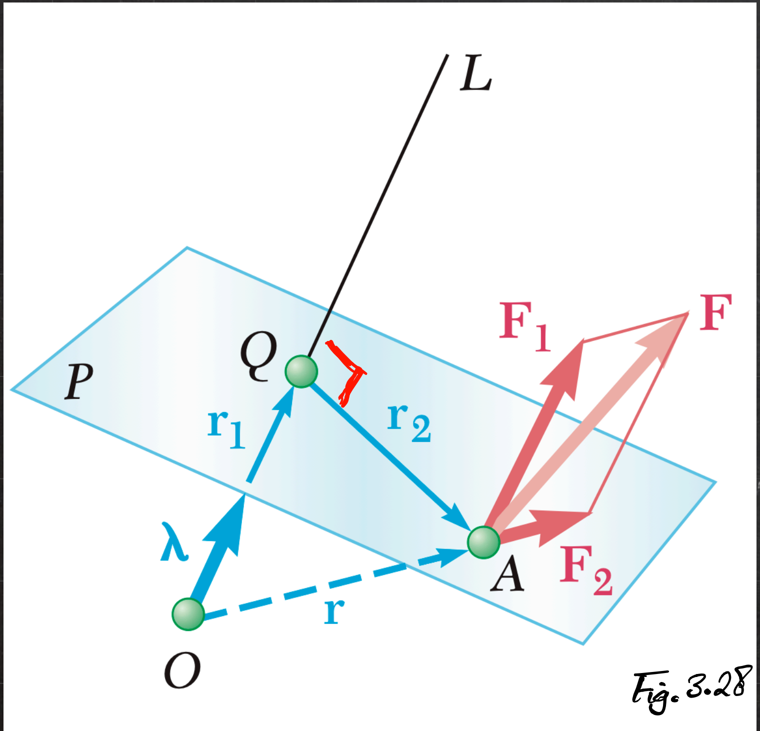


Fig. 3.28

$$M_{OL} = \lambda \cdot M_O$$
$$= \lambda \cdot [\mathbf{r} \times \mathbf{F}]$$

SCALAR

PARALLEL TO λ

$$M_{OL} = \lambda \cdot [\mathbf{r}_2 \times \mathbf{F}_2]$$

−

INDICATES DIRECTION

+

$$\Rightarrow |M_{OL}| = |\mathbf{r}_2 \times \mathbf{F}_2|$$

MECHANICS AND MATERIALS I

2

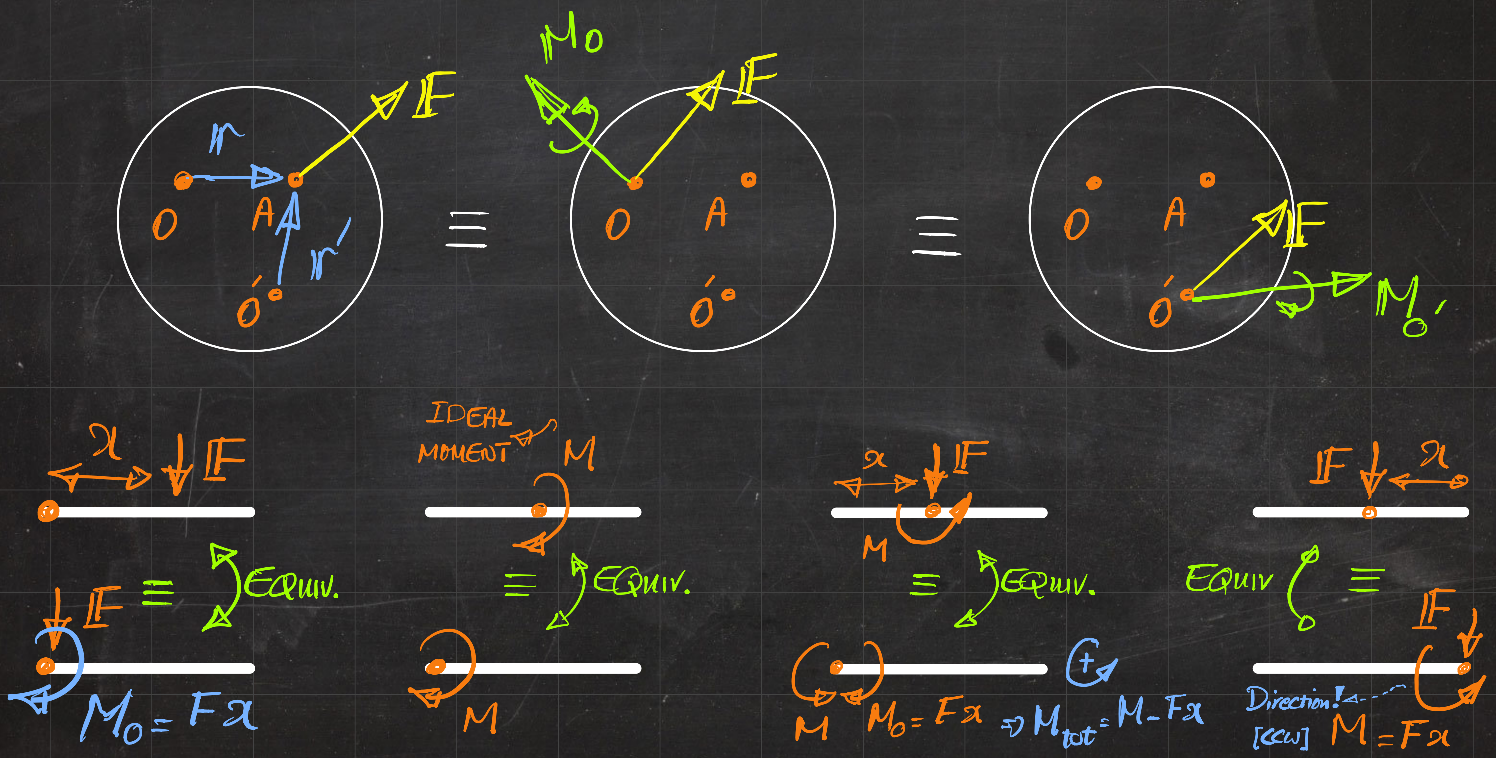
Appendix I

MECHANICS AND MATERIALS I

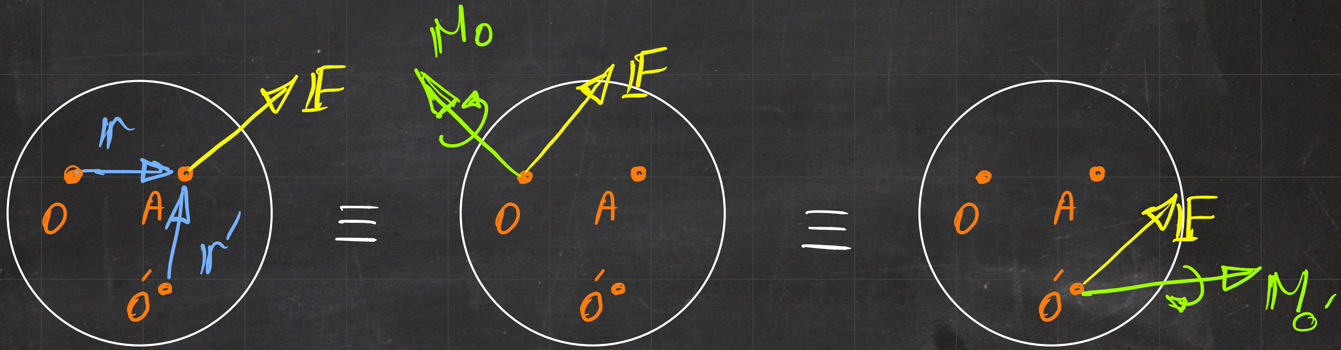
2

Appendix II

STATICALLY EQUIVALENT SYSTEM OF FORCES

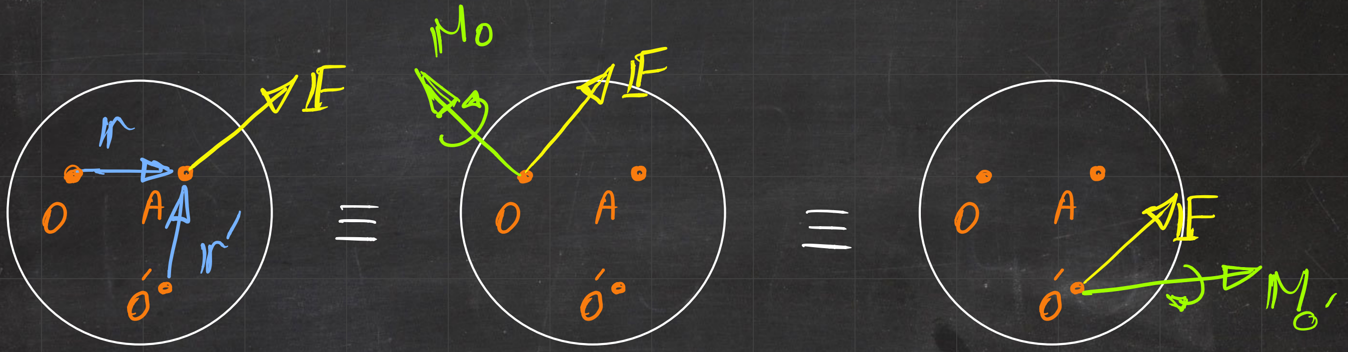


STATICALLY EQUIVALENT SYSTEM OF FORCES



$$\sum \mathbb{M}_A = \emptyset$$

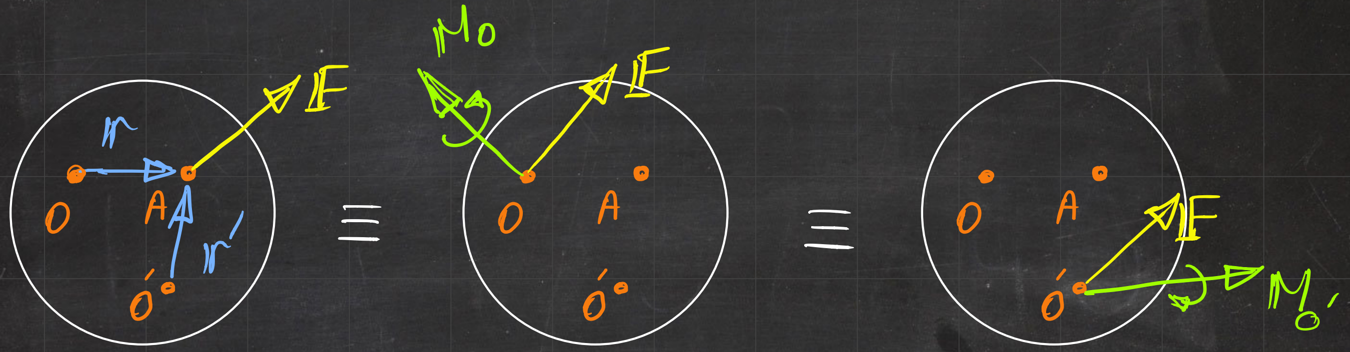
STATICALLY EQUIVALENT SYSTEM OF FORCES



$$\sum \mathbb{M}_A = \emptyset$$

$$\sum \mathbb{M}_A = \vec{AO} \times \mathbb{F} + \mathbb{M}_O$$

STATICALLY EQUIVALENT SYSTEM OF FORCES



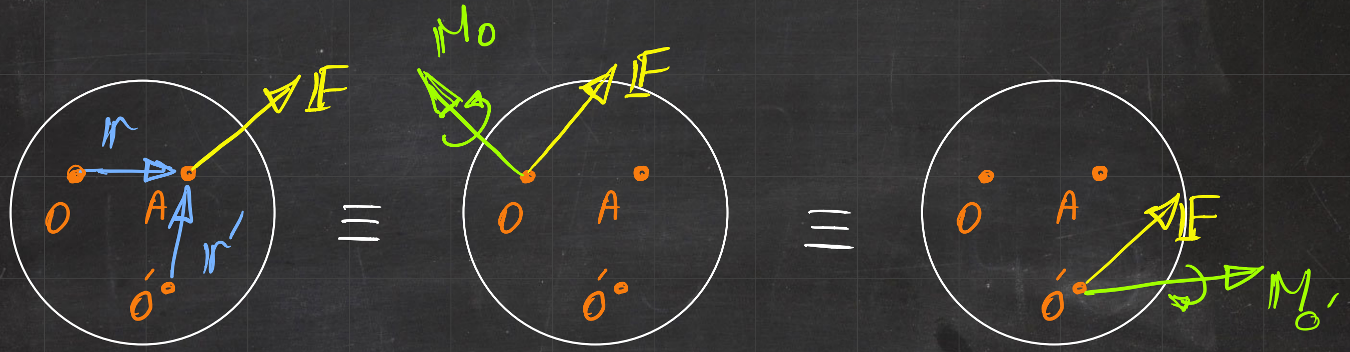
$$\sum M_A = \emptyset$$

$$\sum M_A = \vec{AO} \times F + M_O$$

$$= \emptyset$$

EQUIVALENT

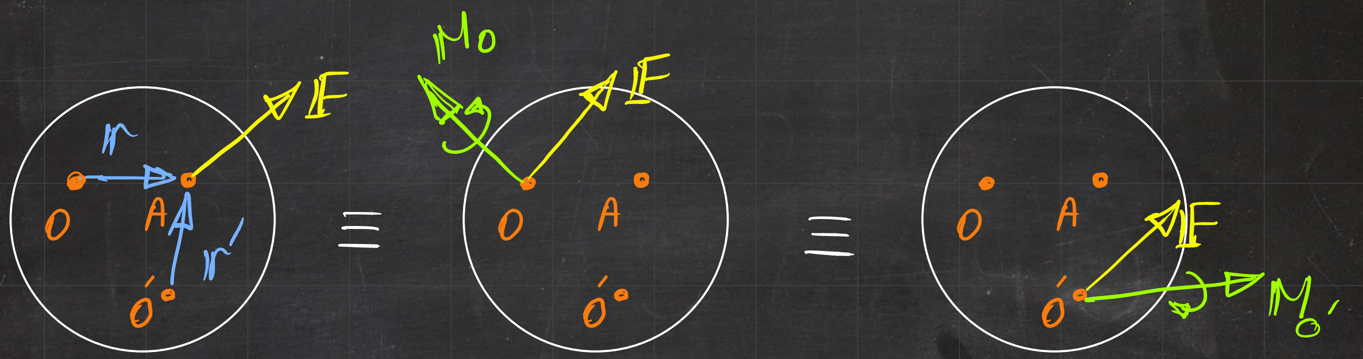
STATICALLY EQUIVALENT SYSTEM OF FORCES



$$\sum M_A = 0 \qquad \sum M_A = \vec{AO} \times F + M_O$$

EQUIVALENT $\qquad = 0 \qquad M_O = -\vec{AO} \times F$

STATICALLY EQUIVALENT SYSTEM OF FORCES



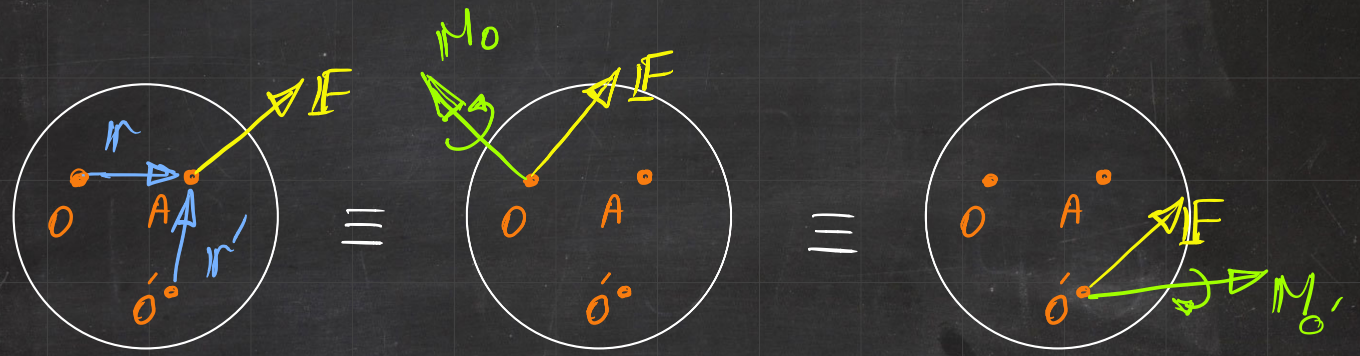
$$\sum \mathbb{M}_A = \emptyset \qquad \sum \mathbb{M}_A = \vec{AO} \times \mathbb{F} + \mathbb{M}_O \qquad \sum \mathbb{M}_A = \vec{AO'} \times \mathbb{F} + \mathbb{M}_{O'}$$

EQUIVALENT

$$= \emptyset$$

$$\mathbb{M}_O = -\vec{AO} \times \mathbb{F}$$

STATICALLY EQUIVALENT SYSTEM OF FORCES

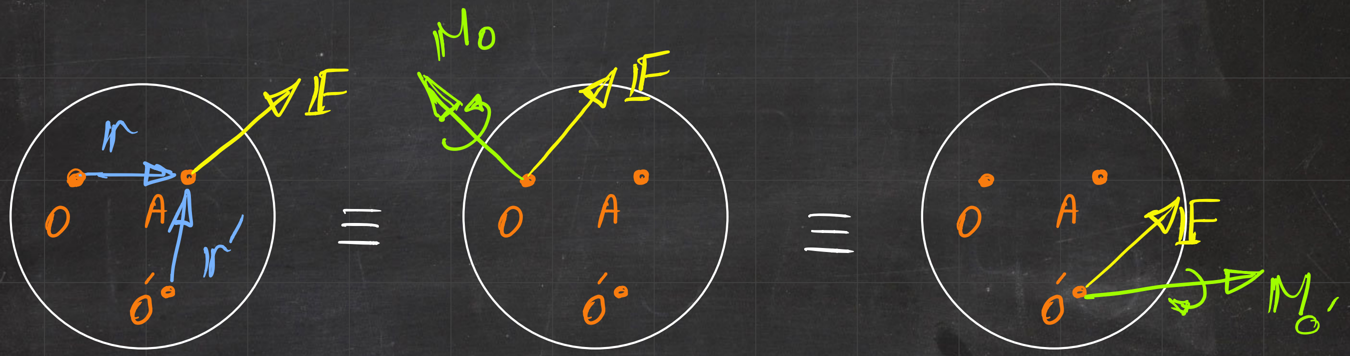


$$\sum \mathbb{M}_A = \emptyset \qquad \sum \mathbb{M}_A = \vec{AO} \times \mathbb{F} + \mathbb{M}_O \qquad \sum \mathbb{M}_A = \vec{AO'} \times \mathbb{F} + \mathbb{M}_{O'}$$

EQUIVALENT

$$= \emptyset \qquad \mathbb{M}_O = -\vec{AO} \times \mathbb{F} \qquad = \emptyset$$

STATICALLY EQUIVALENT SYSTEM OF FORCES

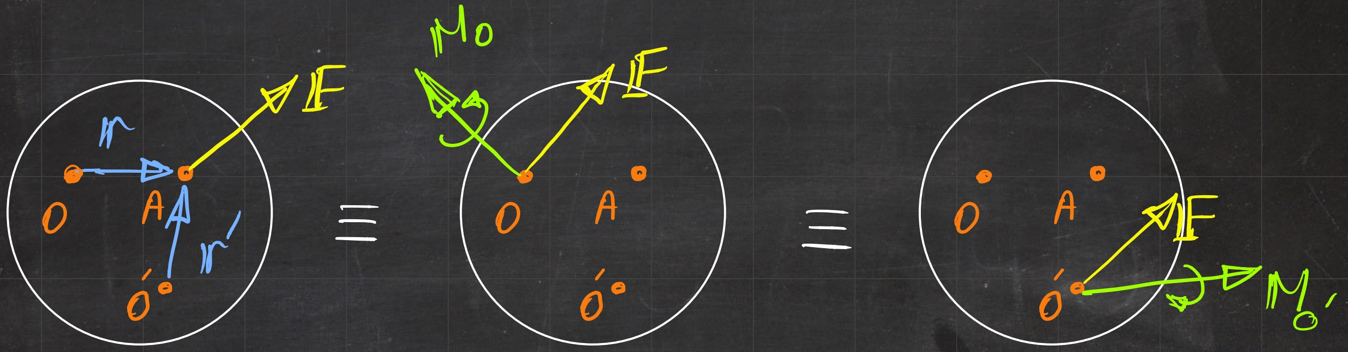


$$\sum M_A = \emptyset \qquad \sum M_A = \vec{AO} \times F + M_O \qquad \sum M_A = \vec{AO'} \times F + M_{O'}$$

EQUIVALENT

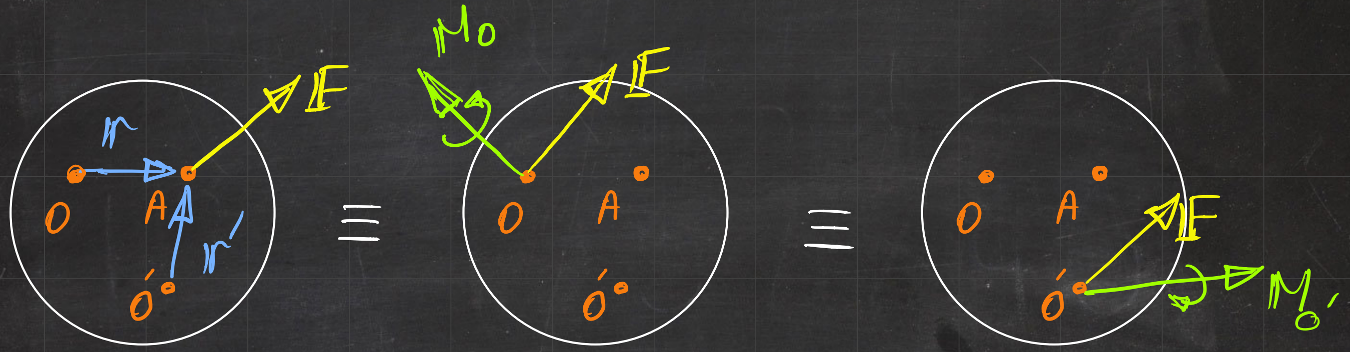
$$= \emptyset \qquad M_O = -\vec{AO} \times F \qquad = \emptyset \qquad M_{O'} = -\vec{AO'} \times F$$

STATICALLY EQUIVALENT SYSTEM OF FORCES



$$\Sigma \mathbb{M}_{O'} = \vec{O'A} \times \mathbb{F}$$

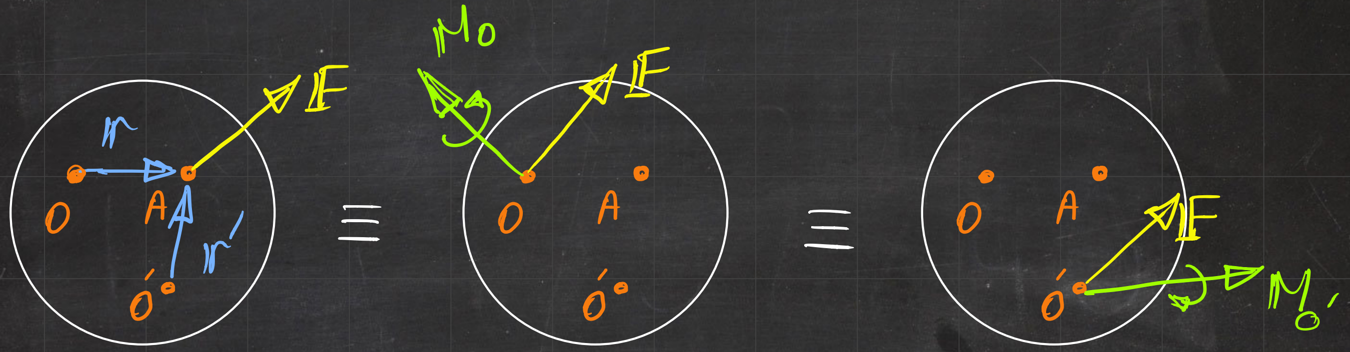
STATICALLY EQUIVALENT SYSTEM OF FORCES



$$\sum \mathbb{M}_{O'} = \overrightarrow{O'A} \times \mathbb{F}$$

$$\sum \mathbb{M}_{O'} = \overrightarrow{O'O} \times \mathbb{F} + \mathbb{M}_O$$

STATICALLY EQUIVALENT SYSTEM OF FORCES

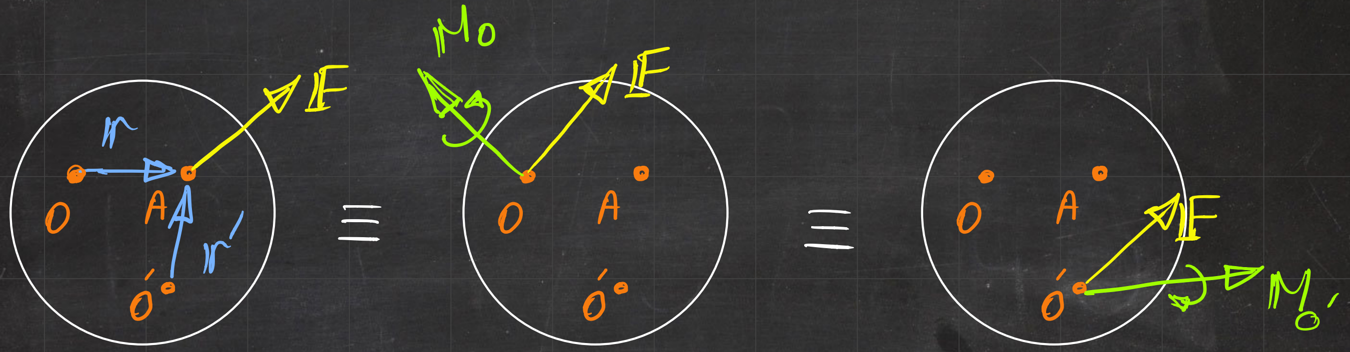


$$\sum \mathbb{M}_{O'} = \vec{O'A} \times \mathbb{F} \qquad \sum \mathbb{M}_{O'} = \vec{O'O} \times \mathbb{F} + \mathbb{M}_O$$

EQUIVALENT

$$= \vec{O'A} \times \mathbb{F}$$

STATICALLY EQUIVALENT SYSTEM OF FORCES



$$\sum \mathbb{M}_{O'} = \vec{O'A} \times \mathbb{F}$$

$$\underbrace{\sum \mathbb{M}_{O'}}_{= \vec{O'A} \times \mathbb{F}} = \vec{O'O} \times \mathbb{F} + \mathbb{M}_O$$

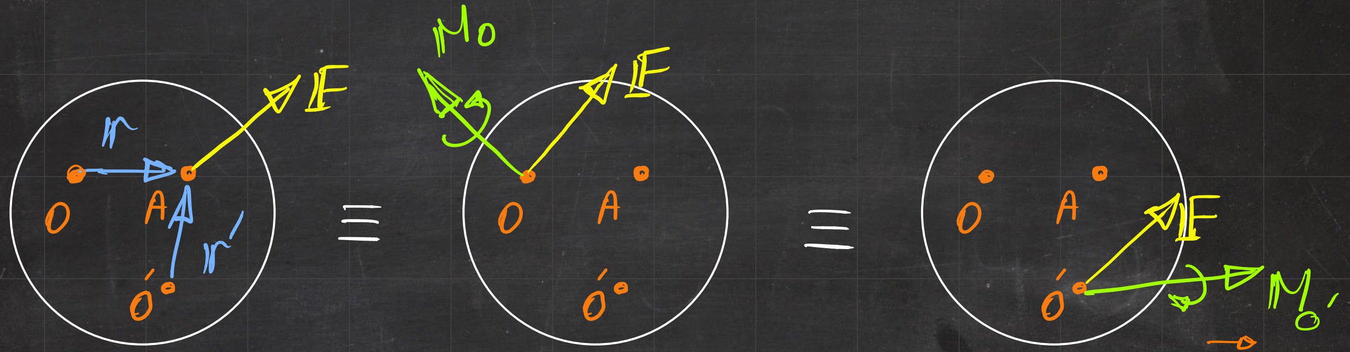
EQUIVALENT

$$= \vec{O'A} \times \mathbb{F}$$

$$\Downarrow$$

$$\vec{O'A} \times \mathbb{F} = \vec{O'O} \times \mathbb{F} + \mathbb{M}_O$$

STATICALLY EQUIVALENT SYSTEM OF FORCES



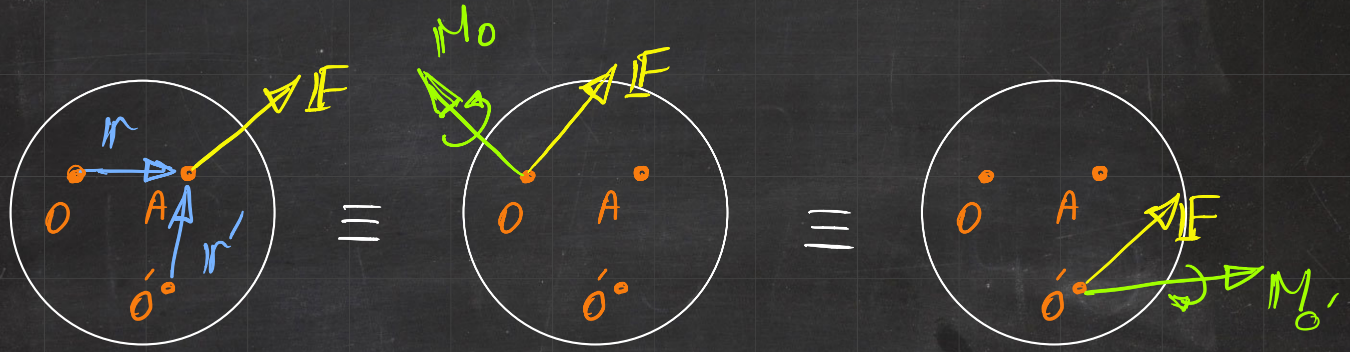
$$\sum M_{O'} = \vec{O'A} \times F \qquad \sum M_{O'} = \vec{O'O} \times F + M_O \qquad M_O = \underbrace{[\vec{O'A} - \vec{O'O}]}_{\vec{OA}} \times F$$

$$\Downarrow \qquad \Uparrow \qquad \Rightarrow M_O = \vec{OA} \times F$$

EQUIVALENT $= \vec{O'A} \times F$

$$\vec{O'A} \times F = \vec{O'O} \times F + M_O \qquad = -\vec{AO} \times F$$

STATICALLY EQUIVALENT SYSTEM OF FORCES

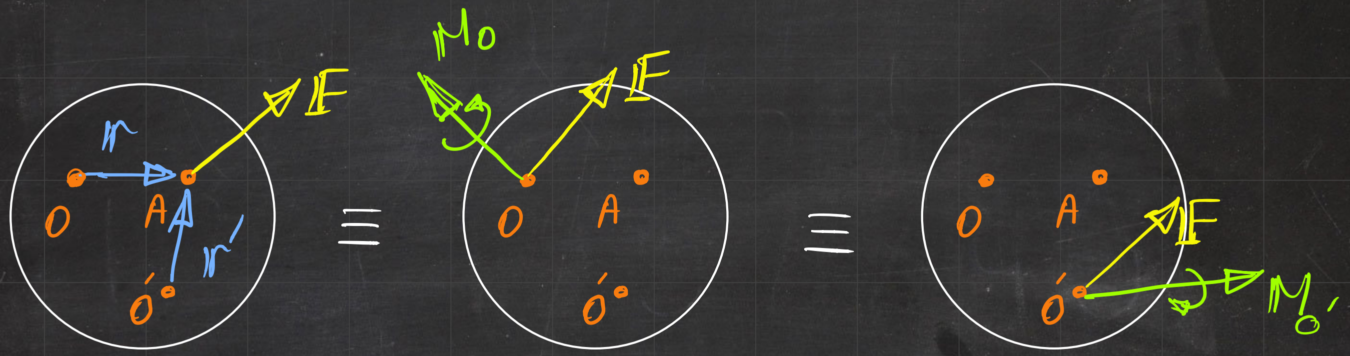


$$\sum M_{O'} = \vec{O'A} \times F \qquad \sum M_{O'} = \vec{O'O} \times F + M_O$$

$$\Downarrow$$

EQUIVALENT $\quad = \vec{O'A} \times F \qquad M_O = -\vec{AO} \times F$

STATICALLY EQUIVALENT SYSTEM OF FORCES

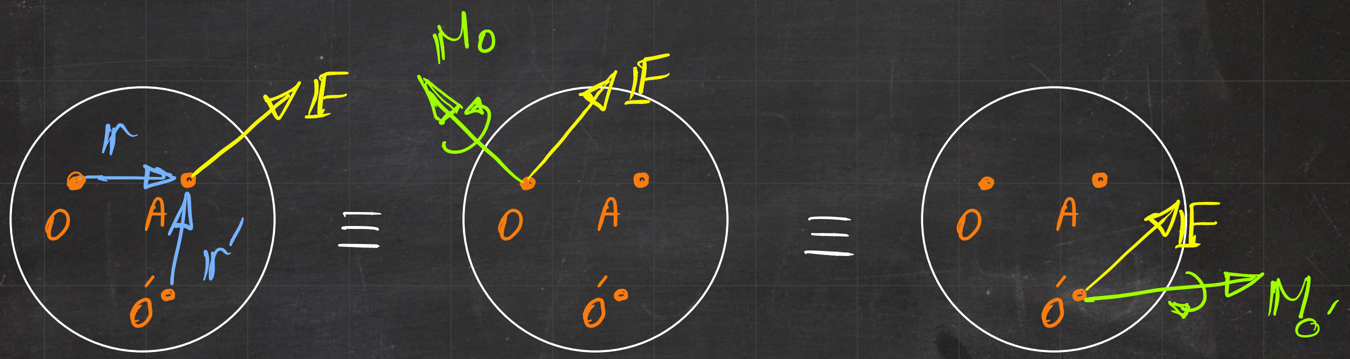


$$\sum \mathbf{M}_{O'} = \overrightarrow{O'A} \times \mathbf{F} \qquad \sum \mathbf{M}_{O'} = \overrightarrow{O'O} \times \mathbf{F} + \mathbf{M}_O \qquad \sum \mathbf{M}_{O'} = \mathbf{M}_{O'}$$

$$\Downarrow$$

EQUIVALENT $\quad = \overrightarrow{O'A} \times \mathbf{F} \quad$ $\mathbf{M}_O = -\overrightarrow{AO} \times \mathbf{F}$

STATICALLY EQUIVALENT SYSTEM OF FORCES

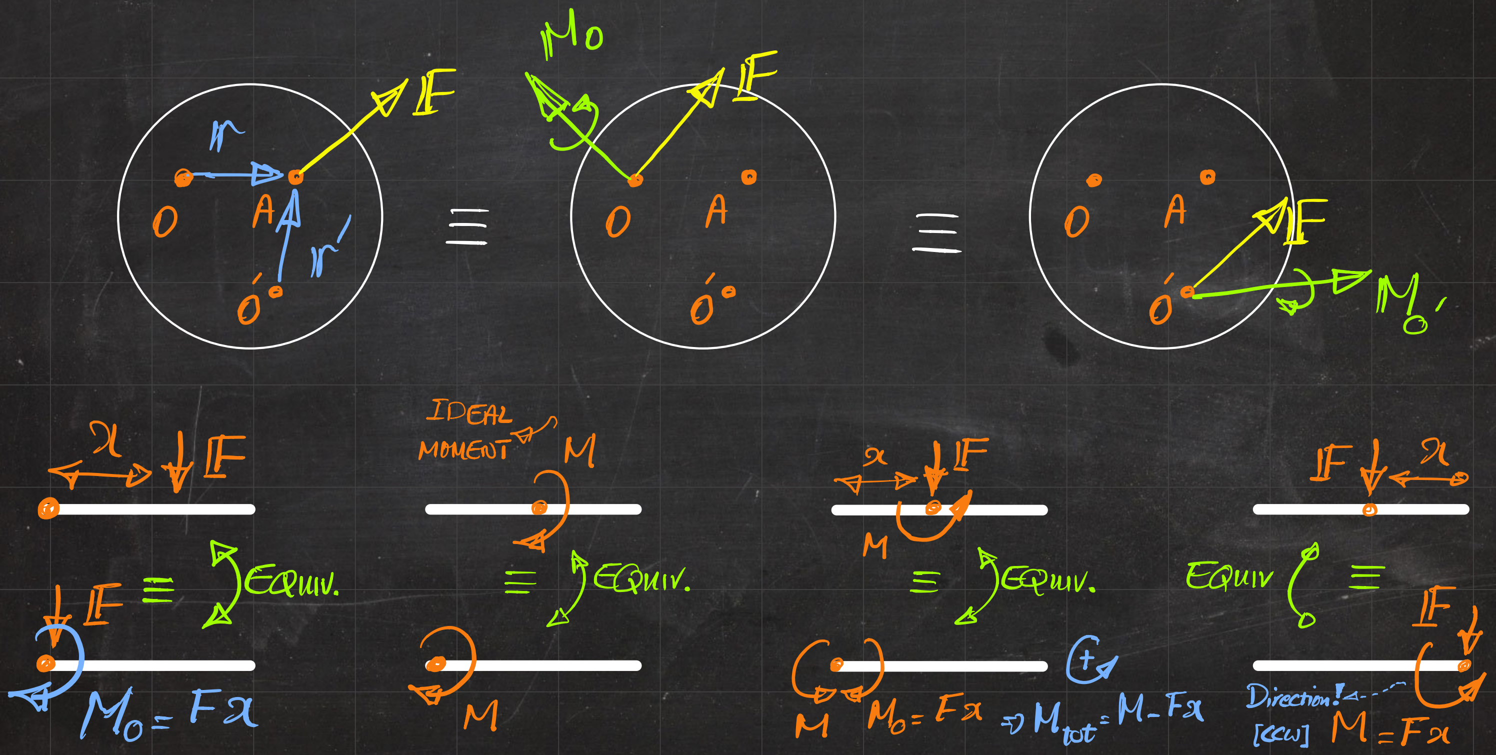


$$\Sigma \mathbb{M}_{O'} = \vec{O'A} \times \mathbb{F} \qquad \Sigma \mathbb{M}_{O'} = \vec{O'O} \times \mathbb{F} + \mathbb{M}_O \qquad \Sigma \mathbb{M}_{O'} = \mathbb{M}_{O'}$$

EQUIVALENT

$$= \vec{O'A} \times \mathbb{F} \qquad \mathbb{M}_O = -\vec{AO} \times \mathbb{F} \qquad = \vec{O'A} \times \mathbb{F} = -\vec{AO'} \times \mathbb{F}$$

STATICALLY EQUIVALENT SYSTEM OF FORCES



MECHANICS AND MATERIALS I

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Appendix II